Numerical Modeling of Hybrid Fiber-Reinforced Concrete (HyFRC)

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Abstract-A model for numerical simulation of mechanical response of concrete reinforced with slipping and non slipping metallic fibers in hybrid form is presented in this paper. Constitutive law used to model plain concrete behaviour is based on plasticity and damage theories, and is capable to determine localized crack opening in three dimensional (3-D) systems. Behaviour law used for slipping metallic fibers is formulated based on effective stress carried by these fibers after when concrete matrix is cracked. A continuous approach is proposed to model the effect of addition of non-slipping metallic fibers in plain concrete. This approach considers the constitutive law of concrete matrix with increased fracture energy in tension obtained experimentally in direct tension tests on Fiber Reinforced Concrete (FRC). To simulate the mechanical behaviour of hybrid fiber-reinforced concrete (HyFRC), proposed approaches to model nonslipping metallic fibers and constitutive law of plain concrete and slipping fibers are used simultaneously without any additive equation. All the parameters used by the proposed model have physical meanings and are determined through experiments or drawn from literature. The model was implemented in Finite Element (FE) Code "CASTEM" and tested on FRC prismatic notched specimens in flexure. Model prediction showed good agreement with experimental results.

Keywords-Concrete, Metallic Fibers, Hybrid, Modeling, Fracture Energy, Crack Opening

I. INTRODUCTION

Plain concrete when reinforced with randomly distributed short fibers to reduce its brittleness and increase the resistance to cracking is known as Fiber-Reinforced Concrete (FRC). Currently in practice, most of the FRCs contain only one type of fiber. It is established that failure process in concrete is of gradual and multi-scale nature. Under the application of load, the pre-existing micro-cracks in concrete grow and eventually join together to form macro- cracks. A macro-crack propagates at a stable rate until it attains conditions of unstable propagation which results in a rapid fracture. The gradual and multi-scale nature of fracture in concrete indicates that a given fiber can provide reinforcement only at one level and within a limited range of strains [i]. For optimal result therefore different types of fibers may be combined and the resulting composite is known as Hybrid Fiber-Reinforced Concrete (HyFRC).

Initiation, growth and coalescence of micro-cracks within the concrete matrix control the failure mode. The localized micro-stress fields are induced within the concrete matrix due to the geometrical anisotropy of micro-cracks which causes anisotropy and unilateral effect of damage [ii]. Fracture mechanics based modeling of concrete structure is still considered to be a challenging subject in several structural engineering laboratories of the world. The problem is approached in different ways and one of them is the fracture energy derivative methods which are closer to the fracture mechanics theory, and thanks to mesh adaptation which provides a geometrical description of the cracks. However, their implementation in commercial finite element code needs significant adaptations. As a consequence, alternatives methods have been proposed during the last decades; among them are the damage mechanics based models [iii-vii] or the softening plasticity ones [viii-ix]. Coupling between plasticity and damage is also widely used in research programs dealing with cyclic loading like seismic analyses [x-xi].

Addition of fibers in plain concrete further complicates the fracture modeling. Most of the existing numerical models of FRC deal with concrete reinforced with single type of fiber and these models can be employed successfully to predict the mechanical performance of FRC. Numerical model presented in this paper deals with plain concrete reinforced with two types of metallic fibers (slipping and non-slipping) in hybrid form. When failure of the FRC is due to pulling out of fibers, the fiber is considered as slipping fibers and when the failure of FRC is due to the fiber rupture, the fibers are characterized as non-slipping fibers [xii].

II. MODEL PRESENTATION

The response of FRC containing slipping and nonslipping metallic fibers before and after cracking in direct tension is shown in Fig.1. Behaviour of FRC before cracking mainly depends upon the properties of plain concrete and the post-cracking stage is governed by the characteristics of fibers.

The effect of addition of slipping and non-slipping fibers in mono and hybrid forms on the behaviour of concrete in direct tension was studied experimentally. The curves shown in Fig. 1 were obtained from the results of this experimental study. The properties of slipping and non-slipping fibers have been reported in [xii]. The constitutive laws of plain concrete and fibers are generally combined for numerical simulation of FRC. In case of Hybrid Fiber-reinforced concrete (HyFRC) where two or more than two types of fibers are mixed, the effect of each type of fibers is considered separately to model its behaviour. Non-slipping fibers effectively control the micro-cracking mechanism in the concrete matrix and as a result, damage is diffused over large area. In this way, localization of macro-crack is delayed and fracture energy is significantly increased compared to plain concrete (Fig. 2). In this figure G_f^{T} represents fracture energy in tension.

In plain concrete reinforced with slipping fibers and subjected to direct tension, after micro-crack localization, an important value of post-peak stress is exhibited after drop in the stress (Fig. 1). The value of post-peak stress in FRC containing slipping fibers is greater than the value observed in control composite. Moreover, stress carrying ability after cracking is possible up to significant crack opening until the fibers are slipped from the concrete. The slipping fibers act when concrete matrix is cracked and their stress carrying ability changes with the increase of crack opening.



