Technical Journal, University of Engineering and Technology (UET) Taxila, Pakistan *Vol. 27 No. 1-2022*
ISSN:1813-1786 (Print) 2313-7770 (Online)

Complex Intuitionistic Fuzzy N-Soft Sets and Their Applications in Decision Making Algorithm

U. U. Rehman[1], T. Mahmood[2]

[1,2] *Department of Mathematics and Statistics, International Islamic University Islamabad, Pakistan.*

[1]ubaid5@outlook.com.

Abstract- Complex intuitionistic fuzzy sets and N-soft sets are the generalized models of fuzzy sets and soft sets respectively, to deal with unknown and complicated data in the problems of real life. This study initiates a novel concept of complex intuitionistic fuzzy N-soft sets, which is a mixture of two different models, called complex intuitionistic fuzzy sets and N-soft sets. complex intuitionistic fuzzy N-soft sets can be seen as modified N-soft sets based on the complex intuitionistic fuzzy soft set. complex intuitionistic fuzzy N-soft set is a more generalized tool as compared to the existing ones. Moreover, we initiate some basic properties along with their basic operations and illustrate them with the help of examples. Further, we initiate the relationship of our novel model with existing models like complex intuitionistic fuzzy soft sets and soft sets. Additionally, to show the credibility and effectiveness of our initiated model, we apply it to decision-making problems of real life. We also define an algorithm to solve DM problems by using complex intuitionistic fuzzy N-soft sets. Finally, we compare our novel approach with the existing model intuitionistic fuzzy N-soft set to show the supremacy of our initiated concept.

Keywords- Complex intuitionistic fuzzy N-soft sets; Intuitionistic fuzzy N-soft sets; N-soft sets, soft sets; Decision making.

I. INTRODUCTION

In all sciences such as information sciences, management sciences, and all fields of real life, the researchers and decision-makers face problems to cope with uncertainty and vagueness. Various attempts have been made to cope with this problem, the first attempt was made by [1] in 1965 to deal with the uncertainty. He introduced the notion of fuzzy sets (FSs) which is the generalization of the crisp set. The notion of FS is very flexible to express vague and uncertain data. In FS the membership grade (MG) lies in $[0, 1]$. In any case, it may not generally be right to assume that the non-membership grade (NMG) of an element in FS is equivalent to one minus its MG, because there may likewise be some grade of indeterminacy. Consequently, numerous authors modified the conception of FS such as [2] modified FS to intuitionistic FSs (IFSs), [3] modified to spherical FS, [4] expanded to T-spherical FS [5]. Also various researchers diagnosed numerous similarity measures and aggregation operators on the various modified conception of FS such as [6] diagnosed similarity measures in the setting of interval-valued picture FS, [7], diagnosed aggregation operators for picture FS. Historically, the set of real numbers was extended to the set of complex numbers, this extension motivated [8] to define the notion of complex FSs (CFSs). [9] initiated the notion of complex hesitant FSs. After this [10] initiated the concept of complex dual hesitant FSs.

The majority of our conventional techniques for modeling, computing, and reasoning are crisp but researchers in environmental science, economics, medical science, engineering, sociology, and many other fields daily cope with the complicated information which is not crisp. So in the problems where uncertainties involve the researchers are not able to use these conventional techniques. While the theory of probability, fuzzy sets (FSs) [1], interval mathematics [11-12], intuitionistic FS [2], and rough sets [13-17], are supposed as mathematical techniques to cope with the uncertainties. But each of the theories has some difficulties as given by [18]. To overcome these difficulties [18] introduced a completely different notion for modeling uncertainty and vagueness, called soft set (SS) theory. A SS is the parameterized family of subsets of a universal set. In SS the issue of setting MG, and other associated issues, clearly does not occur. This makes the SS very easy and suitable to apply to real-life problems. SS has applications in different fields, such as operations research, perron integration, measurement theory, game theory, etc. [19] defined some operations in SS theory. A survey of DM methods based on two classes of hybrid SS models were interpreted by [20]. [21] defined SS theoretic approach for dimensionality reduction. A linguistic value SS based approach to multiple criteria group DM was given by [22]. [23] defined similarity in SS theory. [24] explored T-soft equality relation. VIKOR and TOPSIS methods for SS were diagnosed by [25] and [26]. The belief interval-valued SS was established by [27]. Some researchers involve FS theory in SS theory [28] interpreted the notion of fuzzy SS (FSS). An intuitionistic FSS (IFSS) was given by [29]. The theory of complex FSS (CFSS) was given by [30]. [31] gave the idea of complex IFSS (CIFSS).

A close observation of the models with SSs shows that most researchers in SS theory worked on binary evaluation i.e; either $\{0,1\}$ or $[0, 1]$ [32, 33]. Although, there are a lot of real-life problems where data is in the discrete structure or non-binary, for example, hotels compared by websites, dramas, etc. which cannot handle by existing methods of SS. From another point of view, [34] defined the ternary voting system. The notion of multi-SS was given by [35]. Further, [36] interpreted a notion of N-soft set (N-SS). Then, [37] utilized N-SS in the setting of rough set. [38] diagnosed N-soft topology. To cope with uncertain data [39] defined fuzzy N-SS (FN-SS). [40] diagnosed the multi-fuzzy N-SS. [41] also gave the notion of intuitionistic FN-SS (IFN-SS). [42] diagnosed complex fuzzy N-SS. IFN-SS is an efficient tool to cope with uncertainty in a single dimension. IFN-SS cannot help in DM where two-dimensional information is involved. IFN-SS cannot carry additional information for example a family wants to select a place for a trip in 4 places. A team “A” of experts gives rating and grading to these places base on the parameters. But what will happen if the family says that they also want to know the view of team “B” of experts. In this case, IFN-SS cannot provide them with this additional information. To overcome this difficulty in this manuscript, we initiate the novel model called complex intuitionistic fuzzy N-soft sets (CIFN-SS) which deals with complicated and two-dimensional information. As the novel model combining CIFSS with N-SS, CIFN-SS could be utilized in a number of various fields. The CIFN-SSs is the generalization so one can get IFN-SSs, N-SSs, and SSs from CIFN-SSs.

This manuscript is settled as follows: In section 2 we review some basic definitions and properties of FSs, IFSs, CFSs, CIFSs, SSs, IFSSs, CFSSs, and IFN-SS. In section 3, we provide our novel model CIFN-SS along with some basic properties and illustrate them with the help of examples. Additionally, we initiate the relationship of our novel model with an existing model like CIFSSs and SS in the same section. In section 4 of this manuscript, we define an algorithm to solve DM problems using CIFN-SS. We also show the effectiveness of our initiated notion and its real-life application in section 4. We do a comparison of our initiated work with the existing method in section 5. In the end, in section 6, the conclusion of the work done in this manuscript is given.

II. PRELIMINARIES

In this Section, of the manuscript, we revise some fundamental definitions of FSs, IFS, CFSs, CIFSs, SSs, N-SSs, and intuitionistic FN-SSs (IFN-SS). The basic properties of the above methods are also discussed in detail. Throughout this article, the symbols $\mathfrak{B} \neq \emptyset$ be universe set and $\mathcal{U}$ be the set of parameters.

Definition 1: [1] A fuzzy set (FS) is denoted and given as

$\mathbb{F} = \{(x, \gamma^M(x)) : x \in \mathfrak{B}\}$,

where $\gamma^M : \mathfrak{B} \to [0,1]$ with a condition $0 \leq \gamma^M(x) \leq 1$.

*Definition 2***:** [1] Let $\mathbb{F} = (x_i, \gamma_{\mathbb{F}}^M(x_i))$ and $\mathbb{G} = (x_i, \gamma_{\mathbb{G}}^M(x_i)), i = 1,2,3, \dots, n$ are two fuzzy numbers (FNs). Then

1. $\mathbb{F} = \mathbb{G} \Leftrightarrow \gamma_{\mathbb{F}}^M(x_i) = \gamma_{\mathbb{G}}^M(x_i), i = 1,2, \dots, n;$
2. $\mathbb{F} \subseteq \mathbb{G} \Leftrightarrow \gamma_{\mathbb{F}}^M(x_i) \leq \gamma_{\mathbb{G}}^M(x_i), i = 1,2, \dots, n;$
3. $\mathbb{F}^c = (x_i, 1 - \gamma_{\mathbb{F}}^M(x_i));$
4. $\mathbb{F} \cup \mathbb{G} = \mathbb{F} \vee \mathbb{G} = (x_i, \max(\gamma_{\mathbb{F}}^M(x_i), \gamma_{\mathbb{G}}^M(x_i))), i = 1,2,3, \dots, n;$
5. $\mathbb{F} \cap \mathbb{G} = \mathbb{F} \wedge \mathbb{G} = (x_i, \min(\gamma_{\mathbb{F}}^M(x_i), \gamma_{\mathbb{G}}^M(x_i))), i = 1,2,3, \dots, n.$

Definition 3: [2] An intuitionistic FS (IFS) is denoted and given as

$\mathbb{I} = \left\{\left(x, (\gamma^M(x), \gamma^N(x))\right) : x \in \mathfrak{B}\right\}$,

where $\gamma^M : \mathfrak{B} \to [0,1], \gamma^N : \mathfrak{B} \to [0,1]$ represents the membership grade (MG) and non-membership grade (NMG) for each $x \in \mathfrak{B}$, with a condition that $0 \leq \gamma^M(x) + \gamma^N(x) \leq 1$.

Definition 4: [2] Let $\mathbb{I} = \left(x_i, (\gamma_{\mathbb{I}}^M(x_i), \gamma_{\mathbb{I}}^N(x_i))\right)$ and $\mathbb{K} = \left(x_i, (\gamma_{\mathbb{K}}^M(x_i), \gamma_{\mathbb{K}}^N(x_i))\right), i = 1,2,3, \dots, n$ are two IF numbers (IFNs). Then

1. $\mathbb{I} = \mathbb{K} \Leftrightarrow \gamma_{\mathbb{I}}^M(x_i) = \gamma_{\mathbb{K}}^M(x_i) \; and \; \gamma_{\mathbb{I}}^N(x_i) = \gamma_{\mathbb{K}}^N(x_i), i = 1,2, \dots, n;$
2. $\mathbb{I} \subseteq \mathbb{K} \Leftrightarrow \gamma_{\mathbb{I}}^M(x_i) \leq \gamma_{\mathbb{K}}^M(x_i) \; and \; \gamma_{\mathbb{I}}^N(x_i) \geq \gamma_{\mathbb{K}}^N(x_i) i = 1,2, \dots, n;$
3. $\mathbb{I}^c = x_i, (\gamma_{\mathbb{I}}^N(x_i), \gamma_{\mathbb{I}}^M(x_i));$
4. $\mathbb{I} \cup \mathbb{K} = \mathbb{I} \vee \mathbb{K} = (x_i, \max(\gamma_{\mathbb{I}}^M(x_i), \gamma_{\mathbb{K}}^M(x_i)), \min(\gamma_{\mathbb{I}}^N(x_i), \gamma_{\mathbb{K}}^N(x_i))), i = 1,2,3, \dots, n;$

5. $\mathbb{I}\cap\mathbb{K} = \mathbb{I}\wedge\mathbb{K} = \left(x_i, \min\left(\gamma_{\mathbb{I}}^M(x_i), \gamma_{\mathbb{K}}^M(x_i)\right), \max\left(\gamma_{\mathbb{I}}^N(x_i), \gamma_{\mathbb{K}}^N(x_i)\right)\right)$, $i = 1,2,3,\dots,n$.

