Technical Journal, University of Engineering and Technology (UET) Taxila, Pakistan *Vol. 28 No. 2-2023*
ISSN:1813-1786 (Print) 2313-7770 (Online)

Comparing Performance of a New Statistical Model with Contemporary Models of Unbound Basecourse Permanent Deformation

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Abstract- The use of unbound granular materials (UGM) in road pavements is very common in New Zealand (NZ) and sparsely populated countries as they are economical and provide a good load bearing foundation for the wearing course. The load bearing capacity of UGM materials defines the performance of these types of pavements, which are normally used on the lower traffic volumed roads spectrum. The most common performance test for unbound granular materials is the repeated load triaxial (RLT) test that simulates the dynamic loading of traffic on UGM in actual pavements. Given that this test can be a good indicator of the actual performance of these materials under simulated cyclic loading, forecasting models have been developed using RLT data. In this paper, various statistical models based on single input criteria (number of loading cycles, or stress state) as well as combined effects of loading and stress criteria are assessed for the RLT test data obtained in this research. A comparison of how existing models fit the empirical data is undertaken. Furthermore, an alternative model is proposed that takes account of both the number of loading cycles and stress conditions in the sample. This new model shows promising results, especially since it is able to predict aggregate performance for different moisture and drainage conditions.

Keywords- Repeated Load Triaxial Test, Unbound Granular Materials, Permanent Deformation, Regression Models

I. INTRODUCTION

1.1. Background

Road engineers since Roman and even earlier periods have realised the importance of providing sufficient drainage for road pavements [1-2]. This is particularly important for roads constructed using unbound granular base courses such as crushed rock. These roads are often associated with shear failures in wet conditions, thus emphasising the importance of using appropriate aggregates that are not susceptible to change in high moisture conditions especially for the upper pavement layers and using watertight surfacing such as an asphalt surface or thin chip or sprayed seal surfaces.

In a recent New Zealand Transport Agency (NZTA) research project undertaken at the Canterbury Accelerated Pavement Testing Indoor Facility (CAPTIF), two main aspects were investigated. Firstly, experiments were undertaken to establish how much water seeps through the surface layers that were constructed according to different sealing techniques [3]. The second aspect was to investigate the performance of road building aggregate in wet conditions, while being subjected to expected traffic loading [4, 33-34]. The unbound aggregates after being compacted and placed at the CAPTIF facility were subjected to loading in wet surface flow conditions. Previously CAPTIF research had been only in dry conditions. In addition, the properties of these aggregate types were also tested in the laboratory in order to understand its geological properties [3] plus its performance behaviour according to repeated load triaxial tests.

This paper documents the statistical modelling of the data obtained from the testing of the aggregate using the Repeated Load Triaxial (RLT) test in which the materials are subjected to axial stress simulating the vehicle load on the material at various stress states. Different stress states were investigated since that represents different depths of the material within the pavement structure, plus it may also represent different levels of saturation.

1.2. Objectives and Scope

The main objectives of the over-all research project are twofold. Firstly, to investigate the permeability characteristics of different surface technologies and secondly, to investigate the performance of various base course materials with different engineering properties at different moisture levels. Ultimately, the outcome of this research is to develop the ability to forecast the performance of both the surface and pavement layers under different moisture conditions and varying aggregate properties.

This research documents results from RLT tests and the forecasting of the expected plastic deformation.

Different models were assessed as part of this work and an alternative model is proposed. A review of the empirically derived models that are either principally derived from predicting the deformation from the number of loadings or from the applied stresses. In the context of this research these models are referred to as First Generation Models. Second Generation Models incorporate both the loading cycles and the stresses in forecasting the permanent deformation. These models were tested based upon the data from the RLT tests conducted in this research. A brief assessment of the strength for the First Generation Models is presented, while results from model tests of the Second Generation Models are discussed in more detail.

II. Experimental Methodology

2.1 Aggregate Properties

The three unbound granular materials used in this research are Greywacke obtained from the South Island of New Zealand. An example material is selected to demonstrate the modelling process. This material in its various forms is the most commonly used as basecourse material. The particle size distribution (gradation) curve presented in Figure 1 shows a small percentage of clay fractions in the material, thus it has been classified as largely non-plastic. Other engineering properties of the material are listed in Table 1. According to these properties, the material complies with the specifications for basecourse according to the TNZ M/4 [5] standard.

