Experimental Analysis of Tooth Breakage Effect on the Vibration Characteristics of Spur Gears

M. A. Akram¹, M. N. Nawaz², A. Mehmood¹, H. M. Ali¹, A. Anum¹, S. Akmal⁶, L. A. Nizam⁵, S. Khushnood⁷

¹, ³, ⁴, ⁵, ⁷, ⁸ Mechanical Engineering Department, University of Engineering and Technology, Taxila, Pakistan.
², ⁶, ⁷ Mechanical Engineering Department, HITEC University, Taxila, Pakistan.

Abstract—Various industrial applications have wider use of gears. Vibration analysis has been generally utilized as a part of condition monitoring of rotary machinery and engineering structures in order to prevent failure, increase reliability and decrease maintenance cost of the system. The aim of this study is to analyze the vibration produced due to the single as well as double tooth breakage of spur gear at five different RPMs. First, the test model was developed and then specific fault was introduced in driven gear at 25, 50, 75 and 100% of single and double tooth breakage and data was acquired for individual fault using wireless tri-axial accelerometer. The recorded data was then analyzed using different statistical techniques and results have been plotted. The analysis shows that maximum vibration amplitude is produced at 75% of tooth breakage and tends to reduce at 100% of tooth breakage due to increase in the Transmission Error (TE). The Power Spectral Density (PSD) analysis suggests that amplitude of Gear Mesh Frequency (GMF) first rises up to 75% of tooth breakage and then suddenly falls down due to complete tooth removal but sidebands of Gear Mesh Frequency (GMF) tends to rise as tooth breakage increases. Also the other statistical parameters Root Mean Square (RMS), Crest Factor (CF), Kurtosis and Skewness are analyzed and are very important for complete understanding of the phenomenon.

Keywords—Spur Gears, Tooth Breakage, Vibration Amplitude, Statistical Techniques, Probability Density Moments, Spectral Analysis (PSD).

I. INTRODUCTION

Various industrial applications have wider use of rotary machines like industrial gearboxes, aircraft engines, power station, etc. Gearboxes are widely used to transmit power from prime mover to load. Most of rotor elements in rotor dynamics systems are highly susceptible to transverse vibrations due to fatigue which can result in injuries and severe damage to machinery even catastrophic failure. Vibration analysis has been generally utilized as a part of the condition monitoring of rotary machinery and engineering structures. It is especially suited to the early detection of faults in different rotating components created due to transverse vibrations. Vibration signals generated due to certain machine component defects like rotor unbalance, looseness, shaft bow and cracks, rotor hub, coupling defects, misalignment, belt and pulley defects are stationary.

The common types of gear damage mainly consist of cracking, wear, pitting, scuffing and spelling [i]. Gearbox generate more complex transient vibration signals generated by tooth meshing, gear and pinion shaft rotation, however measured vibration signals in multistage gearboxes are complex and transient due to impulses, random noises and high frequency faults[ii]. Every fault has specific signature in measured signal and to identify possible fault in machinery, vibration analysis is the most convenient and cheapest method. Advanced signal processing techniques are being applied on vibration signals to detect, locate and identify faults and based on them life of machine can also be predicted include wavelet transform, neural network, fuzzy logic, genetic programming, genetic algorithms, and machine support vectors [iii–iv].

Vibration signals collected from gearbox are processed using waveform data processing techniques like Time domain analysis (Time waveform, Time Synchronous Averaging), Frequency domain analysis (Fast Fourier Transform) and Joint Time Frequency domain analysis (Short Time Fourier Transform, Wavelet Analysis) [v]. The development of Fast Fourier Transform algorithm and advanced digital processor made real time spectrum analyzed [vi]. For time domain methods, statistical parameters like root mean square (RMS) value, kurtosis, skewness, crest factor value of the measured time domain, raw vibration signal can be used for fault detection [vii].

The frequency domain methods were generally used for rotational machinery fault detection and diagnostics but back draw was that spectral analysis assumes the signal to be stationary but in real sense localized machine faults introduced non-stationary property into the vibration signal. The time domain method endeavors to analyze the amplitude and phase information of the vibration signal in order to observe
the fault and in analyzing the impulsive signals from gear defects and bearing. Waveform analysis comprises of recording the time history of the data. It is helpful for analysis of non-steady conditions and short transient impulses.