Definition 5: [8] A complex FS (CFS) is denoted and given as

$\mathbb{S} = \left\{\left(x, \mu^M(x)\right): x \in \mathfrak{B}\right\}$

where $\mu^M = \gamma^M e^{i2\pi\left(\omega_{\gamma^M}\right)}$ with a condition $0 \le \gamma^M, \omega_{\gamma^M} \le 1$.

Definition 6: [8] Let $\mathbb{F} = \left(x_i, \gamma_{\mathbb{F}}^M(x_i) e^{\iota 2\pi\left(\omega_{\gamma_{\mathbb{F}}^M(x_i)}\right)}\right)$ and $\mathbb{G} = \left(x_i, \gamma_{\mathbb{G}}^M(x_i) e^{\iota 2\pi\left(\omega_{\gamma_{\mathbb{G}}^M(x_i)}\right)}\right), i = 1,2,3,\dots,n$ are two complex fuzzy numbers (CFNs). Then

1. $\mathbb{F} = \mathbb{G} \Leftrightarrow \mu_{\mathbb{F}}^M(x_i) = \mu_{\mathbb{G}}^M(x_i)$ i.e. $\gamma_{\mathbb{F}}^M(x_i) = \gamma_{\mathbb{G}}^M(x_i), \omega_{\gamma_{\mathbb{F}}^M}(x_i) = \omega_{\gamma_{\mathbb{G}}^M}(x_i), i = 1,2,\dots,n$;
2. $\mathbb{F} \subseteq \mathbb{G} \Leftrightarrow \mu_{\mathbb{F}}^M(x_i) \le \mu_{\mathbb{G}}^M(x_i)$ i.e. $\gamma_{\mathbb{F}}^M(x_i) \le \gamma_{\mathbb{G}}^M(x_i), \omega_{\gamma_{\mathbb{F}}^M}(x_i) \le \omega_{\gamma_{\mathbb{G}}^M}(x_i), i = 1,2,\dots,n$;
3. $\mathbb{F}^c = 1 - \mu^M = (1 - \gamma_{\mathbb{F}}^M) e^{i2\pi\left(1-\left(\omega_{\gamma_{\mathbb{F}}^M}\right)\right)}$;
4. $\mathbb{F}\cup\mathbb{G} = \mathbb{F}\vee\mathbb{G} = \left(x_i, \max\left(\mu_{\mathbb{F}}^M(x_i), \mu_{\mathbb{G}}^M(x_i)\right)\right) = \left(x_i, \max\left(\gamma_{\mathbb{F}}^M(x_i), \gamma_{\mathbb{G}}^M(x_i)\right) e^{\iota 2\pi \max\left(\omega_{\gamma_{\mathbb{F}}^M}(x_i), \omega_{\gamma_{\mathbb{G}}^M}(x_i)\right)}\right), i = 1,2,3,\dots,n$;
5. $\mathbb{F}\cap\mathbb{G} = \mathbb{F}\wedge\mathbb{G} = \left(x_i, \min\left(\mu_{\mathbb{F}}^M(x_i), \mu_{\mathbb{G}}^M(x_i)\right)\right) = \left(x_i, \min\left(\gamma_{\mathbb{F}}^M(x_i), \gamma_{\mathbb{G}}^M(x_i)\right) e^{\iota 2\pi \min\left(\omega_{\gamma_{\mathbb{F}}^M}(x_i), \omega_{\gamma_{\mathbb{G}}^M}(x_i)\right)}\right), i = 1,2,3,\dots,n$;

Definition 7: [43] A complex intuitionistic FS (CIFS) is denoted and given as

$\mathbb{T} = \left\{\left(x, \left(\mu^M(x), \mu^N(x)\right)\right): x \in \mathfrak{B}\right\}$

where $\mu^M = \gamma^M e^{i2\pi\left(\omega_{\gamma^M}\right)}$, $\mu^N = \gamma^N e^{i2\pi\left(\omega_{\gamma^N}\right)}$ are complex MG (CMG) and complex NMG (CNMG) respectively, with a condition $0 \le \gamma^M, \omega_{\gamma^M}, \gamma^N, \omega_{\gamma^N} \le 1$.

Definition 8: [43] Let $\mathbb{F} = \left(x_i, \left(\gamma_{\mathbb{F}}^M(x_i) e^{\iota 2\pi\left(\omega_{\gamma_{\mathbb{F}}^M(x_i)}\right)}, \gamma_{\mathbb{F}}^N(x_i) e^{\iota 2\pi\left(\omega_{\gamma_{\mathbb{F}}^N(x_i)}\right)}\right)\right)$ and $\mathbb{G} = \left(x_i, \left(\gamma_{\mathbb{G}}^M(x_i) e^{\iota 2\pi\left(\omega_{\gamma_{\mathbb{G}}^M(x_i)}\right)}, \gamma_{\mathbb{G}}^N(x_i) e^{\iota 2\pi\left(\omega_{\gamma_{\mathbb{G}}^N(x_i)}\right)}\right)\right)$, $i = 1,2,3,\dots,n$ are two CIF numbers (CIFNs). Then

1. $\mathbb{F}^c = \left(x_i, \left(\mu_{\mathbb{F}}^N(x_i), \mu_{\mathbb{F}}^M(x_i)\right)\right) = \left(x_i, \left(\gamma_{\mathbb{F}}^N(x_i) e^{\iota 2\pi\left(\omega_{\gamma_{\mathbb{F}}^N(x_i)}\right)}, \gamma_{\mathbb{F}}^M(x_i) e^{\iota 2\pi\left(\omega_{\gamma_{\mathbb{F}}^M(x_i)}\right)}\right)\right)$, $i = 1,2,3,\dots,n$
2. $\mathbb{F}\cup\mathbb{G} = \mathbb{F}\vee\mathbb{G} = \left(x_i, \begin{pmatrix}\max\left(\mu_{\mathbb{F}}^M(x_i), \mu_{\mathbb{G}}^M(x_i)\right), \\ \min\left(\mu_{\mathbb{F}}^N(x_i), \mu_{\mathbb{G}}^N(x_i)\right)\end{pmatrix}\right) = \left(x_i, \begin{pmatrix}\max\left(\gamma_{\mathbb{F}}^M(x_i), \gamma_{\mathbb{G}}^M(x_i)\right) \\ .e^{\iota 2\pi \max\left(\omega_{\gamma_{\mathbb{F}}^M}(x_i), \omega_{\gamma_{\mathbb{G}}^M}(x_i)\right)}, \\ \min\left(\gamma_{\mathbb{F}}^N(x_i), \gamma_{\mathbb{G}}^N(x_i)\right) \\ .e^{\iota 2\pi \min\left(\omega_{\gamma_{\mathbb{F}}^N}(x_i), \omega_{\gamma_{\mathbb{G}}^N}(x_i)\right)}\end{pmatrix}\right), i = 1,2,3,\dots,n$
3. $\mathbb{F}\cap\mathbb{G} = \mathbb{F}\wedge\mathbb{G} = \left(x_i, \begin{pmatrix}\min\left(\mu_{\mathbb{F}}^M(x_i), \mu_{\mathbb{G}}^M(x_i)\right) \\ , \max\left(\mu_{\mathbb{F}}^N(x_i), \mu_{\mathbb{G}}^N(x_i)\right)\end{pmatrix}\right) = \left(x_i, \begin{pmatrix}\min\left(\gamma_{\mathbb{F}}^M(x_i), \gamma_{\mathbb{G}}^M(x_i)\right) \\ e^{\iota 2\pi \min\left(\omega_{\gamma_{\mathbb{F}}^M}(x_i), \omega_{\gamma_{\mathbb{G}}^M}(x_i)\right)}, \\ \max\left(\gamma_{\mathbb{F}}^N(x_i), \gamma_{\mathbb{G}}^N(x_i)\right) \\ e^{\iota 2\pi \max\left(\omega_{\gamma_{\mathbb{F}}^N}(x_i), \omega_{\gamma_{\mathbb{G}}^N}(x_i)\right)}\end{pmatrix}\right), i = 1,2,3,\dots,n$

Definition 9: [18] A pair $(J, \mathcal{V})$ represents a SS over $\mathfrak{B}$ where $J: \mathcal{V} \to P(\mathfrak{B})$, be set-valued function and $\mathcal{V} \subseteq \mathcal{U}$.

Definition 10: [19] Let $(J_1, \mathcal{V}_1)$ and $(J_2, \mathcal{V}_2)$ be two SSs with $\mathcal{V}_1 \cap \mathcal{V}_2 \ne \emptyset$, then their restricted union and intersection are denoted and given as follows:

$$(J_1, \mathcal{V}_1) \cup_r (J_2, \mathcal{V}_2) = (\alpha, \mathcal{V}_1 \cap \mathcal{V}_2)\ \forall\ \mathscr{v} \in \mathcal{V}_1 \cap \mathcal{V}_2 \Longrightarrow \alpha(\mathscr{v}) = J_1(\mathscr{v}) \cup J_2(\mathscr{v}).$$

$$(J_1, \mathcal{V}_1) \cap_r (J_2, \mathcal{V}_2) = (\beta, \mathcal{V}_1 \cap \mathcal{V}_2) \text{ where } \beta(\mathscr{v}) = J_1(\mathscr{v}) \cap J_2(\mathscr{v}) \forall \mathscr{v} \in \mathcal{V}_1 \cap \mathcal{V}_2.$$

Definition 11: [19] Let $(J_1, \mathcal{V}_1)$ and $(J_2, \mathcal{V}_2)$ be two SSs with $\mathcal{V}_1 \cap \mathcal{V}_2 \ne \emptyset$, then their extended union and intersection are denoted and given as follows:

$$(J_1, \mathcal{V}_1) \cup_e (J_1, \mathcal{V}_2) = (\alpha, \mathcal{V}_1 \cup \mathcal{V}_2) \text{ where } \forall\ \mathscr{v} \in \mathcal{V}_1 \cup \mathcal{V}_2 \Longrightarrow \alpha(\mathscr{v}) = \begin{cases} J_1(\mathscr{v}) & if\ \mathscr{v} \in \mathcal{V}_1 \backslash \mathcal{V}_2 \\ J_2(\mathscr{v}) & if\ \mathscr{v} \in \mathcal{V}_2 \backslash \mathcal{V}_1 \\ J_1(\mathscr{v}) \cup J_2(\mathscr{v}) & \mathscr{v} \in \mathcal{V}_1 \cup \mathcal{V}_2 \end{cases}$$

$$(J_1, \mathcal{V}_1) \cap_e (J_1, \mathcal{V}_2) = (\beta, \mathcal{V}_1 \cup \mathcal{V}_2) \text{ where } \forall \mathscr{v} \in \mathcal{V}_1 \cup \mathcal{V}_2 \Longrightarrow \beta\ (\mathscr{v}) = \begin{cases} J_1(\mathscr{v}) & if\ \mathscr{v} \in \mathcal{V}_1 \backslash \mathcal{V}_2 \\ J_2(\mathscr{v}) & if\ \mathscr{v} \in \mathcal{V}_2 \backslash \mathcal{V}_1 \\ J_1(\mathscr{v}) \cap J_2(\mathscr{v}) & \mathscr{v} \in \mathcal{V}_1 \cup \mathcal{V}_2 \end{cases}$$

Definition 12: [36] A triplet $(J, \mathcal{V}, N)$ represents an N-soft set (N-SS) over $\mathfrak{B}$ if $J: \mathcal{V} \to 2^{\mathfrak{B}\times\mathfrak{D}} = P(\mathfrak{B} \times \mathfrak{D}), \mathcal{V} \subseteq Q$ with the property that for each $\mathscr{v} \in \mathcal{V}$ and $\exists$ a unique $\left((\mathfrak{b}, \mathfrak{o}_{\mathscr{v}}) \in P(\mathfrak{B} \times \mathfrak{D})\right)$ such that $(\mathfrak{b}, \mathfrak{o}_{\mathscr{v}}) \in$

$J(v)$, $\mathfrak{b} \in \mathfrak{B}, \mathfrak{o}_v \in \mathfrak{O} = \{0, 1, 2, \dots, N-1\}$ be ordered grades set.