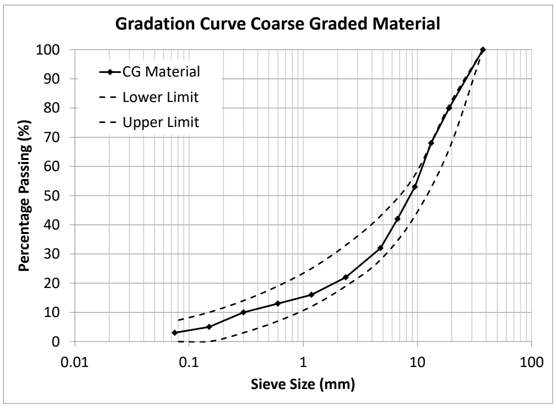

Figure 1 Gradation Curve for Unbound Material with Upper and Lower Limits

Table.1: Engineering Properties of the Basecourse Material used

Engineering Property	AP-40
Unified Classification System	GW
AASHTO Classification System	A-1-a
Cone Penetration Limit (Moisture in %age)	21
Sand Equivalent (% age of sand to clay)	36
Clay Index (volume in ml of methylene blue absorbed by 1 g of material)	1.4
Permeability (Average of Head Difference 2 kPa and 5 kPa, m/s)	0.00001

Notes: Cone Penetration Limit: Test 3.2, NZS 4407:1991
Sand Equivalent: Test 3.6, NZS 4407:1991
Clay Index: Test 3.5, NZS 4407:1991
Permeability: Triaxial test with back pressure technique

2.2 RLT Testing Method

2.2.1 Background to the RLT Tests

In an RLT test, a specimen is compacted and placed in a triaxial cell (shown in Figure 2) where it is confined with air or water that applies the confining stress (σ3). A load cell is placed at the top of the sample which applies the axial load on the sample which is known as the deviator stress (σ1). The application and release of vertical load over the compacted sample completes one loading cycle. The sample deforms with each load application and a portion of the deformation is recovered when the loading is relaxed. The un-recovered portion of the deformation contributes to the permanent plastic deformation of the sample. This plastic deformation behavior of the UGM causes failure in the field when the deformation accumulates in the basecourse with each passing vehicle. The permanent deformation or plastic deformation is the property of UGM that the material settles in increments with each loading cycle. Hence it is very important to formulate the phenomenon of UGM plastic deformation.

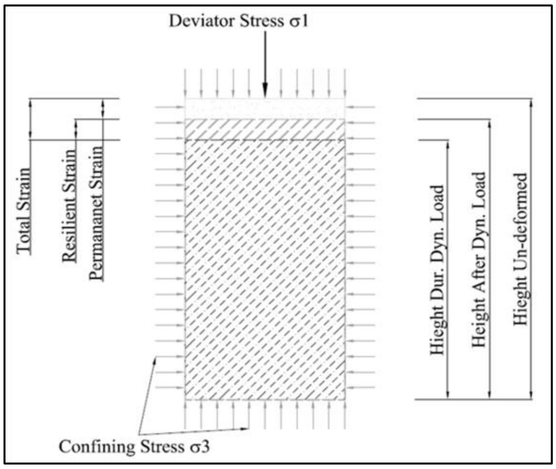

Figure.2: Repeated load triaxial test

Note that later sections refer to the deviator stress (q) and mean normal stress (p), given by Equations 1and 2 [6, 32].

$$q = \sigma_1 - \sigma_3 \quad (1)$$

$$p = \frac{\sigma_1 + 2\sigma_3}{3} \quad (2)$$

2.2.2 Testing Undertaken for this Research

The samples are compacted in layers into a cylindrical shape 290 mm height and 150 mm in diameter. All samples were compacted at optimum moisture content (4.8%) using a vibratory hammer and a maximum dry density (2.34t/m^3) was achieved. The RLT tests were conducted at different levels of saturation of this material. In addition, some of the samples were subjected to an increase in saturation in the triaxial cell by applying back pressure. This was carried out to, as best that could be best practically undertaken, saturate the basecourse material. The three different moisture conditions for testing were:

1. Samples were compacted and tested at optimum moisture under *drained* conditions;
2. Samples were compacted at optimum moisture. Following compaction, the moisture was increased in the triaxial cell through backpressure technique. *Drained* conditions were allowed during the loading cycles; and
3. Lastly some samples were prepared in the same way and saturated to more than 90 percent, the samples were then tested in *un-drained* conditions.