Statistical analysis can be performed on the basis of time domain data. Probability density is probability to find instantaneous values within a certain amplitude interval, divided by the size of the total interval. Indices like Peak value, root-mean square value (RMS) and their ratio crest factor (CF) are usually used to quantify the time signal[viii]. Peak value is half the difference between maximum and minimum vibration levels and is represented by relation in Eq. (1)

\[ \text{Peak value} = A_{\text{max}} \]

(1)

The peak level is not solid in detecting damage continuously operating systems because it is not statistical quantity.

The RMS esteem is more reliable for steady-state applications. Normalized second statistical moment of the signal (standard deviation) peak is RMS (Root Mean Square) and is represented by relation in Eq. (2)

\[ \text{RMS} = \sqrt{\frac{\sum_{n=1}^{N} (A(n))^2}{N}} \]

(2)

Where, \( A(n) \) is the amplitude level of the \( n \)th digitized point and \( N \) is the point range in time domain. The RMS of the signal is usually used to portray the steady-state amplitude of a time varying signal. The most straightforward way to deal with defects in the time domain is using the RMS approach. However, in case of early stages of bearing and gear damage, the RMS level may not show prominent changes in the value.

The crest factor is the proportion of the peak value of input signal to the RMS level and is represented by relation in Eq. (3). Therefore, the crest factor value will increase with increase in peaks in the time series signal as in case of impulsive vibration sources such as a defect on the outer race of a bearing and its other faults, or tooth breakage on a gear. This approach is not considered a very sensitive technique. It is very important to note that with one gear tooth damage, for one revolution of damaged gear, its RMS value will not increase but peak value will increase. So Crest factor will also increase but as damage becomes severe, RMS value will also increase with increase in peak value, in that case crest factor value will decrease. This uniqueness of this feature enables to detect very tiny surfaces damages [ix-x].

\[ \text{Crest Factor} = \frac{\text{Peak Level}}{\text{RMS level}} \]

(3)

In the presence of significant impulsiveness, crest factors are more reliable and are insensitive to operating speed and gear load provided generated vibration of gear is above background noise level.

Typical values of crest factors for good conditioned gear range from 3.5 to 4.0. Generally crest factors above 4.5 are indicative of damage and values for gear with impulsive defects are higher, ranging up to ~10.0.

Kurtosis is the normalized fourth statistical moment of the signal [xi] and is represented by relation in Eq. (4). It checks the relative peakness or flatness of a distribution as compared to a normal distribution. It is utilized to provide a measure of the impulsive nature of the signal. Raising the signal to the fourth power effectively amplifies isolated peaks in the signal. This feature should indicate an error due to the increased level of vibration because of gear wears and breaks. A normal distribution has a kurtosis value of 3 and it shows the good condition.

\[ K = \frac{\sum_{n=1}^{N} (y(n)-\mu)^4}{N\sigma^4} \]

(4)

Here \( y(n) = \) data \( (n = 1, 2, 3, ..., N); \) \( N = \) total number of data samples collected; \( \mu = \) mean; and \( \sigma = \) standard deviation [xii-xiii].

Skewness is the most precise way to measure symmetry or the lack of symmetry of collected data and to find data distribution. Distribution of data is said to be symmetric if data set looks the same to the left and right of the center point, and is represented by relation in Eq. (5)[xiv]

\[ S = \frac{\sum_{n=1}^{N} (y(n)-\mu)^3}{N\sigma^3} \]

(5)

Here \( y(n) = \) data \( (n = 1, 2, 3, ..., N); \) \( N = \) total number of data samples; \( \mu = \) mean; and \( \sigma = \) standard deviation; \( N \) is the range of point in time domain.

FFT algorithmic rule was developed by Cooley and Tukey (1965). It should be noted that FFT is not a totally different transform from the DFT, however rather simply a method of computing the DFT with a substantial reduction within the range of calculations needed. The quick Fourier Transform (FFT) is a distinct Fourier Transform algorithmic rule that reduces the range of computations required for N points from 2N² to 2Nlog₂(N), wherever log is that the base-2 index. Fourier Transform of auto correlation functions is spectral density. Normally the time history of signal is not periodic so it is not possible to obtain Fourier transform from this time data. Each analysis has its own importance depending upon vibration signal characteristics. FFTs are used to analyze vibration signals with finite number of dominant frequency components but for random vibration signals, power spectral densities (PSD) are used. In a FFT, PSD is processed by multiplying each frequency bin by its complex conjugate which results in the real spectrum of amplitude in g²/Hz [xv-xvi]. PSD core purpose to form it adept than a FFT for random vibration analysis is that this amplitude value is then normalized to the frequency bin width to urge units of g²/Hz. By normalizing the result we get dispose of the
dependency on bin width so that we can compare vibration levels in signals of different lengths [vii].