Definition 13: [36] Let $(J_1, \mathcal{V}_1, N_1)$ and $(J_2, \mathcal{V}_2, N_2)$ be two N-SSs with $\mathcal{V}_1 \cap \mathcal{V}_2 \neq \emptyset$, then their restricted union and intersection are denoted and given as follows:

$$(J, \mathcal{V}_1, N_1) \cup_R (J_2, \mathcal{V}_2, N_1) = (\delta, \mathcal{V}_1 \cap \mathcal{V}_2, \max(N_1, N_2))$$

where $\forall v \in \mathcal{V}_1 \cap \mathcal{V}_2$ & $\mathfrak{b} \in \mathfrak{B} \Longrightarrow (\mathfrak{b}, \mathfrak{o}_v) \in \delta(v) \Leftrightarrow \mathfrak{o}_v = \max(\mathfrak{o}_v^1, \mathfrak{o}_v^2), if\ (\mathfrak{b}, \mathfrak{o}_v^1) \in J_1(v)$ & $(\mathfrak{b}, \mathfrak{o}_v^2) \in J_2(v)$.

$$(J, \mathcal{V}_1, N_1) \cap_R (G, B, N_1) = (\eta, A \cap B, \min(N_1, N_2))$$

where $\forall v \in \mathcal{V}_1 \cap \mathcal{V}_2$ & $\mathfrak{b} \in \mathfrak{B} \Longrightarrow (\mathfrak{b}, \mathfrak{o}_v) \in \eta(v) \Leftrightarrow \mathfrak{o}_v = \min(\mathfrak{o}_v^1, \mathfrak{o}_v^2), if\ (\mathfrak{b}, \mathfrak{o}_v^1) \in J_1(v)$ & $(\mathfrak{b}, \mathfrak{o}_v^2) \in J_2(v)$.

Definition 14: [36] Let $(J_1, \mathcal{V}_1, N_1)$ and $(J_2, \mathcal{V}_2, N_2)$ be two N-SSs. Then their extended union and intersection are denoted and given as:

$$(J_1, \mathcal{V}_1, N_1) \cup_E (J_2, \mathcal{V}_2, N_2) = (\delta, \mathcal{V}_1 \cup \mathcal{V}_2, \max(N_1, N_2))$$

where $\delta(v) =$

$$\begin{cases} J(v) & if\ v \in \mathcal{V}_1 \backslash \mathcal{V}_2 \\ G(v) & if\ v \in \mathcal{V}_2 \backslash \mathcal{V}_1 \\ (\mathfrak{b}, \mathfrak{o}_v)/\mathfrak{o}_v = \max(\mathfrak{o}_v^1, \mathfrak{o}_v^2) & \begin{pmatrix} (\mathfrak{b}, \mathfrak{o}_v^1) \in J_1(v) \\ (\mathfrak{b}, \mathfrak{o}_v^2) \in J_2(v) \end{pmatrix} \end{cases}$$

$$(J_1, \mathcal{V}_1, N_1) \cap_e (J_2, \mathcal{V}_2, N_2) = (\eta, \mathcal{V}_1 \cup \mathcal{V}_2, \max(N_1, N_2))$$

where $\eta(v) =$

$$\begin{cases} J(v) & if\ v \in \mathcal{V}_1 \backslash \mathcal{V}_2 \\ G(v) & if\ v \in \mathcal{V}_2 \backslash \mathcal{V}_1 \\ (\mathfrak{b}, \mathfrak{o}_v)/\mathfrak{o}_v = \min(\mathfrak{o}_v^1, \mathfrak{o}_v^2) & \begin{pmatrix} (\mathfrak{b}, \mathfrak{o}_v^1) \in J_1(v) \\ (\mathfrak{b}, \mathfrak{o}_v^2) \in J_2(v) \end{pmatrix} \end{cases}.$$

Definition 15: [29] A pair $(J', \mathcal{V})$ characterized an intuitionistic fuzzy SS (IFSS) over $\mathfrak{B}$ if $J': \mathcal{V} \to IFS(\mathfrak{B}), \mathcal{V} \subseteq \mathcal{U}$, where $IFS(\mathfrak{B})$ contains the set of all IFS of $\mathfrak{B}$.

Definition 16: [30] A pair $(J'', \mathcal{V})$ represents a complex fuzzy SS (CFSS) over $\mathfrak{B}$ if $J'': \mathcal{V} \to CFS(\mathfrak{B}), \mathcal{V} \subseteq \mathcal{U}$, where $CFS(\mathfrak{B})$ contains the set of all complex FS of $\mathfrak{B}$.

Definition 17: [31] A pair $(J''', \mathcal{V})$ characterize a complex IFSS (CIFSS) over $\mathfrak{B}$ if $J''': \mathcal{V} \to CIFS(\mathfrak{B}), \mathcal{V} \subseteq \mathcal{U}$, where $CIFS(\mathfrak{B})$ contains the set of all complex intuitionistic FS of $\mathfrak{B}$.

Definition 18: [41] Let $\mathcal{V} \subseteq \mathcal{U}$ and $\mathfrak{O} = \{0, 1, 2, \dots, N-1\}$ be an ordered grades set with $N \in \{2, 3, \cdots\}$. A pair $(\gamma, ((J, \mathcal{V}, N)))$ is said to be IFN-SS where $(J, \mathcal{V}, N)$ is an N-SS on $\mathfrak{B}$, and γ maps each parameter $v \in \mathcal{V}$ with an IFS $\mathcal{B}$ on $J(v) \subseteq P(\mathfrak{B} \times \mathfrak{O})$. This means that $\gamma: \mathcal{V} \to IF^{\mathfrak{B} \times \mathfrak{O}}$, where $IF^{\mathfrak{B} \times \mathfrak{O}}$ is the collection of all IFS over $\mathfrak{B} \times \mathfrak{O}$.

Definition 19: [41] Let $(\gamma_1, (J_1, \mathcal{V}, N_1))$ and $(\gamma_1, (J_2, \mathcal{V}_2, N_2))$ be two IFN-SS. Then their restricted union and intersection are designated and defined by:

$$(\gamma_1, (J_1, \mathcal{V}_1, N_1)) \cup_{\mathbb{R}} (\gamma_2, (J_2, \mathcal{V}_2, N_2)) = (\vartheta, \mathcal{V}_1 \cap \mathcal{V}_2, \max(N_1, N_2)), \text{ where}$$

$$\forall v_k \in \mathcal{V}_1 \cap \mathcal{V}_2 \ \&\ \mathfrak{b}_j \in \mathfrak{B}, \left(\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}\right), a, b\right) \in \vartheta(v_k)$$

$$\Leftrightarrow \left(\mathfrak{o}_{v_k} = \max\left(\mathfrak{o}_{v_k}^1, \mathfrak{o}_{v_k}^2\right), a = \max\left(\gamma_1^M\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}^1\right), \gamma_2^M\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}^2\right)\right), b\right.$$
$$= \min\left(\gamma_1^N\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}^1\right), \gamma_2^N\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}^2\right)\right) if\ \left(\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}\right), \gamma_1^M\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}^1\right), \gamma_1^N\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}^1\right)\right)$$
$$\left.\in J_1(v_k) \text{ and } \left(\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}\right), \gamma_1^M\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}^1\right), \gamma_1^N\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}^1\right)\right) \in J_1(v_k)\right)$$

$$(\gamma_1, (J_1, \mathcal{V}_1, N_1)) \cap_{\mathbb{R}} (\gamma_2, (J_2, \mathcal{V}_2, N_2)) = (\varepsilon, \mathcal{V}_1 \cap \mathcal{V}_2, \min(N_1, N_2)), \text{ where}$$

$$\forall v_k \in \mathcal{V}_1 \cap \mathcal{V}_2 \ \&\ \mathfrak{b}_j \in \mathfrak{B}, \left(\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}\right), a, b\right) \in \varepsilon(v_k)$$

$$\Leftrightarrow \left(\mathfrak{o}_{v_k} = \min\left(\mathfrak{o}_{v_k}^1, \mathfrak{o}_{v_k}^2\right), a = \min\left(\gamma_1^M\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}^1\right), \gamma_2^M\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}^2\right)\right), b\right.$$
$$= \max\left(\gamma_1^N\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}^1\right), \gamma_2^N\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}^2\right)\right) if\ \left(\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}\right), \gamma_1^M\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}^1\right), \gamma_1^N\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}^1\right)\right)$$
$$\left.\in J_1(v_k) \text{ and } \left(\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}\right), \gamma_1^M\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}^1\right), \gamma_1^N\left(\mathfrak{b}_j, \mathfrak{o}_{v_k}^1\right)\right) \in J_1(v_k)\right)$$