The RLT deviator stress (q) was applied at a frequency of 4 Hz. The stress states used for the stage tests were taken from the draft New Zealand Standard TNZ T15 [7]. These stresses have been recommended from the previous CAPTIF projects and have been formulated in the new standard. The stress paths for these stress states are shown in Figure 3. A total of 50,000 loads were applied for each stress level. The values of different stresses at different test stages are shown in Table 2.

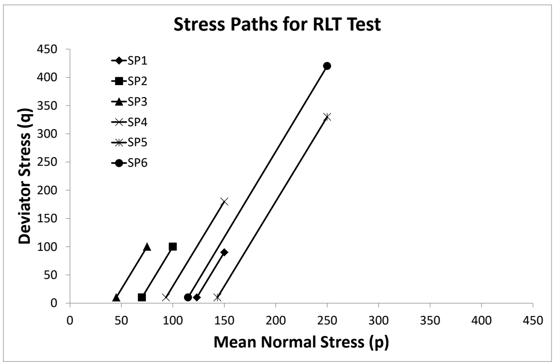

Figure.3 Stress paths for Repeated Load Triaxial Test

Table 2 Stress States for RLT Test

Stress State	Axial Stress σ_1 (kPa)	Confining Stress σ_3 (kPa)	Deviator Stress q (kPa)	Mean Normal Stress p (kPa)
1	210	120	90	150
2	166.67	66.67	100	100
3	141.67	41.67	100	75
4	270	90	180	150
5	470	140	330	250
6	530	110	420	250

III. Assessing First Generation Permanent Deformation Models

3.1 Background to Different Modelling Approaches

To date, the RLT test remains the most popular and accurate test method for the estimation of permanent deformation of aggregates. It successfully simulates the repeated load characteristics associated with road pavement material under traffic loading. In addition to that it can also simulate the aggregate being under different levels of confining stress similar to road aggregates being under different stress due to their varying depth within a pavement. Many researchers have attempted to model permanent deformation of aggregates on the basis of RLT results by using both empirical and mechanistic models [8-17]. Most of the empirical models use input parameters such as the number of repeated axial loadings, confining stress, deviator stress, and elastic deformation to predict the permanent deformation of granular materials used in basecourse construction. The plasticity theories also involve some empirical models, that utilise finite element modelling to predict the deformation of the unbound granular materials. This technique is commonly known as constitutive modeling. A number of constitutive models follow the shake down theory concept [9, 18-21] while some of the others are based on high cycle plasticity theory [22-23] and fuzzy set plasticity theory models [11, 13, 24].

3.2 Forecasting Permanent Deformation on the Basis of Loading Cycles (N)

Forecasting permanent deformation (PD) on the basis of load repetitions is certainly one of the favoured methods of previous researchers. Table 3 lists some of these models and the parameters utilised in the models. Theyse models were tested on the basis of RLT test data for this research and a qualitative assessment of its strengths and limitations are discussed in subsequent paragraphs based upon how the model output fits the RLT data.

The Paute model [25] (Equation 3) uses three parameters: $\varepsilon_{p(100)}$ is the PD after 100 load cycles; and, A and B are model constants. This model performs well for the materials that achieve a stable condition after a certain number of loads. It however does not follow the pattern seen in empirical test data for the various materials/stress states where the permanent deformation increases with the number of loading cycles.

The Sweere model [26] (Equation 4) and Barksdale model [15] (Equation 5) both have two regression parameters: a; and b. These models effectively predict PD up to a certain load repetition. From this point onwards, it constantly under-predicts the PD. It is suspected that the dataset for these models only covered lower repetition ranges.

The Wolff model [27] (Equation 6) and These model [28] (Equation 7) tend to constantly over-predict the PD values. The model given by Pérez [17] (Equation 8) is the combination of Sweere Model (Equation 4) and Wolff Model (Equation 6). This combined model seems to be more accurate in predicting the PD for the RLT test data.