Fig. 1. Response of FRC in direct tension



Fig. 2. Increased fracture energy in tension of concrete in the presence of non-slipping metallic fibers

2.1 Constitutive Law of Plain Concrete and Slipping Fibers

Detailed description of constitutive law of plain concrete and slipping fibers has already been published by the authors [xiii]. The model of the plain concrete supplies the crack opening and damage intensity anisotropically. Fracture energy is used explicitly by the model to regularize the post-peak behaviour and to manage the possible orthotropic crack re-closure. Constitutive law of slipping fibers considers the effective stress transferred through the crack by the slipping fibers and is computed using Equation 1.

$$\widetilde{\sigma}_{f_{\mathrm{I}}} = \mathbf{K}_{\mathrm{f}} \cdot \boldsymbol{w}_{\mathrm{I}} \tag{1}$$

Where K_f is the fiber rigidity; w_I is CMOD. Equation 1 indicates that fibers act only after the localization of crack as w_I is localized crack opening. A damage variable d_f is introduced to take into account the progressive fiber matrix de-bonding. The value of d_f varies from 0 to 1, $d_f = 1$ indicates that fiber is fully de-bonded from the concrete.

Damage theory is adopted here to model behaviour of slipping fibers rather than a plasticity formulation, because the slipping fibers buckle if they are loaded in compression and consequently, they become unable to transmit stress as illustrated in Fig. 3. If a plastic formulation is adopted to simulate their de-bonding, a compressive stress would appear in fibres during the crack re-closure process, while with a damage formulation the stress in the fiber remains positive so long as the crack opening w_1 exists, and this stress vanishes when the crack is reclosed. This approximation is more specifically representative for the fibers bridging the widely opened cracks; in this case fibers will easily buckle. At the beginning of the slipping process a plasticity formulation could also be adopted, but during this part of the behaviour the

fiber's contribution towards the global behaviour remains small compared to the residual stress in concrete and a combination of plasticity and damage would make the model complicated without significant benefits in terms of global behaviour of composite. Therefore, only the damage formulation is used in the present model.

The stress carrying capacity of the fibers taking into account the damage at matrix-fiber interface is computed using Equation 2.

$$\vec{\sigma}_{f_1} = \vec{\tilde{\sigma}}_{f_1} \cdot \left(\mathbf{l} - \mathbf{d}_{\mathbf{f}} \right) \tag{2}$$

The stress carried by the slipping fibers is added to the apparent stress in concrete as described in Equation 3 [xiii].

$$\vec{\sigma} = (1 - D^c)(1 - D^f)S^0\vec{\varepsilon} + (1 - D^c)D^f(S^0(\vec{\varepsilon} - \vec{\varepsilon}^f) + \vec{\sigma}_f)$$
(3)

NOMENCLATURE

- $\vec{\sigma}$ Apparent stress
- $\vec{\tilde{\sigma}}$ Effective stress (healthy part of the material)
- $\tilde{\tilde{\sigma}}^{f}$ Stress state in tension crack
- S^{0} Stiffness matrix of material without damage
- D^{\prime} Tensile damage tensor
- D^{c} Compressive damage tensor

- $\vec{\varepsilon}^{f}$ In-elastic strain
- $\vec{\varepsilon}$ Total strain
- $\bar{\sigma}_{f}$ Effective stress carried by the fibers
- w_I Crack mouth opening displacement (CMOD)

2.2 Constitutive Law of Non-Slipping Fibers

General trend of tensile stress versus CMOD curve of FRC containing non-slipping fibers shown in Fig.1 is almost similar to that of plain concrete but with greater area under the stress-strain curve which means greater fracture energy value compared to plain concrete. Constitutive behaviour law used for concrete reinforced with non-slipping fibers is of the same form as that for plain concrete but with a greater value of fracture energy in tension obtained experimentally.

2.3 Modeling of Hybrid FRC

To model the behaviour of hybrid fiber-reinforced concrete (HyFRC) containing non-slipping and slipping fibers, Equation 3 is used but the value of fracture energy in tension used for $\bar{\sigma}$ is taken similar to that for the concrete reinforced with non-slipping fibers obtained experimentally. In this approach, no additional parameters/or equations are needed to model the behaviour of HyFRC.



b) Buckling of partially De-bonded fiber during crack reclosure

Fig. 3. De-bonding and buckling phenomena considered in damage model

III. FITTING OF MODEL PARAMETERS

Mechanical tests on concrete matrix reinforced with 20 kg/m³ of slipping and non-slipping fibers in mono and hybrid form were performed to obtain values of different model parameters. The composition of concrete mix is given in Table I. The values of different model parameters obtained through experimentation are listed in Table II.