Definition 20: [41] Let $(\gamma_1, (J_1, \mathcal{V}_1, N_1))$ and $(\gamma_2, (J_2, \mathcal{V}_2, N_2))$ be two IFN-SSs. Then their extended union and intersection are denoted and defined by:

$$(\gamma_1, (J_1, \mathcal{V}_1, N_1)) \cup_{\mathbb{E}} (\gamma_2, (J_2, \mathcal{V}_2, N_2)) = (\vartheta, \mathcal{V}_1 \cup \mathcal{V}_2, \max(N_1, N_2)) \text{ where } \forall\ v_k \in \mathcal{V}_1 \cup \mathcal{V}_2, \mathfrak{b}_j \in \mathfrak{B}$$

$$\vartheta(\mathcal{v}_k) = \begin{cases} \gamma_1(\mathcal{v}_k) & if\ \mathcal{v}_k \in \mathcal{V}_1 \backslash \mathcal{V}_2 \\ \gamma_2(\mathcal{v}_k) & if\ \mathcal{v}_k \in \mathcal{V}_2 \backslash \mathcal{V}_1 \\ \left((\mathfrak{b}_j, \mathfrak{o}_{\mathcal{v}_k}), a, b\right) & \begin{pmatrix} s.t\ \mathfrak{o}_{\mathcal{v}_k} = \max(\mathfrak{o}^1_{\mathcal{v}_k}, \mathfrak{o}^2_{\mathcal{v}_k}), a = \max\left(\gamma_1^M(\mathfrak{b}_j, \mathfrak{o}^1_{\mathcal{v}_k}), \gamma_2^M(\mathfrak{b}_j, \mathfrak{o}^2_{\mathcal{v}_k})\right), \\ b = \min\left(\gamma_1^N(\mathfrak{b}_j, \mathfrak{o}^1_{\mathcal{v}_k}), \gamma_2^N(\mathfrak{b}_j, \mathfrak{o}^2_{\mathcal{v}_k})\right) if\ \left((\mathfrak{b}_j, \mathfrak{o}_{\mathcal{v}_k}), \gamma_1^M(\mathfrak{b}_j, \mathfrak{o}^1_{\mathcal{v}_k}), \gamma_1^N(\mathfrak{b}_j, \mathfrak{o}^1_{\mathcal{v}_k})\right) \\ \in J_1(\mathcal{v}_k) \text{and } \left((\mathfrak{b}_j, \mathfrak{o}_{\mathcal{v}_k}), \gamma_1^M(\mathfrak{b}_j, \mathfrak{o}^1_{\mathcal{v}_k}), \gamma_1^N(\mathfrak{b}_j, \mathfrak{o}^1_{\mathcal{v}_k})\right) \in J_1(\mathcal{v}_k) \end{pmatrix} \end{cases}$$

$\left(\gamma_1, (J_1, \mathcal{V}_1, N_1)\right) \cap_{\mathbb{E}} \left(\gamma_1, (J_2, \mathcal{V}_2, N_2)\right) = (\varepsilon, \mathcal{V}_1 \cup \mathcal{V}_2, \min(N_1, N_2))$ where $\forall\ \mathcal{v}_k \in \mathcal{V}_1 \cup \mathcal{V}_2, \mathfrak{b}_j \in \mathfrak{B}$

$$\varepsilon(\mathcal{v}_k) = \begin{cases} \gamma_1(\mathcal{v}_k) & if\ \mathcal{v}_k \in \mathcal{V}_1 \backslash \mathcal{V}_2 \\ \gamma_2(\mathcal{v}_k) & if\ \mathcal{v}_k \in \mathcal{V}_2 \backslash \mathcal{V}_1 \\ \left((\mathfrak{b}_j, \mathfrak{o}_{\mathcal{v}_k}), a, b\right) & \begin{pmatrix} s.t\ \mathfrak{o}_{\mathcal{v}_k} = \min(\mathfrak{o}^1_{\mathcal{v}_k}, \mathfrak{o}^2_{\mathcal{v}_k}), a = \min\left(\gamma_1^M(\mathfrak{b}_j, \mathfrak{o}^1_{\mathcal{v}_k}), \gamma_2^M(\mathfrak{b}_j, \mathfrak{o}^2_{\mathcal{v}_k})\right), \\ b = \max\left(\gamma_1^N(\mathfrak{b}_j, \mathfrak{o}^1_{\mathcal{v}_k}), \gamma_2^N(\mathfrak{b}_j, \mathfrak{o}^2_{\mathcal{v}_k})\right) if\ \left((\mathfrak{b}_j, \mathfrak{o}_{\mathcal{v}_k}), \gamma_1^M(\mathfrak{b}_j, \mathfrak{o}^1_{\mathcal{v}_k}), \gamma_1^N(\mathfrak{b}_j, \mathfrak{o}^1_{\mathcal{v}_k})\right) \\ \in J_1(\mathcal{v}_k) \text{and } \left((\mathfrak{b}_j, \mathfrak{o}_{\mathcal{v}_k}), \gamma_1^M(\mathfrak{b}_j, \mathfrak{o}^1_{\mathcal{v}_k}), \gamma_1^N(\mathfrak{b}_j, \mathfrak{o}^1_{\mathcal{v}_k})\right) \in J_1(\mathcal{v}_k) \end{pmatrix} \end{cases}$$

III. Construction of Complex Intuitionistic Fuzzy N-Soft Sets

In this part of the manuscript, we interpret the novel idea of CIFN-SSs and some basic properties of CIFN-SSs. Further, we will discuss its functional representation in this section. Throughout this article, $\mathfrak{B}$ be a universe set, $\mathcal{U}$ be a set of parameters and $\rho = (\mu^M, \mu^N)$, where $\mu^M = \gamma^M e^{i2\pi(\omega_{\gamma^M})}$ be a CMG and $\mu^N = \gamma^N e^{i2\pi(\omega_{\gamma^N})}$ be a (CNMG).

We interpret the concept of CIFN-SSs.

Definition 21: Let $\mathfrak{B} \neq \emptyset$ be a set of objects, $\mathcal{U}$ be the set of parameters and $\mathcal{V} \subseteq \mathcal{U}$. Let $\mathfrak{O} = \{0, 1, 2, \ldots, N-1\}$ be an ordered grades set with $N \in \{2, 3, \cdots\}$. A pair $(\rho, \mathcal{H})$ is said to be CIFN-SS when $\mathcal{H} = (J, \mathcal{V}, N)$ is an N-SS on $\mathfrak{B}$, and ρ maps each parameter $\mathcal{v} \in \mathcal{V}$ with a CIFS $\mathcal{B}$ on $J(\mathcal{v}) \subseteq P(\mathfrak{B} \times \mathfrak{O})$. This means that $\rho: \mathcal{V} \to CIJ^{\mathfrak{B} \times \mathfrak{O}}$, where $CIJ^{\mathfrak{B} \times \mathfrak{O}}$ represent the collection of all CIFS over $\mathfrak{B} \times \mathfrak{O}$.

With the help of the following example, we will explain the notion of our proposed model. Moreover, we will see a helpful tabular representation for CIFN-SSs.

Example 1: Let $\mathfrak{B} = \{\mathfrak{b}_1, \mathfrak{b}_2, \mathfrak{b}_3, \mathfrak{b}_4, \mathfrak{b}_5, \mathfrak{b}_6\}$ be set of 6 public transport buses and $\mathcal{V} = \{\mathcal{v}_1 = Income\ of\ bus, \mathcal{v}_2 = number\ of\ passanger, \mathcal{v}_3 = short\ route, \mathcal{v}_4 = saving\ fuel\}$ be set of parameters, based on these parameters an expert allocate grading to the buses. The information obtained from real data is given in table (1)

Table 1. Information obtained from real data for example 1.

$\mathfrak{B}$	$\mathcal{v}_1$	$\mathcal{v}_2$	$\mathcal{v}_3$	$\mathcal{v}_4$
$\mathfrak{b}_1$	××	×××	×	×
$\mathfrak{b}_2$	××××	××	××	××
$\mathfrak{b}_3$	∘	×	×××	××××
$\mathfrak{b}_4$	××××	×××	××××	×××××
$\mathfrak{b}_5$	∘	×××××	∘	××
$\mathfrak{b}_6$	××	×	×××	××××

where

Five cross marks represent ‘ Outstanding’,
Four cross marks represent ‘Excellent’,
Three cross marks represent ‘Very Good’,
Two cross marks represent ‘Good’,
One cross mark represents ‘Normal’,
The hole represents ‘Poor’,
The set $\mathfrak{O} = \{0, 1, 2, 3, 4, 5\}$ can undoubtedly link with the cross marks presented in table (1), where
0 denotes “∘”,
1 denotes “×”,
2 denotes “××”,
3 denotes “×××”,
4 denotes “××××”,
5 denotes “×××××”,
The tabular representation of 6-SS is described in table (2).

Table 2. The tabular form of 6-SS interpreted in for example 1.

$\mathfrak{B}$	$\mathcal{v}_1$	$\mathcal{v}_2$	$\mathcal{v}_3$	$\mathcal{v}_4$
$\mathfrak{b}_1$	2	3	1	1
$\mathfrak{b}_2$	4	2	2	2
$\mathfrak{b}_3$	0	1	3	4
$\mathfrak{b}_4$	4	3	4	5
$\mathfrak{b}_5$	0	5	0	2
$\mathfrak{b}_6$	2	1	3	4

The grading criteria for CMG and CNMG of elements of the set $\mathfrak{B}$ are presented below.
$0.0 \leq \Delta\mu^M(\mathfrak{b}) < 0.15$ when $\mathfrak{o} = 0$;
$0.15 \leq \Delta\mu^M(\mathfrak{b}) < 0.35$ when $\mathfrak{o} = 1$;
$0.35 \leq \Delta\mu^M(\mathfrak{b}) < 0.55$ when $\mathfrak{o} = 2$;
$0.55 \leq \Delta\mu^M(\mathfrak{b}) < 0.75$ when $\mathfrak{o} = 3$;
$0.75 \leq \Delta\mu^M(\mathfrak{b}) < 0.90$ when $\mathfrak{o} = 4$;
$0.90 \leq \Delta\mu^M(\mathfrak{b}) \leq 1.0$ when $\mathfrak{o} = 5$;
Where $\Delta\mu^M(\mathfrak{b}) = \frac{\gamma^M(\mathfrak{b})+\omega_{\gamma^M(\mathfrak{b})}}{2}$, and $0 \leq \mu^M + \mu^N \leq 1$. Therefore, the CIF6-SS are given below by using def (21).