Table 3 Permanent Deformation Models using Number of Loads as Predictor

Model	Equation	Parameters	Eq. No.
Paute Model [25]	$\varepsilon_p = \varepsilon_{p(100)} + A\left[1-\left(\frac{N}{100}\right)^{-B}\right]$	A, B, $\varepsilon_{p(100)}$	3
Sweere Model [26]	$\varepsilon_p = aN^b$	a, b	4
Barksdale Model [15]	$\varepsilon_p = a + b\log N$	a, b	5
Wolff and Visser Model [27]	$\varepsilon_{1p} = (mx+a)(1-e^{-bx})$	m, a, b	6
Theyse Model [28]	$pd = mN + a(1-e^{-bN})$	m, a, b	7
Pérez Model [17]	$pd = a1\,N^{b1} + (mx + a2)(1 - e^{-b2x})$	m,a1,a2,b1,b2	8

3.3 Forecasting Permanent Deformation Models Based on Stress

Some of the models that used stress states to predict the permanent deformation of the UGM are listed in Table 4. The axial (σ_1) and confining (σ_3) stresses are directly used in some of the models, while in other models, the deviator stress and average stresses are used to predict the permanent deformation of UGM.

Table 4 Permanent Deformation Models using Stresses as Predictors

Model	Equation	Parameters	Eq. No.
Hyde Model [29]	$\varepsilon_1^p = a\frac{q}{\sigma_3}$	a	9
Shenton Model [29]	$\varepsilon_1^p = K\left(\frac{q_{max}}{\sigma_3}\right)^{\alpha}$	K, α	10
Lekarp Model [14]	$\frac{\varepsilon_1^p(N_{ref})}{(L/p_0)} = a\left(\frac{q}{p}\right)_{max}^b$	a, b, Nref, L=√(q^2+p^2) p0=100 kPa	11
Paute Model [25]	$A = \frac{\frac{q}{p+p^*}}{b\left(m-\frac{q}{p+p^*}\right)}$	b, m p* is the stress parameter defined by the intersection of the static failure	12

The Hyde model [29] (Equation 9) and Shenton model [29] (Equation 10) use deviator stress and confining stresses to predict the PD. The Hyde model is a linear model and tends to fit the trend of this research dataset well, but does not fit the absolute data points.

The Shenton [29] model is a non-linear model that when fitted to the RLT data both the absolute forecasts and the trend fitting performed well. The Lekarp Model [14] (Equation 11) was based on the shake down approach. It tended to predict the PD trend well but not the absolute values from this dataset. The Paute model [25] (Equation 12) had the best correlation with this research's dataset for both the trend and absolute values compared to all the other models presented in Table 4.

It has been concluded from these two sections that both the number of loadings and the stresses are relevant in the forecasting of permanent deformation. The next section discusses tests that have been conducted on models that incorporate both these aspects.

IV. Testing Second Generation Models on The Basis of Data From This Research

The permanent deformation in the UG material has been predicted by the regression models which take into account both the effect of stresses and number of repeated loading cycles. Diagnostic model tests were undertaken on these models for the RLT data completed for different load cycles repeated on each individual stress state.

Two of the most recent available models (from Gidel [29] and Werkmeister [30]) have been selected from the literature and are compared in greater detail. In both cases model coefficients were determined using a least square approach fitted on the dataset from this research. Results from these tests are presented in the following sections.

4.1 Werkmeister Model

The shakedown theory of the UGM has been discussed in detail by Werkmeister [30]. There are three stages defined by Werkmeister: 1) Range A - Plastic Shakedown Range; 2) Range B - Intermediate Response - Plastic Creep; and 3) Range C - Incremental Collapse. The material from the RLT tests most closely represents that from the Range B of the shakedown state. The model presented by Werkmeister that links the permanent deformations with the number of loading cycles and stress states is expressed in Equation 13:

$$\varepsilon_p(N) = \left[\left(a_1\sigma_3^{a_2}\right)\left(\frac{\sigma_1}{\sigma_3}\right)^2 + \left(a_3\sigma_3^{a_4}\right)\frac{\sigma_1}{\sigma_3}\right](N)^{\left[\left(b_1\sigma_3^{b_2}\right)\frac{\sigma_1}{\sigma_3}+\left(b_3\sigma_3^{b_4}\right)\right]} \quad 13$$

where:

$\varepsilon_p(N)$ = Permanent deformation at number of loading 'N' (µm)

σ_1 and σ_3 = axial and confining stresses respectively (kPa)
a_1, a_2, a_3, a_4
b_1, b_2, b_3, b_4 = regression parameters

The Werkmeister model Equation 13, is an eight parameter model involving axial and confining stresses. In this model, the parameters which addressed the range ‘B’ behavior have been used as there are some other equations for range ‘A’. The outcome from the fitted model is presented in Figure 4. It shows the fitted model compared to actual data points for three different moisture conditions. The estimates of the regression parameters for Equation 13 are shown in Table 5.

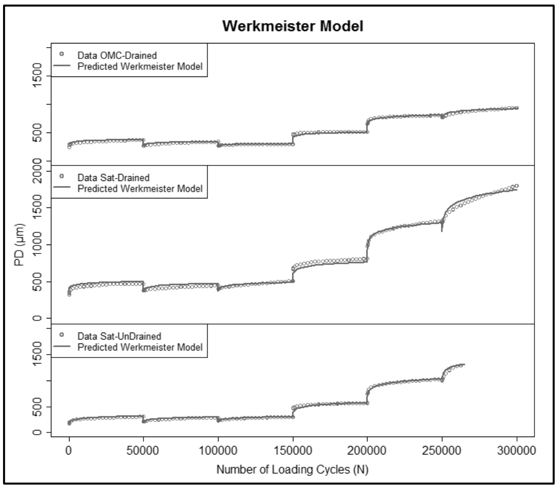

Figure 4: Fitted Werkmeister Model in Different Moisture States of an Unbound Material

Table 5 Regression constants for Werkmeister Model

Stress-St	a1	a2	a3	a4
11x	1.38E-06	3.00	5.00	0.6855
12x	8.85E-07	3.00	6.05133	0.71815
13x	0.318616	1.00143	2.00	-2.00
	b1	**b2**	**b3**	**b4**
11x	-0.1840	-0.895	0.0733	-0.101
12x	0.01222	-0.0678	0.0015	0.545
13x	-4.76E-5	1.20129	0.015	0.417

The first digit in stress states in Table 5 show the number of material which is selected i.e., CG material. The second digit shows the moisture state i.e., 1 shows OMC Drained, 2 shows Saturated Drained and 3 shows Saturated Undrained conditions. The third digit or ‘x’ shows for all stress states. This model predicts the behaviour of the material while changing the stress states in repeated load triaxial (RLT) tests. The Werkmeister model tends to slightly under-estimate the PD values towards the end of each stress state.

4.2 *Gidel Model*

Gidel [29] presented the model which used the mean average stress ‘p’ and deviator stress ‘q’ to predict the plastic deformation in the unbound granular materials (Equation 14). This model contains two portions; the first part predicts the deformation with respect to the number of loadings applied while the second part is used to shift the model according to a change in mean and deviator stresses. Gidel fitted two types of stress models to the data: one fitted the data hyperbolically with change in stress state and the other fitted the data exponentially. The model that changed hyperbolically with the stress state fitted better in the RLT test data and is incorporated in the final model.

$$\varepsilon_p = [a(1 - N^{-b})]\left[\left(\frac{L}{p_a}\right)^n \frac{1}{m + \frac{s}{p} - \frac{q}{p}}\right] \qquad 14$$

where:
ε_p = Permanent deformation (µm)
N = Number of loading cycles
q and p = deviator and mean stresses respectively (kPa)
L = Length of stress path ($L = \sqrt{q^2 + p^2}$)
p_a = 100 kPa
a, b, n, m, s = Regression parameters

The model was fitted to the RLT test results obtained from the tested materials at three different moisture conditions (refer to Figure 5 and Table 6). The fit in Figure 5 shows that the Gidel model generally follows the change in stress state well compared to the RLT test data. However, it is shown that at a high stress state the model fit is less accurate, especially for the saturated drained moisture condition.