In real applications, mostly background noises disturb vibration signal containing symptoms. To overcome this problem, we need to increase signal to noise ratio. Time synchronous average (TSA) technique was used. TSA separates the vibration signal of the component and suppressing the background noise. TSA is the most powerful tool in vibration signal pre-processing and is the most popular technique for gear fault detection. The back draws of TSA include requirement of additional synchronous signal; the tachometer signal and it also needs several cycles of vibration signals in order to achieve the required signal to noise ratio [iii, xvii].

Another way to increase the signal to noise ratio is to use modal based methods to remove periodic signal or the background noise from the raw vibration signal. These methods include auto regressive modal based methods, adaptive noise cancellation and narrow band interference cancellation. The back draw of this method is that these techniques need to use structure of the model [xviii].

The most recent new technique joint time frequency analysis methods are developed for rotational machinery fault detection and diagnostics. Its fundamental thought is to figure a way to represent the energy density of the signal in time and frequency domain simultaneously. Simultaneous time and frequency information of non-stationary signal can provide vital diagnostic information. Short Time Fourier Transform (STFT), Winger Ville Distribution (WVD), Continuous Wavelet Analysis (CWT) and Hilbert Huang Transform (HT) are the most important time frequency analysis techniques for transient signal. In transient signal analysis technique like STFT, windowing function is used to first broke signal into different stationary segments by and then FFT is applied on each segment of signal. This technique faces poor time frequency resolution problems. To overcome weak frequency resolution problems, WVD has developed to enhance the time frequency resolution however the technique produces cross terms in time frequency domain. In CWT, variable window is used to further improve time frequency resolution [vii, xix-xxi].

Adaptive wavelet filter was introduced by Lin J. and Zuo M. J in 2003. In light of Morlet Wavelet, the parameters in the Morlet wavelet function are optimized based on the kurtosis maximization principle [xxii]. Adaptive wavelet was utilized because the parameters are not fixed. It is found to be very effective in early fatigue tooth crack detection from symptoms of vibration signals of a gearbox [xxiii].

The objective of this paper is to analyze the vibrations produced by various faults of gear. First, the test model was designed and developed and then different faults have been created in the test model to study its vibration characteristics. Amplitude analysis, Time Waveform analysis, Spectrum analysis and Statistical parameters have been used for the detection and the analysis of these faults.

II. EXPERIMENTAL TEST RIG

Tests were performed on a single stage spur gear, which is running at variable speed of five different RPM (1330, 1386, 1424, 1445 and 1462 rpm) and regulator was used to vary the speed of the shaft carrying the gear.

Gears were manufactured on horizontal milling machine with a module of 2.

The defect was produced by removing the one tooth of the spur gear with tooth breakage percentage of 25, 50, 75 and 100 % respectively. Same is the case for second tooth breakage.

<table>
<thead>
<tr>
<th>TABLE I</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DESIGN SPECIFICATION</strong></td>
</tr>
<tr>
<td><strong>Power</strong></td>
</tr>
<tr>
<td><strong>Torque</strong></td>
</tr>
<tr>
<td><strong>Diameter of shaft</strong></td>
</tr>
<tr>
<td><strong>Gear Ratio</strong></td>
</tr>
<tr>
<td><strong>Pressure Angle</strong></td>
</tr>
<tr>
<td><strong>Diameter of Pinion</strong></td>
</tr>
<tr>
<td><strong>Diameter of Gear</strong></td>
</tr>
</tbody>
</table>

Fig. 1 shows experimental test rig. Experimental setup with single phase, 0.5 HP induction motor used to drive the system.

For vibration measurements G-Link tri-axial accelerometer developed by MICROSTRAIN Corporation was used to collect the data. Accelerometer is placed at different bearing location in
axial & radial direction of gear in order to analyze fault with different tooth breakage and its effect with different position and RPM. The readings obtained from accelerometer were processed using SIGVIEW software. Signals are recorded for vertical, horizontal and axial direction and then are analyzed to identify fault.