$$\rho(v_1) = \begin{Bmatrix} \left((\mathfrak{b}_1, 2), \left(0.4e^{i2\pi(0.45)}, 0.2e^{i2\pi(0.5)}\right)\right), \left((\mathfrak{b}_2, 4), \left(0.8e^{i2\pi(0.75)}, 0.1e^{i2\pi(0.2)}\right)\right), \\ \left((\mathfrak{b}_3, 0), \left(0.05e^{i2\pi(0.1)}, 0.5e^{i2\pi(0.6)}\right)\right), \left((\mathfrak{b}_4, 4), \left(0.75e^{i2\pi(0.88)}, 0.2e^{i2\pi(0.1)}\right)\right) \\ \left((\mathfrak{b}_5, 0), \left(0.1e^{i2\pi(0.14)}, 0.4e^{i2\pi(0.36)}\right)\right), \left((\mathfrak{b}_6, 2), \left(0.48e^{i2\pi(0.5)}, 0.3e^{i2\pi(0.3)}\right)\right) \end{Bmatrix}$$

$$\rho(v_2) = \begin{Bmatrix} \left((\mathfrak{b}_1, 3), \left(0.6e^{i2\pi(0.7)}, 0.1e^{i2\pi(0.15)}\right)\right), \left((\mathfrak{b}_2, 2), \left(0.37e^{i2\pi(0.45)}, 0.6e^{i2\pi(0.3)}\right)\right), \\ \left((\mathfrak{b}_3, 1), \left(0.2e^{i2\pi(0.28)}, 0.7e^{i2\pi(0.5)}\right)\right), \left((\mathfrak{b}_4, 3), \left(0.9e^{i2\pi(0.4)}, 0.09e^{i2\pi(0.5)}\right)\right) \\ \left((\mathfrak{b}_5, 5), \left(0.9e^{i2\pi(0.92)}, 0.05\ e^{i2\pi(0.03)}\right)\right), \left((\mathfrak{b}_6, 1), \left(0.23e^{i\pi(0.33)}, 0.52e^{i2\pi(0.63)}\right)\right) \end{Bmatrix}$$

$$\rho(v_3) = \begin{Bmatrix} \left((\mathfrak{b}_1, 1), \left(0.2e^{i2\pi(0.31)}, 0.7e^{i2\pi(0.4)}\right)\right), \left((\mathfrak{b}_2, 2), \left(0.42e^{i2\pi(0.5)}, 0.4e^{i2\pi(0.4)}\right)\right), \\ \left((\mathfrak{b}_3, 3), \left(0.8e^{i2\pi(0.6)}, 0.1e^{i2\pi(0.3)}\right)\right), \left((\mathfrak{b}_4, 4), \left(0.95e^{i2\pi(0.8)}, 0.04e^{i2\pi(0.15)}\right)\right) \\ \left((\mathfrak{b}_5, 0), \left(0.07e^{i2\pi(0.1)}, 0.9e^{i2\pi(0.7)}\right)\right), \left((\mathfrak{b}_6, 3), \left(0.6e^{i2\pi(0.57)}, 0.25e^{i2\pi(0.4)}\right)\right) \end{Bmatrix}$$

$$\rho(v_4) = \begin{Bmatrix} \left((\mathfrak{b}_1, 1), \left(0.18e^{i2\pi(0.29)}, 0.45e^{i2\pi(0.65)}\right)\right), \left((\mathfrak{b}_2, 2), \left(0.38e^{i2\pi(0.48)}, 0.33e^{i2\pi(0.23)}\right)\right), \\ \left((\mathfrak{b}_3, 4), \left(0.77e^{i2\pi(0.85)}, 0.17e^{i2\pi(0.1)}\right)\right), \left((\mathfrak{b}_4, 5), \left(0.96e^{i2\pi(0.96)}, 0.03\ e^{i2\pi(0.03)}\right)\right) \\ \left((\mathfrak{b}_5, 2), \left(0.35e^{i2\pi(0.53)}, 0.61\ e^{i2\pi(0.27)}\right)\right), \left((\mathfrak{b}_6, 4), \left(0.82e^{i2\pi(79)}, 0.1e^{i2\pi(0.15)}\right)\right) \end{Bmatrix}$$

The tabular representation of CIF6-SS is given in table (3).

Table 3. The tabular representation of CIF6-SS interpreted in example 1.

$(\rho, (J, \mathcal{V}, 6))$	v_1	v_2	v_3	v_4
$\mathfrak{b}_1$	$2, \begin{pmatrix} 0.4e^{i2\pi(0.45)}, \\ 0.2e^{i2\pi(0.5)} \end{pmatrix}$	$3, \begin{pmatrix} 0.6e^{i2\pi(0.7)}, \\ 0.1e^{i2\pi(0.15)} \end{pmatrix}$	$1, \begin{pmatrix} 0.2e^{i2\pi(0.31)}, \\ 0.7e^{i2\pi(0.4)} \end{pmatrix}$	$1, \begin{pmatrix} 0.18e^{i2\pi(0.29)}, \\ 0.45e^{i2\pi(0.65)} \end{pmatrix}$
$\mathfrak{b}_2$	$4, \begin{pmatrix} 0.8e^{i2\pi(0.75)}, \\ 0.1e^{i2\pi(0.2)} \end{pmatrix}$	$2, \begin{pmatrix} 0.37e^{i2\pi(0.45)}, \\ 0.6e^{i2\pi(0.3)} \end{pmatrix}$	$2, \begin{pmatrix} 0.42e^{i2\pi(0.5)}, \\ 0.4e^{i2\pi(0.4)} \end{pmatrix}$	$2, \begin{pmatrix} 0.38e^{i2\pi(0.48)}, \\ 0.33e^{i2\pi(0.23)} \end{pmatrix}$
$\mathfrak{b}_3$	$0, \begin{pmatrix} 0.05e^{i2\pi(0.1)}, \\ 0.5e^{i2\pi(0.6)} \end{pmatrix}$	$1, \begin{pmatrix} 0.2e^{i2\pi(0.28)}, \\ 0.7e^{i2\pi(0.5)} \end{pmatrix}$	$3, \begin{pmatrix} 0.8e^{i2\pi(0.6)}, \\ 0.1e^{i2\pi(0.3)} \end{pmatrix}$	$4, \begin{pmatrix} 0.77e^{i2\pi(0.85)}, \\ 0.17e^{i2\pi(0.1)} \end{pmatrix}$
$\mathfrak{b}_4$	$4, \begin{pmatrix} 0.75e^{i2\pi(0.88)}, \\ 0.2e^{i2\pi(0.1)} \end{pmatrix}$	$3, \begin{pmatrix} 0.9e^{i2\pi(0.4)}, \\ 0.09e^{i2\pi(0.5)} \end{pmatrix}$	$4, \begin{pmatrix} 0.95e^{i2\pi(0.8)}, \\ 0.04e^{i2\pi(0.15)} \end{pmatrix}$	$5, \begin{pmatrix} 0.96e^{i2\pi(0.96)}, \\ 0.03e^{i2\pi(0.03)} \end{pmatrix}$
$\mathfrak{b}_5$	$0, \begin{pmatrix} 0.1e^{i2\pi(0.14)}, \\ 0.4e^{i2\pi(0.36)} \end{pmatrix}$	$5, \begin{pmatrix} 0.9e^{i2\pi(0.92)}, \\ 0.05e^{i2\pi(0.03)} \end{pmatrix}$	$0, \begin{pmatrix} 0.07e^{i2\pi(0.1)}, \\ 0.9e^{i2\pi(0.7)} \end{pmatrix}$	$2, \begin{pmatrix} 0.35e^{i2\pi(0.53)}, \\ 0.61e^{i2\pi(0.27)} \end{pmatrix}$
$\mathfrak{b}_6$	$2, \begin{pmatrix} 0.48e^{i2\pi(0.5)}, \\ 0.3e^{i2\pi(0.3)} \end{pmatrix}$	$1, \begin{pmatrix} 0.23e^{i2\pi(0.33)}, \\ 0.52e^{i2\pi(0.63)} \end{pmatrix}$	$3, \begin{pmatrix} 0.6e^{i2\pi(0.57)}, \\ 0.25e^{i2\pi(0.4)} \end{pmatrix}$	$4, \begin{pmatrix} 0.82e^{i2\pi(0.79)}, \\ 0.1e^{i2\pi(0.15)} \end{pmatrix}$

For better understanding, let assignment $4, \begin{pmatrix} 0.82e^{i2\pi(0.79)}, \\ 0.1e^{i2\pi(0.15)} \end{pmatrix}$ in the bottom-right cell of table (3) shows that when 4 is the assessment grade w.r.t saving fuel, the bus $\mathfrak{b}_6$ belongs to the parameterized sub-universe with $0.82e^{i2\pi(0.79)}$ CMG and $0.1e^{i2\pi(0.15)}$ NMG.

Remark 1:

- Any CIF2-SS $(\rho, (J, \mathcal{V}, 2))$ can be linked with CIFSS. We associate CIF2-SS $\rho: \mathcal{V} \to CIJ^{\mathfrak{B}\times\{0,1\}}$, with a CIFSS $(J', \mathcal{V})$ where
$J'(\boldsymbol{v_k}) = \left\{\left(\mathfrak{b}, \mu^M(\mathfrak{b}), \mu^N(\mathfrak{b})\right) \mid \left((\mathfrak{b}, 1), \left(\mu^M(\mathfrak{b}), \mu^N(\mathfrak{b})\right)\right) \in J(v_k)\right\}$,

and $CIJ^{\mathfrak{B}\times\{0,1\}}$ represent the collection of all CIF subsets over $\mathfrak{B}\times\{0,1\}$.

- In def (21), grade $0 \in \mathfrak{D}$ describes the lowest score, it doesn't mean that the information is not complete or absence of assessment.
- One can set any scale for membership values to select a grade. It is not mandatory to use the same scale which we established in example (1).
- We can take any CIFN-SS as complex intuitionistic N+1-SS. Further, we can be considered CIFN-SS as CIFN^*-SS with $N^* > N$. For example, the CIF6-SS in example (1) can be considered as CIF7-SS over the same fuzzy parameterizations and parameters.

Definition 22: A CIFN-SS $(\rho,\mathcal{H})$ is said to be efficient if $\rho(\mathscr{v}_k) = \left(\left(\mathfrak{b}_j, N-1\right), \begin{pmatrix} 1.0e^{i2\pi(1.0)}, \\ 0.0e^{i2\pi(0.0)} \end{pmatrix}\right)$ for some $\mathscr{v}_k \in \mathcal{V}, \mathfrak{b}_j \in \mathfrak{B}$,

Example 2: One can note that the CIF6-SS given in example (1) is not efficient.

Definition 23: If $(\rho,\mathcal{H})$ is an efficient CIFN-SS over $\mathfrak{B}$, then the minimized efficient CIFV-SS of CIFN-SS on $\mathfrak{B}$ is designated by $(\rho_V,\mathcal{H}_V)$, where $\mathcal{H}_V = (J_V,\mathcal{V},V)$ is given as $V = \max_{j,k} J(\mathscr{v}_k)(\mathfrak{b}_j) + 1$, $\mu^M(\mathfrak{b}_j), \mu^N(\mathfrak{b}_j) = \left(1.0e^{i2\pi(1.0)}, 0.0e^{i2\pi(0.0)}\right)$, $J_v(\mathscr{v}_k)(\mathfrak{b}_j) = J(\mathscr{v}_k)(\mathfrak{b}_j)$ for all $\mathscr{v}_k \in \mathcal{V}, \mathfrak{b}_j \in \mathfrak{B}$.

Proposition 1: Every efficient CIFN-SS corresponds with minimized efficient CIFN-SS.

Proof: Use definition (22) and definition (23).

Now we give some algebraic properties linked with CIFN-SS. We do start with equality

Definition 24: Let $(\rho_1,\mathcal{H}_1)$ and $(\rho_2,\mathcal{H}_2)$ be two CIFN-SSs over $\mathfrak{B}$. Then $(\rho_1,\mathcal{H}_1)$ and $(\rho_2,\mathcal{H}_2)$ are called equal iff $\rho_1 = \rho_2$ $\left((\mu_1^M,\mu_1^N) = (\mu_2^M,\mu_2^N) \Rightarrow \mu_1^M = \mu_2^M, \mu_1^N = \mu_2^N\right)$ and $\mathcal{H}_1 = \mathcal{H}_2$.