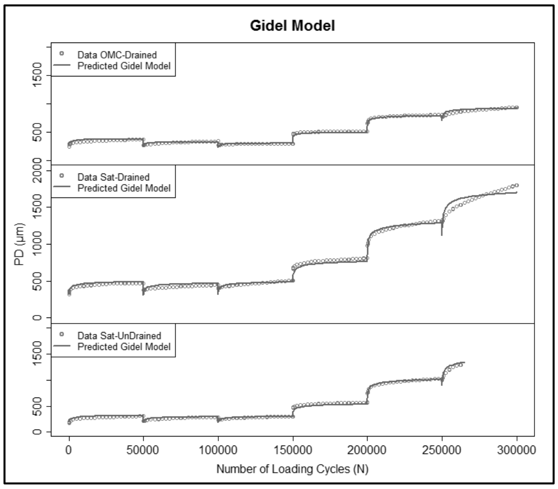

Figure 5 Fitted Gidel Model in Different Moisture States of an Unbound Material

Table 6 Regression constants for Gidel Model

Stress-St	a	b	n	m	s
11x	4278.824	0.279572	0.811331	17.83414	-16.1542
12x	1244.709	0.181785	0.890353	4.370875	-25.6755
13x	687.5761	0.165184	1.072105	3.993461	-17.3861

4.3 *Suggested Hussain Model*

Taking the lessons from Gidel [29], an alternative Hussain model is proposed. Gidel’s model was used as a base model but the 'N' term from Equation 14 is replaced by Equation 8, resulting in:

$$\varepsilon_p = [a_1 N^{b1} + (m_1 N + a_2)(1 - e^{-b_2 N})]\left[\left(\frac{L}{p_a}\right)^n \frac{1}{m_2 + \frac{s}{p}}\right] \quad 15$$

ε_p = Permanent deformation (µm)
N = Number of loading cycles
q and p = deviator and mean stresses respectively (kPa)
L = Length of stress path ($L = \sqrt{q^2 + p^2}$)
p_a = 100 kPa
a_1, b_1, a_2, b_2
m_1, n, m_2, s = Regression parameters

Therefore, the new model is based on the combination of two models; a) the model presented by Pérez which predicts the permanent deformations with respect to number of loads only; and b) the second part of this model is taken from the second portion of the Gidel model that predicts the UGM behaviour with respect to stress states. The resulting model outcome is presented in Figure 6 and Table 7.

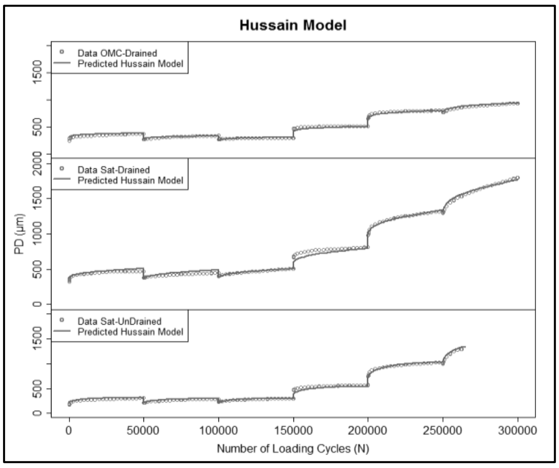

Figure 6 Fitted New Hussain Model in Different Moisture States of an Unbound Material

Table 7 Regression constants for Hussain Model

Stress-St	a	b	A	B
11x	3073.17	0.0205	110.5577	0.00017
12x	734.366	0.0239	43.73904	0.00014
13x	341.292	0.0236	79.72744	0.00012
	m1	**n**	**m2**	**s**
11x	0.0037	0.8108	17.68	-15.2418
12x	0.002346	0.8902	4.35	-25.6831
13x	0.00029	1.0409	3.63	-6.46421

V. MODEL DIAGNOSTIC COMPARISONS

In addition to the promising visual results depicted in the previous three graphs a detailed statistical comparison was also undertaken in order to assess the predictive power of the three modelling approaches. This section reports on the:

- Graphical fit for the maximum stress state;
- Akaike's Information Criterion;
- Residual standard error and R-squared.