Bearing 4 (B4). It was observed that with increase in severity of fault amplitude of vibration was increased with increase in RPM. Amplitude of vibration increased up to 75% for each tooth breakage with increase in RPM and after a complete tooth removal amplitude of vibration was prominently decreased at B2. We can observe that overall vibration amplitude at B4 is less than B2 and there is increase in vibration amplitude with RPM.

(a)

Development of vibration-based condition monitoring system for gearbox remained one of the most important research topics for quantitative analysis and effect of gear tooth damage on gearbox vibration. Resultant was calculated for horizontal, vertical and axial direction for amplitude analysis at B2 and B4 for five different RPM to study extent of fault, how it varies from one position of bearing to another. It is very important to note that at 1464 RPM, for 1st 100% tooth breakage, Fig 3 (a) and 3(b) show that amplitude is highest for B2 and it decreases from 41 to 24 m/s² for bearing 4.

(b)
defects than more vibrating gearbox. It is only helpful when fault is less severe and when fault becomes prominent, vibration amplitude decreases and for that case one have to use more powerful techniques as discussed earlier.

Load is for the most part transmitted by one or two pairs of teeth in spur gears, and at the contact around the pitch point the Transmission Error (TE) increases and it decreases at the contact on the top and on the root of the gear tooth respectively. The effective reason for gradual increase in the TE is that single tooth contact takes places around pitch point and that pitch point is between the highest and lowest point of single tooth contact because of single tooth gear tooth deformation. On single tooth, area of contact is larger than that on double tooth contact area. So because of increase of single tooth contact area, elastic deformation is more than double tooth contact elastic deformation. In result, TE of single tooth contact is more than double tooth contact. But it is very important to note that when tooth is completely removed, peak to peak value of vibration will be more than that of undamaged gearbox case but it will be less than that of partially broken tooth that is not completely removed [xxiv-xxv]. Vibration of amplitude decreases as we move away from origin of fault. Means vibration amplitude is found more near the location of fault and its transmission error decrease from first to second distant bearing.

It is concluded that research work was to consider whether the loss of a tooth would result in more or less vibration with different percentage tooth breakage. If a tooth is not there, there is no vibration from it but it would require that other teeth are taking the load. This is what it is expected, depending upon how the teeth mesh when there is a tooth damaged versus partially removed versus completely removed tooth. If the gear can still turn and stay in mesh with a tooth meshing then the vibration will change but perhaps not as originally thought. If the meshing was disrupted then the vibration will change a great deal more. The time waveform and frequency spectrum are the perfect tool for gearbox fault detection [xxvi].

B. Time Waveform Analysis

Time waveform is the most powerful to detect gearbox faults. Below are time waveform graphs for 25, 75 and 100 % for 1462 RPM for first and second symmetric broken teeth and we can see that with increase in severity of fault generated impacts increase in amplitude. Upon each revolution there will be one impact if one tooth is defected and two impacts per revolution in case if two teeth are defected.
Here results are discussed in case of highest 1462 RPM for time waveform. For 1 broken tooth of 25% impulse amplitude will be small and it will occur once per revolution. With increase in severity of fault impulse amplitude will increase up to 100% of tooth removal; amplitude will be maximum. If there is 2nd tooth breakage we will see same behavior but there will be two impulses per revolution. This behavior will be repeated throughout signal and this indicates that two teeth are defected symmetric to each other. Similarly if three tooth breakage, we will see 3 impulses per revolution. So time waveform is the most efficient tool to analyze broken gear tooth defects [xxvii-xxviii].

C. Spectrum Analysis

Power Spectrum Density technique has shown a much better illustration for fault identification. Gearbox vibration signals are usually periodic and noisy. Time-frequency domain averaging technique with success removes the noise from the signal. PSD is very effective tool to analyze broken gear tooth defects. Experiment was performed for 5 variable speeds and up to two broken teeth but in this case, we will compare 1386 and 1462 RPM results. For corresponding RPM, we have Gear Pinion Frequency (GPF) \(X_p=23.1\text{Hz}\), 24.36 Hz, Gear Rotational Frequency (GRF)
(Xg=11.55 Hz, 12.18 Hz), and Gear Mesh frequency (GMF=277.2 Hz, 292.8 Hz). Amplitude of GMF, GNF, GPF increase up to 75% of tooth breakage and then decreases upon complete tooth removal but sidebands level will increase throughout with fault severity.