We interpret the complement of CIFN-SS as below

Definition 25: Let $(\rho,\mathcal{H})$ be a CIFN-SS over $\mathfrak{B}$, where $\mathcal{H} = (J,\mathcal{V},N)$ is an N-SS. Then the CIF weak complement of the CIFN-SS is designated by $\left(\rho^c,(J^c,\mathcal{V},N)\right)$, where $(J^c,\mathcal{V},N)$ is a weak complement, i.e. $J^c(\mathscr{v}_k) \cap J(\mathscr{v}_k) = \emptyset, \forall \mathscr{v}_k \in \mathcal{V}$, and ρ^c maps any parameter in $\mathcal{V}$ with a CIFS $\mathcal{B}$ on $J^c(\mathscr{v}_k)$, i.e. $\rho^c(\mathscr{v}_k) = \left\{\begin{pmatrix} (\mathfrak{b}_j,\mathfrak{o}_{\mathscr{v}_k}), \\ \mu^N(\mathfrak{b}_j,\mathfrak{o}_{\mathscr{v}_k}), \mu^M(\mathfrak{b}_j,\mathfrak{o}_{\mathscr{v}_k}) \end{pmatrix} \mid \left((\mathfrak{b}_j,\mathfrak{o}_{\mathscr{v}_k}) \in \mathfrak{B}\times\mathfrak{D}\right)\right\}$.

Example 3: A CIF weak complement of CIF6-SS in example (1) is presented in table (4).

Table 4. The CIF weak complement of CIF6-SS interpreted in example 1.

$(\rho^c,(J^c,\mathcal{V},6))$	$\mathscr{v}_1$	$\mathscr{v}_2$	$\mathscr{v}_3$	$\mathscr{v}_4$
$\mathfrak{b}_1$	$1,\left(0.2e^{i2\pi(0.5)}, 0.4e^{i2\pi(0.45)}\right)$	$4,\left(0.1e^{i2\pi(0.15)}, 0.6e^{i2\pi(0.7)}\right)$	$5,\left(0.7e^{i2\pi(0.4)}, 0.2e^{i2\pi(0.31)}\right)$	$4,\left(0.45e^{i2\pi(0.65)}, 0.18e^{i2\pi(0.29)}\right)$
$\mathfrak{b}_2$	$5,\left(0.1e^{i2\pi(0.2)}, 0.8e^{i2\pi(0.75)}\right)$	$0,\left(0.6e^{i2\pi(0.3)}, 0.37e^{i2\pi(0.45)}\right)$	$5,\left(0.4e^{i2\pi(0.4)}, 0.42e^{i2\pi(0.5)}\right)$	$2,\left(0.33e^{i2\pi(0.23)}, 0.38e^{i2\pi(0.48)}\right)$
$\mathfrak{b}_3$	$1,\left(0.5e^{i2\pi(0.6)}, 0.05e^{i2\pi(0.1)}\right)$	$0,\left(0.7e^{i2\pi(0.5)}, 0.2e^{i2\pi(0.28)}\right)$	$5,\left(0.1e^{i2\pi(0.3)}, 0.8e^{i2\pi(0.6)}\right)$	$1,\left(0.17e^{i2\pi(0.1)}, 0.77e^{i2\pi(0.85)}\right)$
$\mathfrak{b}_4$	$5,\left(0.2e^{i2\pi(0.1)}, 0.75e^{i2\pi(0.88)}\right)$	$4,\left(0.09e^{i2\pi(0.5)}, 0.9e^{i2\pi(0.4)}\right)$	$5,\left(0.04e^{i2\pi(0.15)}, 0.95e^{i2\pi(0.8)}\right)$	$2,\left(0.03e^{i2\pi(0.03)}, 0.96e^{i2\pi(0.96)}\right)$
$\mathfrak{b}_5$	$1,\left(0.4e^{i2\pi(0.36)}, 0.1e^{i2\pi(0.14)}\right)$	$4,\left(0.05e^{i2\pi(0.03)}, 0.9e^{i2\pi(0.92)}\right)$	$5,\left(0.9e^{i2\pi(0.7)}, 0.07e^{i2\pi(0.1)}\right)$	$1,\left(0.61e^{i2\pi(0.27)}, 0.35e^{i2\pi(0.53)}\right)$
$\mathfrak{b}_6$	$3,\left(0.3e^{i2\pi(0.3)}, 0.48e^{i2\pi(0.5)}\right)$	$0,\left(0.52e^{i2\pi(0.63)}, 0.23e^{i2\pi(0.33)}\right)$	$5,\left(0.25e^{i2\pi(0.4)}, 0.6e^{i2\pi(0.57)}\right)$	$5,\left(0.1e^{i2\pi(0.15)}, 0.82e^{i2\pi(0.79)}\right)$

Definition 26: Let $\left(\rho,(J,\mathcal{V},N)\right)$ be a CIFN-SS over $\mathfrak{B}$. Then the top CIF weak complement of $\left(\rho,(J,\mathcal{V},N)\right)$ is $\left(\rho^c,(J^{\mathbb{T}},\mathcal{V},N)\right)$, where

$$J^{\mathbb{T}}(\mathscr{v}_k) = \begin{cases} \begin{pmatrix} (\mathfrak{b}_j, N-1), \\ \left(\mu^N(\mathfrak{b}_j,\mathfrak{o}_{\mathscr{v}_k}), \mu^M(\mathfrak{b}_j,\mathfrak{o}_{\mathscr{v}_k})\right) \end{pmatrix} & if\ \mathfrak{o}_{\mathscr{v}_k} < N-1 \\ \begin{pmatrix} (\mathfrak{b}_j, 0), \\ \left(\mu^N(\mathfrak{b}_j,\mathfrak{o}_{\mathscr{v}_k}), \mu^M(\mathfrak{b}_j,\mathfrak{o}_{\mathscr{v}_k})\right) \end{pmatrix} & if\ \mathfrak{o}_{\mathscr{v}_k} = N-1 \end{cases}$$

Example 4: The top CIF weak complement of the CIF6-SS in example (1) is presented in table (5).

Table 5. The top CIF weak complement of the CIF6-SS interpreted in example 1.

$\left(\rho^c,(J^c,\mathcal{V},6)\right)$	$\mathscr{v}_1$	$\mathscr{v}_2$	$\mathscr{v}_3$	$\mathscr{v}_4$
$\mathfrak{b}_1$	$5,\left(0.2e^{i2\pi(0.5)}, 0.4e^{i2\pi(0.45)}\right)$	$5,\left(0.1e^{i2\pi(0.15)}, 0.6e^{i2\pi(0.7)}\right)$	$5,\left(0.7e^{i2\pi(0.4)}, 0.2e^{i2\pi(0.31)}\right)$	$5,\left(0.45e^{i2\pi(0.65)}, 0.18e^{i2\pi(0.29)}\right)$
$\mathfrak{b}_2$	$5,\left(0.1e^{i2\pi(0.2)}, 0.8e^{i2\pi(0.75)}\right)$	$5,\left(0.6e^{i2\pi(0.3)}, 0.37e^{i2\pi(0.45)}\right)$	$5,\left(0.4e^{i2\pi(0.4)}, 0.42e^{i2\pi(0.5)}\right)$	$5,\left(0.33e^{i2\pi(0.23)}, 0.38e^{i2\pi(0.48)}\right)$
$\mathfrak{b}_3$	$5,\left(0.5e^{i2\pi(0.6)}, 0.05e^{i2\pi(0.1)}\right)$	$5,\left(0.7e^{i2\pi(0.5)}, 0.2e^{i2\pi(0.28)}\right)$	$5,\left(0.1e^{i2\pi(0.3)}, 0.8e^{i2\pi(0.6)}\right)$	$5,\left(0.17e^{i2\pi(0.1)}, 0.77e^{i2\pi(0.85)}\right)$
$\mathfrak{b}_4$	$5,\left(0.2e^{i2\pi(0.1)}, 0.75e^{i2\pi(0.88)}\right)$	$5,\left(0.09e^{i2\pi(0.5)}, 0.9e^{i2\pi(0.4)}\right)$	$5,\left(0.04e^{i2\pi(0.15)}, 0.95e^{i2\pi(0.8)}\right)$	$0,\left(0.03e^{i2\pi(0.03)}, 0.96e^{i2\pi(0.96)}\right)$
$\mathfrak{b}_5$	$5,\left(0.4e^{i2\pi(0.36)}, 0.1e^{i2\pi(0.14)}\right)$	$0,\left(0.05e^{i2\pi(0.03)}, 0.9e^{i2\pi(0.92)}\right)$	$5,\left(0.9e^{i2\pi(0.7)}, 0.07e^{i2\pi(0.1)}\right)$	$5,\left(0.61e^{i2\pi(0.27)}, 0.35e^{i2\pi(0.53)}\right)$
$\mathfrak{b}_6$	$5,\left(0.3e^{i2\pi(0.3)}, 0.48e^{i2\pi(0.5)}\right)$	$5,\left(0.52e^{i2\pi(0.63)}, 0.23e^{i2\pi(0.33)}\right)$	$5,\left(0.25e^{i2\pi(0.4)}, 0.6e^{i2\pi(0.57)}\right)$	$5,\left(0.1e^{i2\pi(0.15)}, 0.82e^{i2\pi(0.79)}\right)$

Definition 27: Let $\left(\rho,(J,\mathcal{V},N)\right)$ be a CIFN-SS over $\mathfrak{B}$. Then the bottom CIF weak complement of $\left(\rho,(J,\mathcal{V},N)\right)$ is $\left(\rho^c,(J^{\mathbb{B}},\mathcal{V},N)\right)$, where

$$J^{\mathbb{B}}(\mathscr{v}_k) = \begin{cases} \left(\begin{array}{c}(\mathfrak{b}_j,0),\\ \left(\mu^N(\mathfrak{b}_j,\mathfrak{o}_{\mathscr{v}_k}),\mu^M(\mathfrak{b}_j,\mathfrak{o}_{\mathscr{v}_k})\right)\end{array}\right) & if\ \mathfrak{o}_{\mathscr{v}_k}>0 \\ \left(\begin{array}{c}(\mathfrak{b}_j,N-1),\\ \left(\mu^N(\mathfrak{b}_j,\mathfrak{o}_{\mathscr{v}_k}),\mu^M(\mathfrak{b}_j,\mathfrak{o}_{\mathscr{v}_k})\right)\end{array}\right) & if\ \mathfrak{o}_{\mathscr{v}_k}=0 \end{cases}$$

Example 5: The bottom CIF weak complement of the CIF6-SS in example (1) is presented in table (6).

Table 6. The bottom CIF weak complement of the CIF6-SS interpreted in example 1.