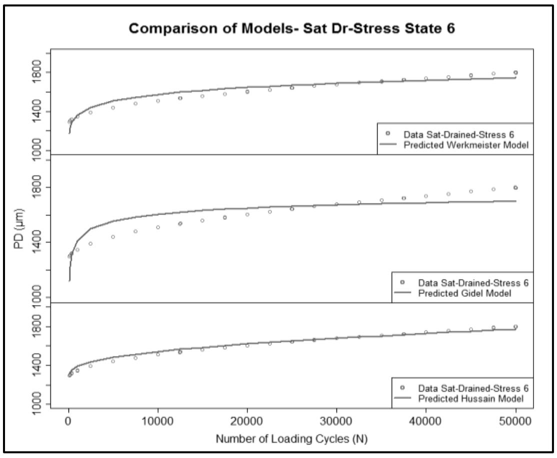

Figure 7 Comparison of Fits for the presented models

5.1 Graphical Fit

From previous work it has been established that most existing models are capable of predicting PD relatively accurate at low stress states. However, it becomes more challenging to forecast the PD at higher stress states. Figure 7 shows the comparison of the three models compared to the actual data for the highest stress state. It appears that the alternative model format is more accurately fitting this dataset.

5.2 Akaike's Information Criterion (AIC)

The AIC value can be defined as "an estimate of the distance from the model fit to the true but unknown model that generated the data" [31]. It is a function of:

- Number of observations;
- Residual sum of squares;
- Best estimate of the parameter; and,
- Number of regression parameters.

The AIC values for some of the models are shown in Table 8.

Table 8 Comparison of Criteria for Goodness of Fit

Stress -State	Werkmeister Model	Gidel Model	Hussain Model
Akaike Information Value			
11x	11617.54	12416.42	12012.7
12x	13811.84	14286.48	13237.2
13x	12662.54	12883.04	12728.4
Residual Standard Error			
11x	15.9346	21.27998	18.37602
12x	35.16625	41.77889	28.58223
13x	40.09072	43.87764	41.17145
R-Squared Values			
11x	0.995318	0.991633	0.993774
12x	0.99401	0.991527	0.996043
13x	0.984363	0.981223	0.983508

Note that if a model has additional number of parameters, it increases the AIC value. However, for the Hussain model, the increase in number of parameters (from Gidel Model) has not increased AIC value, suggesting a potential better fit to the data. The comparison of the respective AIC values revealed that the models did not differ significantly.

5.3 Residual Standard Error (RSE) and R Squared (Coefficient of Determination)

RSE is the second measure for assessing the best fit of the nonlinear model. Lower values of RSE indicate a better model fit. It is a function of:

- the 'distance' between the forecasted and actual data points;
- the number of observations; and,
- the number of regression constants used in the equation.

The values of RSE (Table 8) show that there is not a significant difference in the model accuracies, with the alternative model resulting in the best outcome. One of the most commonly used regression fit assessment for linear models is the R- Square. The R-Squared also confirms the Hussain model equally best fitting the RLT data from this experiment as the other two.

VI. CONCLUSIONS

This research is a part of a CAPTIF project which is full scale indoor testing facility. The materials used in this research were tested using RLT tests. This paper has reviewed two regression models that used the number of loading cycles and the stress states to predict the permanent deformation in the UGM. The assessments of available models resulted in the compilation of an alternative model that combines principles used by Pérez and Gidel [17, 29]. The loading cycle component of the Pérez model was combined with the stress state model suggested by Gidel. Conclusions from the results are:

1. Aggregates used in road pavement layers are subjected to different stress states (as a result of its depth and relative position to the wheel loading) plus the number of loading cycles. Therefore, second generation models, taking account of both the stress state and loading cycles, more closely represent the actual field situation compared to the first generation models that uses either one of these factors in isolation to predict deformation;
2. The Hussain model presented in this paper resulted in a closer fit with the research dataset, especially at higher stress levels when compared to other existing models.

The method used to fit the new model along with the other compared models give the regression parameters for the unbound granular materials to be used in the basecourse and sub-base layers. The deformation can be predicted at various stress states in these materials which is an important criterion to judge the material performance in real in field pavements. The model work results in an improved model that simulates the PD at different stress states. In addition, it can also reflect the varying behaviour of the material at different moisture conditions, which contributes to the understanding of material behaviour in in-service pavements which are often subjected to different moisture regimes.

VII. ACKNOWLEDGEMENTS

The authors acknowledge input from Haran Arampamoorthy, John Patrick, and Alan Fussel. The author also acknowledges the Higher Education Commission (HEC) of Pakistan for funding this research which is part of a PhD study at The University of Auckland, New Zealand.

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