Model presented in this paper for HyFRC was implemented in finite element code CASTEM [xiv]. Fitting of all the model parameters was done by simulating direct tension test. Only a quarter of the specimen was simulated taking the advantage of symmetries. Computed damage pattern on FE mesh is shown in Fig. 4. In this figure, ability of the model to detect damage is obvious. Comparison of stress versus CMOD curves obtained through simulation and experiments for both mono and HyFRC is presented in Fig. 5.

TABLE I CONCRETE COMPOSITION (KG/M³)

Cement	322			
Sand	872			
Gravel	967			
Water	193			
Admixtures	1.6			
Fibers	20 for mono FRC			
	20+20 for HyFRC			

IV. VALIDATION OF PROPOSED MODEL

The proposed model was tested on prismatic notched specimens of size 100 x 100 x 500 mm and subjected to three point bending. Values of model parameters fitted through simulation of direct tensile tests were used in model testing. For mono and hybrid FRC, comparison of the numerical simulation results obtained in this study and experimental results [xii] for load versus deflection and load versus CMOD responses is shown in Fig. 6 to Fig. 8. In these Figures, SF stands for slipping fibers and NSF stands for nonslipping fibers. Evolution of fracture/damage on the FE mesh for each concrete is also shown in Fig.9 to Fig. 11.

From the comparison of experimental and model results for HyFRC-20+20, it is evident that the approach adopted in this study to model the behaviour of HyFRC containing slipping and non-slipping fibers gives satisfactory results. Crack localization and propagation in the finite element mesh of HyFRC-20+20 is shown in Fig.13. It is important to mention here that a positive synergy is obtained by the model; the bending strength is increased from 5kN to 7kN without additive equation. This increase is in accordance with experimental data.

TABLE II VALUES OF MODEL PARAMETERS

Model Parameters		Composites			
Name	Unit	Control	FRC-NSF20	FRC-SF20	HyFRC-20+20
Modulus of Elasticty, E	GPa	35	34	34	34
Poisson ratio, $\boldsymbol{\nu}$		0.27	0.27	0.27	0.27
Tensile strength, Rt	MPa	3.5	3.5	3.0	3.5
Strain at Rt , $arepsilon^{peak'}$		$1.0 \mathrm{x} \frac{R_t}{E}$	$1.0 \mathrm{x} \frac{R_i}{E}$	$1.0 \mathrm{x} \frac{R_t}{E}$	$1.0 \mathrm{x} \frac{R_{t}}{E}$
Gf^{t}	MN/m	1.37 x10 ⁻⁴	8.22x10 ⁻⁴	1.37 x10 ⁻⁴	8.22 x10 ⁻⁴
Compressive strength, Rc	MPa	50	52	50	48
Strain at Rc , ε^{peak^c}		2.18 x10 ⁻³	2.18 x10 ⁻³	2.18 x10 ⁻³	2.18 x10 ⁻³
Drucker-Prager Parameter, δ		0.15	0.15	0.15	0.15
Gf^{c}	MN/m	1.37x10 ⁻²	1.37x10 ⁻²	1.37x10 ⁻²	1.37x10 ⁻²
K _f	MPa			11000	11000
$(W_{\mathbf{I}})_{peak}$	m			0.5x10 ⁻²	0.5x10 ⁻²
$(Rt_f)_{\mathbf{I}}$	Mpa			1.2	1.2

SF stands for slipping fibers

NSF stands for non-slipping fibers

20 is quantity of fibers in kg/m^3



Fig. 4. Computed damage pattern (direct tension test simulation)



Fig. 5. Direct tension test simulation results



Fig. 7. Response of FRC-NSF20 in 3PBT



Fig. 8. Response of HyFRC-20+20 in 3PBT



a) Mesh used for three point bending test





b) Damage plotted on the mesh



c) Experimental crack pattern

Fig. 9. Experimental crack pattern and computed damage pattern (NSF-20 in 3PBT)



a) Damage plotted on the mesh



b) Experimental crack pattern

Fig. 10. Experimental crack pattern and computed damage pattern (SF-20 in 3PBT)



Fig. 11. Experimental crack pattern and computed damage pattern (HyFRC 20+20 in 3PBT)

V. CONCLUSIONS

A numerical model to predict behaviour of concrete reinforced with slipping and non-slipping fibers in hybrid form has been presented. The constitutive behaviour law of plain concrete matrix is based on plasticity and damage theories. The particularity of the plain concrete model lies in considering the residual stress in opened crack. Based on this particularity, behaviour law of slipping fibers was developed considering the de-bonding and buckling possibility during crack re-closure process. Modeling of non-slipping fibers considers the plain concrete behaviour law without any modification but with fracture energy in tension greater than the plain concrete to consider the additional energy required to break this type of fibers.

Testing of proposed model on notched prismatic specimens $(100x100x500 \text{ mm}^3)$ subjected to bending produced results in good agreement with experimental ones. The model is able to predict the positive synergy between the two metallic fibers without any modification or re-fitting of behaviour laws.

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