$(\boldsymbol{\rho^c},(\boldsymbol{J^c},\boldsymbol{\mathcal{V}},\boldsymbol{6}))$	$\boldsymbol{\mathscr{v}_1}$	$\boldsymbol{\mathscr{v}_2}$	$\boldsymbol{\mathscr{v}_3}$	$\boldsymbol{\mathscr{v}_4}$
$\mathfrak{b}_1$	$0,\left(0.2e^{i2\pi(0.5)}, 0.4e^{i2\pi(0.45)}\right)$	$0,\left(0.1e^{i2\pi(0.15)}, 0.6e^{i2\pi(0.7)}\right)$	$0,\left(0.7e^{i2\pi(0.4)}, 0.2e^{i2\pi(0.31)}\right)$	$0,\left(0.45e^{i2\pi(0.65)}, 0.18e^{i2\pi(0.29)}\right)$
$\mathfrak{b}_2$	$0,\left(0.1e^{i2\pi(0.2)}, 0.8e^{i2\pi(0.75)}\right)$	$0,\left(0.6e^{i2\pi(0.3)}, 0.37e^{i2\pi(0.45)}\right)$	$0,\left(0.4e^{i2\pi(0.4)}, 0.42e^{i2\pi(0.5)}\right)$	$0,\left(0.33e^{i2\pi(0.23)}, 0.38e^{i2\pi(0.48)}\right)$
$\mathfrak{b}_3$	$5,\left(0.5e^{i2\pi(0.6)}, 0.05e^{i2\pi(0.1)}\right)$	$0,\left(0.7e^{i2\pi(0.5)}, 0.2e^{i2\pi(0.28)}\right)$	$0,\left(0.1e^{i2\pi(0.3)}, 0.8e^{i2\pi(0.6)}\right)$	$0,\left(0.17e^{i2\pi(0.1)}, 0.77e^{i2\pi(0.85)}\right)$
$\mathfrak{b}_4$	$0,\left(0.2e^{i2\pi(0.1)}, 0.75e^{i2\pi(0.88)}\right)$	$0,\left(0.09e^{i2\pi(0.5)}, 0.9e^{i2\pi(0.4)}\right)$	$0,\left(0.04e^{i2\pi(0.15)}, 0.95e^{i2\pi(0.8)}\right)$	$0,\left(0.03e^{i2\pi(0.03)}, 0.96e^{i2\pi(0.96)}\right)$
$\mathfrak{b}_5$	$5,\left(0.4e^{i2\pi(0.36)}, 0.1e^{i2\pi(0.14)}\right)$	$0,\left(0.05e^{i2\pi(0.03)}, 0.9e^{i2\pi(0.92)}\right)$	$5,\left(0.9e^{i2\pi(0.7)}, 0.07e^{i2\pi(0.1)}\right)$	$0,\left(0.61e^{i2\pi(0.27)}, 0.35e^{i2\pi(0.53)}\right)$
$\mathfrak{b}_6$	$0,\left(0.3e^{i2\pi(0.3)}, 0.48e^{i2\pi(0.5)}\right)$	$0,\left(0.52e^{i2\pi(0.63)}, 0.23e^{i2\pi(0.33)}\right)$	$0,\left(0.25e^{i2\pi(0.4)}, 0.6e^{i2\pi(0.57)}\right)$	$0,\left(0.1e^{i2\pi(0.15)}, 0.82e^{i2\pi(0.79)}\right)$

Next, we interpret the concept of union and intersection of CIFN-SS.

Definition 28: Let $(\rho_1,\mathcal{H}_1)$ and $(\rho_2,\mathcal{H}_2)$ be two CIFN-SSs over $\mathfrak{B}$, where $\mathcal{H}_1=(J_1,\mathcal{V}_1,N_1)$ and $\mathcal{H}_2=(J_2,\mathcal{V}_2,N_2)$ are N-SSs over $\mathfrak{B}$. Then their restricted intersection is designated and defined as $(\rho_1,\mathcal{H}_1)\cap_{\mathcal{R}}(\rho_2,\mathcal{H}_2)=(\psi,\mathcal{H}_1\cap_{\mathcal{R}}\mathcal{H}_2)$, where $\mathcal{H}_1\cap_{\mathcal{R}}\mathcal{H}_2=(S,\mathcal{V}_1\cap\mathcal{V}_2,\min(N_1,N_2))\ \forall\ \mathscr{v}_k\in\mathcal{V}_1\cap\mathcal{V}_2$ and $\mathfrak{b}_j\in\mathfrak{B}$, $\left(\left(\mathfrak{b}_j,\mathfrak{o}_{q_k}\right),x,y\right)\in\psi(\mathscr{v}_k)\iff\mathfrak{o}_{\mathscr{v}_k}=\min\left(\mathfrak{o}^1_{\mathscr{v}_k},\mathfrak{o}^2_{\mathscr{v}_k}\right)$ and $x=\min\left(\mu_1^M\left(\mathfrak{b}_j,\mathfrak{o}^1_{q_k}\right),\mu_2^M\left(\mathfrak{b}_j,\mathfrak{o}^2_{q_k}\right)\right)=\left(\min(\gamma_1^M,\gamma_2^M)\,e^{i2\pi\left(\min\left(\omega_{\gamma_1^M},\omega_{\gamma_2^M}\right)\right)}\right)$ $y=\max\left(\mu_1^N\left(\mathfrak{b}_j,\mathfrak{o}^1_{q_k}\right),\mu_2^N\left(\mathfrak{b}_j,\mathfrak{o}^2_{q_k}\right)\right)=\left(\max(\gamma_1^N,\gamma_2^N)\,e^{i2\pi\left(\max\left(\omega_{\gamma_1^N},\omega_{\gamma_2^N}\right)\right)}\right)$ if $\left(\left(\mathfrak{b}_j,\mathfrak{o}^1_{q_k}\right),\mu_1^M\left(\mathfrak{b}_j,\mathfrak{o}^1_{q_k}\right),\mu_1^N\left(\mathfrak{b}_j,\mathfrak{o}^1_{q_k}\right)\right)\in\rho_1(\mathscr{v}_k)$ and $\left(\left(\mathfrak{b}_j,\mathfrak{o}^2_{q_k}\right),\mu_2^M\left(\mathfrak{b}_j,\mathfrak{o}^2_{q_k}\right),\mu_2^N\left(\mathfrak{b}_j,\mathfrak{o}^2_{q_k}\right)\right)\in\rho_2(\mathscr{v}_k)$.

Example 6: Let$(\rho_1,\mathcal{H}_1)$ be CIF4-SS and $(\rho_2,\mathcal{H}_2)$ be CIF5-SS presented in tables (7) and (8). Then their restricted intersection $(\rho_1,\mathcal{H}_1)\cap_{\mathcal{R}}(\rho_2,\mathcal{H}_2)$ is presented in table (9).

Table 7. The tabular form of CIF4-SS considered in example 6.

$(\boldsymbol{\rho_1},(\boldsymbol{J_1},\boldsymbol{\mathcal{V}_1},\boldsymbol{4}))$	$\boldsymbol{\mathscr{v}_1}$	$\boldsymbol{\mathscr{v}_2}$	$\boldsymbol{\mathscr{v}_3}$
$\mathfrak{b}_1$	$0,\left(0.05e^{i2\pi(0.1)}, 0.5e^{i2\pi(0.6)}\right)$	$1,\left(0.3e^{i2\pi(0.38)}, 0.6e^{i2\pi(0.5)}\right)$	$1,\left(0.35e^{i2\pi(0.41)}, 0.5e^{i2\pi(0.4)}\right)$
$\mathfrak{b}_2$	$3,\left(0.85e^{i2\pi(0.85)}, 0.1e^{i2\pi(0.12)}\right)$	$2,\left(0.67e^{i2\pi(0.65)}, 0.2e^{i2\pi(0.3)}\right)$	$2,\left(0.62e^{i2\pi(0.6)}, 0.3e^{i2\pi(0.25)}\right)$
$\mathfrak{b}_3$	$2,\left(0.7e^{i2\pi(0.75)}, 0.2e^{i2\pi(0.15)}\right)$	$3,\left(0.8e^{i2\pi(0.9)}, 0.1e^{i2\pi(0.05)}\right)$	$3,\left(0.87e^{i2\pi(0.9)}, 0.1e^{i2\pi(0.05)}\right)$

Table 8. The tabular form of CIF5-SS considered in example 6.

$(\rho_2,(J_2,\mathcal{V}_2,5))$	$\mathcal{v}_1$	$\mathcal{v}_3$	$\mathcal{v}_4$
$\mathfrak{b}_1$	$4,\begin{pmatrix}0.85e^{i2\pi(0.88)},\\ 0.1e^{i2\pi(0.1)}\end{pmatrix}$	$4,\begin{pmatrix}0.95e^{i2\pi(0.8)},\\ 0.04e^{i2\pi(0.15)}\end{pmatrix}$	$4,\begin{pmatrix}0.96e^{i2\pi(0.96)},\\ 0.03e^{i2\pi(0.03)}\end{pmatrix}$
$\mathfrak{b}_2$	$0,\begin{pmatrix}0.15e^{i2\pi(0.14)},\\ 0.4e^{i2\pi(0.36)}\end{pmatrix}$	$1,\begin{pmatrix}0.35e^{i2\pi(0.25)},\\ 0.5e^{i2\pi(0.3)}\end{pmatrix}$	$3,\begin{pmatrix}0.7e^{i2\pi(0.73)},\\ 0.1e^{i2\pi(0.2)}\end{pmatrix}$
$\mathfrak{b}_3$	$3,\begin{pmatrix}0.68e^{i2\pi(0.6)},\\ 0.3e^{i2\pi(0.3)}\end{pmatrix}$	$2,\begin{pmatrix}0.55e^{i2\pi(0.57)},\\ 0.25e^{i2\pi(0.1)}\end{pmatrix}$	$1,\begin{pmatrix}0.3e^{i2\pi(0.29)},\\ 0.1e^{i2\pi(0.15)}\end{pmatrix}$

Table 9. The restricted intersection of CIF4-SS and CIF5-SS considered in example 6.