Fig. 7. Power Spectral Density (PSD) plots for first tooth breakage percentage of (a) 25% (b) 50% (c) 75% (d) 100%
Fig. 8(b). Comparison of PSD plots for first and second tooth with different tooth breakage
(a) GMF (b) GPF (c) GRF
Fig 8(b) (a, b, c) show comparison of Gear Mesh Frequency (GMF), Gear Pinion Frequency (X_p) and Gear Rotational Frequency (X_g) in PSD analysis. After 1st tooth breakage GRF amplitude suddenly falls showing GPF dominates GRF and fault is in the gear. Spectrum Density techniques have shown a decent illustration for fault identification of Gears.

D. Statistical Parameters
The experimental result indicates that condition indicators like root mean square (RMS), crest factor(CF), peak value, skewness and kurtosis describe the overall vibration level and track the condition of the tested gearbox (healthy or defected) condition very well. Gearbox monitoring Systems are therefore based on condition indicators that describe the overall vibration value.
Fig. 10. Amplitude behaviour with different percentage tooth breakage for 1st and 2nd symmetric teeth breakage with respect to (a) Skewness (b) Kurtosis

Fig. 10 (a, b) show statistical plot for Skewness and Kurtosis. Skewness has ideal value of 0. From fig. 10 (a), upon increase in fault severity, its value increases up to 75% of broken tooth and then decreases upon complete tooth removal. Gearbox has 0 normal values for kurtosis. From fig. 10(b) upon increase in severity of fault, kurtosis will deviate from its normal value. So we can conclude that statistical analysis plays most important role for gear faults diagnosis.

IV. CONCLUSION

In this paper, the experimental study has been carried out to investigate the vibration characteristics of tooth damage in spur gears and its relationship with the RPM of the gear. For 1st tooth breakage, the amplitude of vibration increases with increase in RPM of the gear. Amplitude of vibration increased up to 75% for each tooth breakage with increase in RPM and after complete tooth removal, the amplitude of vibration was prominently decreased because of increase in transmission error. Same behavior was observed for second tooth breakage but vibration amplitudes are higher for 2nd tooth. Overall vibration amplitude at B4 is less than B2. Time waveform analysis suggests that for 1st broken tooth of 25%, the impulse amplitude is small and it will occur once per revolution. With increase in severity of fault (percentage of broken tooth increases), the amplitude rises to maximum. If there is 2nd tooth breakage we will see same behavior but there will be two impulses per revolution. This behavior will be repeated throughout signal and this indicates that two teeth are defected symmetric to each other. Similarly if three tooth breakage, we will see 3 impulses per revolution.

It has been concluded that the amplitude of gear mesh frequency (GMF) increases up to increase in gear tooth breakage and then decreases upon complete tooth
removal but sidebands level will increase throughout with increase in the tooth-breakage percentage. Sidebands will rise in amplitude around gear mesh frequency with frequency of defected gear or pinion. In our case, sidebands rise around GMF with gear rotational frequency indicating fault is within gear. After second tooth removal; gear natural frequency (GNF) peak was raised indicating that the system is moving towards instability. It is concluded from statistical analysis that with increase in fault severity, RMS value will always increase. However, in case of early stages of gear, the RMS level might not show considerable changes within the value. Crest factor value first increases for first tooth damage and upon fault severity, because of increase in peak and RMS value, crest factor decreases. Healthy Gearbox has Kurtosis of 0 normal value, on increase in severity of fault, kurtosis will deviate from its normal value. Skewness has ideal value of 0 and upon increase in fault severity, its value increases up to 75% of broken tooth and then decreases upon complete tooth removal.

ACKNOWLEDGEMENTS

This work is supported technically by Mechanical Engineering Department, HITEC University Taxila and UET Taxila to carry out research. Special thanks to Mr. Ijaz Ahmed and Mr. Jawad Ahmed for providing technical and administrative approach for coduction of experiment.

REFERENCES

[xvii] M. Yang and V. Makis, ARX model-based gearbox fault detection and localization under