$(\psi,(S,\mathcal{V}_1\cap_{\mathcal{R}}\mathcal{V}_2,4))$	$\mathcal{v}_1$	$\mathcal{v}_3$
$\mathfrak{b}_1$	$0,\begin{pmatrix}0.05e^{i2\pi(0.1)},\\ 0.5e^{i2\pi(0.6)}\end{pmatrix}$	$1,\begin{pmatrix}0.35e^{i2\pi(0.41)},\\ 0.5e^{i2\pi(0.4)}\end{pmatrix}$
$\mathfrak{b}_2$	$0,\begin{pmatrix}0.15e^{i2\pi(0.14)},\\ 0.4e^{i2\pi(0.36)}\end{pmatrix}$	$1,\begin{pmatrix}0.35e^{i2\pi(0.25)},\\ 0.5e^{i2\pi(0.3)}\end{pmatrix}$
$\mathfrak{b}_3$	$2,\begin{pmatrix}0.68e^{i2\pi(0.6)},\\ 0.3e^{i2\pi(0.3)}\end{pmatrix}$	$2,\begin{pmatrix}0.55e^{i2\pi(0.57)},\\ 0.25e^{i2\pi(0.1)}\end{pmatrix}$

Definition 29: Let $(\rho_1,\mathcal{H}_1)$ and $(\rho_2,\mathcal{H}_2)$ be two CFIN-SSs over $\mathfrak{B}$, where $\mathcal{H}_1=(J_1,\mathcal{V}_1,N_1)$ and $\mathcal{H}_2=(J_2,\mathcal{V}_2,N_2)$ are N-SSs over $\mathfrak{B}$. Then their extended intersection is designated and given as $(\rho_1,\mathcal{H}_1)\cap_{\mathcal{E}}(\rho_2,\mathcal{H}_2)=(\xi,\mathcal{H}_1\cap_{\mathcal{E}}\mathcal{H}_2)$, where $\mathcal{H}_1\cap_{\mathcal{E}}\mathcal{H}_2=(G,\mathcal{V}_1\cap\mathcal{V}_2,\max(N_1,N_2))\ \forall\ \mathcal{v}_k\in\mathcal{V}_1\cap\mathcal{V}_2$ and $\mathfrak{b}_j\in\mathfrak{B}$, and $\xi(\mathcal{v}_k)$ is presented by

$$\xi(\mathcal{v}_k)=\begin{cases}\rho_1(\mathcal{v}_k) & if\ \mathcal{v}_k\in\mathcal{V}_1-\mathcal{V}_2\\ \rho_2(\mathcal{v}_k) & if\ \mathcal{v}_k\in\mathcal{V}_2-\mathcal{V}_1\\ \left(\left(\mathfrak{b}_j,\mathfrak{o}_{q_k}\right),x,y\right) & \left(\begin{array}{c} such\ that\ \mathfrak{o}_{\mathcal{v}_k}=\min(\mathfrak{o}^1_{\mathcal{v}_k},\mathfrak{o}^2_{\mathcal{v}_k})\ and\ x=\min\left(\mu_1^M\left(\mathfrak{b}_j,\mathfrak{o}^1_{q_k}\right),\mu_2^M\left(\mathfrak{b}_j,\mathfrak{o}^2_{q_k}\right)\right)\\ =\left(\min(\gamma_1^M,\gamma_2^M)\ e^{i2\pi\left(\min\left(\omega_{\gamma_1^M},\omega_{\gamma_2^M}\right)\right)}\right), y=\max\left(\mu_1^N\left(\mathfrak{b}_j,\mathfrak{o}^1_{q_k}\right),\mu_2^N\left(\mathfrak{b}_j,\mathfrak{o}^2_{q_k}\right)\right)\\ =\left(\max(\gamma_1^N,\gamma_2^N)\ e^{i2\pi\left(\max\left(\omega_{\gamma_1^N},\omega_{\gamma_2^N}\right)\right)}\right), where\\ \left(\left(\mathfrak{b}_j,\mathfrak{o}^1_{q_k}\right),\mu_1^M\left(\mathfrak{b}_j,\mathfrak{o}^1_{q_k}\right),\mu_1^N\left(\mathfrak{b}_j,\mathfrak{o}^1_{q_k}\right)\right)\in\rho_1(\mathcal{v}_k)\ and\\ \left(\left(\mathfrak{b}_j,\mathfrak{o}^2_{q_k}\right),\mu_2^M\left(\mathfrak{b}_j,\mathfrak{o}^2_{q_k}\right),\mu_2^N\left(\mathfrak{b}_j,\mathfrak{o}^2_{q_k}\right)\right)\in\rho_2(\mathcal{v}_k)\end{array}\right)\end{cases}$$

Example 7: Let$(\rho_1,\mathcal{H}_1)$ be CIF4-SS shown in table (7) and $(\rho_2,\mathcal{H}_2)$ be CIF5-SS shown in table (8). Then their extended intersection $(\rho_1,\mathcal{H}_1)\cap_{\mathcal{E}}(\rho_2,\mathcal{H}_2)$ is shown in table (10).

Table 10. The extended intersection of CIF4-SS displayed in table 7 and CIF5-SS displayed in table 8.

$(\xi,(G,\mathcal{V}_1\cap_{\mathcal{R}}\mathcal{V}_2,5))$	$\mathcal{v}_1$	$\mathcal{v}_2$	$\mathcal{v}_3$	$\mathcal{v}_4$
$\mathfrak{b}_1$	$0,\begin{pmatrix}0.05e^{i2\pi(0.1)},\\ 0.5e^{i2\pi(0.6)}\end{pmatrix}$	$1,\begin{pmatrix}0.3e^{i2\pi(0.38)},\\ 0.6e^{i2\pi(0.5)}\end{pmatrix}$	$1,\begin{pmatrix}0.35e^{i2\pi(0.41)},\\ 0.5e^{i2\pi(0.4)}\end{pmatrix}$	$4,\begin{pmatrix}0.96e^{i2\pi(0.96)},\\ 0.03e^{i2\pi(0.03)}\end{pmatrix}$
$\mathfrak{b}_2$	$0,\begin{pmatrix}0.15e^{i2\pi(0.14)},\\ 0.4e^{i2\pi(0.36)}\end{pmatrix}$	$2,\begin{pmatrix}0.67e^{i2\pi(0.65)},\\ 0.2e^{i2\pi(0.3)}\end{pmatrix}$	$1,\begin{pmatrix}0.35e^{i2\pi(0.25)},\\ 0.5e^{i2\pi(0.3)}\end{pmatrix}$	$3,\begin{pmatrix}0.7e^{i2\pi(0.73)},\\ 0.1e^{i2\pi(0.2)}\end{pmatrix}$
$\mathfrak{b}_3$	$2,\begin{pmatrix}0.68e^{i2\pi(0.6)},\\ 0.3e^{i2\pi(0.3)}\end{pmatrix}$	$3,\begin{pmatrix}0.8e^{i2\pi(0.9)},\\ 0.1e^{i2\pi(0.05)}\end{pmatrix}$	$2,\begin{pmatrix}0.55e^{i2\pi(0.57)},\\ 0.25e^{i2\pi(0.1)}\end{pmatrix}$	$1,\begin{pmatrix}0.3e^{i2\pi(0.29)},\\ 0.1e^{i2\pi(0.15)}\end{pmatrix}$

Definition 30: Let $(\rho_1,\mathcal{H}_1)$ and $(\rho_2,\mathcal{H}_2)$ be two CIFN-SSs over $\mathfrak{B}$, where $\mathcal{H}_1=(J_1,\mathcal{V}_1,N_1)$ and $\mathcal{H}_2=(J_2,\mathcal{V}_2,N_2)$ are N-SSs over $\mathfrak{B}$. Then their restricted union is designated and given as $(\rho_1,\mathcal{H}_1)\cup_{\mathcal{R}}(\rho_2,\mathcal{H}_2)=(\sigma,\mathcal{H}_1 cup\ _{\mathcal{R}}\mathcal{H}_2)$, where $\mathcal{H}_1\cup_{\mathcal{R}}\mathcal{H}_2=(Y,\mathcal{V}_1\cap\mathcal{V}_2,\max(N_1,N_2))\ \forall\ \mathcal{v}_k\in\mathcal{V}_1\cap$

$\mathcal{V}_2$ and $\mathfrak{b}_j \in \mathfrak{B}$, $\left(\left(\mathfrak{b}_j, \mathfrak{o}_{q_k}\right), x, y\right) \in \psi(v_k) \Leftrightarrow \mathfrak{o}_{v_k} = \max(\mathfrak{o}^1_{v_k}, \mathfrak{o}^2_{v_k})$ and $x = \max\left(\mu_1^M\left(\mathfrak{b}_j, \mathfrak{o}^1_{q_k}\right), \mu_2^M\left(\mathfrak{b}_j, \mathfrak{o}^2_{q_k}\right)\right) = \left(\max(\gamma_1^M, \gamma_2^M)\, e^{i2\pi\left(\max\left(\omega_{\gamma_1^M}, \omega_{\gamma_2^M}\right)\right)}\right)$ $y = \min\left(\mu_1^N\left(\mathfrak{b}_j, \mathfrak{o}^1_{q_k}\right), \mu_2^N\left(\mathfrak{b}_j, \mathfrak{o}^2_{q_k}\right)\right) = \left(\min(\gamma_1^N, \gamma_2^N)\, e^{i2\pi\left(\min\left(\omega_{\gamma_1^N}, \omega_{\gamma_2^N}\right)\right)}\right)$ if $\left(\left(\mathfrak{b}_j, \mathfrak{o}^1_{q_k}\right), \mu_1^M\left(\mathfrak{b}_j, \mathfrak{o}^1_{q_k}\right), \mu_1^N\left(\mathfrak{b}_j, \mathfrak{o}^1_{q_k}\right)\right) \in \rho_1(v_k)$ and $\left(\left(\mathfrak{b}_j, \mathfrak{o}^2_{q_k}\right), \mu_2^M\left(\mathfrak{b}_j, \mathfrak{o}^2_{q_k}\right), \mu_2^N\left(\mathfrak{b}_j, \mathfrak{o}^2_{q_k}\right)\right) \in \rho_2(v_k)$.

Example 8: Let$(\rho_1, \mathcal{H}_1)$ be CIF4-SS and $(\rho_2, \mathcal{H}_2)$ be CIF5-SS presented in tables (7) and (8). Then their restricted union $(\rho_1, \mathcal{H}_1) \cup_{\mathcal{R}} (\rho_2, \mathcal{H}_2)$ is presented in table (11).

Table 10. The restricted union of CIF4-SS displaed in table 7 and CIF5-SS displayed in table 8.

$(\boldsymbol{\sigma}, (\boldsymbol{S}, \boldsymbol{\mathcal{V}_1} \cup_{\mathcal{R}} \boldsymbol{\mathcal{V}_2}, \mathbf{5}))$	$\boldsymbol{v_1}$	$\boldsymbol{v_3}$
$\mathfrak{b}_1$	$4, \begin{pmatrix} 0.85e^{i2\pi(0.88)}, \\ 0.1e^{i2\pi(0.1)} \end{pmatrix}$	$4, \begin{pmatrix} 0.95e^{i2\pi(0.8)}, \\ 0.04e^{i2\pi(0.15)} \end{pmatrix}$
$\mathfrak{b}_2$	$3, \begin{pmatrix} 0.85e^{i2\pi(0.85)}, \\ 0.1e^{i2\pi(0.12)} \end{pmatrix}$	$2, \begin{pmatrix} 0.62e^{i2\pi(0.6)}, \\ 0.3e^{i2\pi(0.25)} \end{pmatrix}$
$\mathfrak{b}_3$	$3, \begin{pmatrix} 0.7e^{i2\pi(0.75)}, \\ 0.2e^{i2\pi(0.15)} \end{pmatrix}$	$3, \begin{pmatrix} 0.87e^{i2\pi(0.9)}, \\ 0.1e^{i2\pi(0.05)} \end{pmatrix}$

Definition 31: Let $(\rho_1, \mathcal{H}_1)$ and $(\rho_2, \mathcal{H}_2)$ be two CFIN-SSs over $\mathfrak{B}$, where $\mathcal{H}_1 = (J_1, \mathcal{V}_1, N_1)$ and $\mathcal{H}_2 = (J_2, \mathcal{V}_2, N_2)$ are N-SSs over $\mathfrak{B}$. Then their extended union is designated and given as $(\rho_1, \mathcal{H}_1) \cup_{\mathcal{E}} (\rho_2, \mathcal{H}_2) = (\tau, \mathcal{H}_1 \cup_{\mathcal{E}} \mathcal{H}_2)$, where $\mathcal{H}_1 \cup_{\mathcal{E}} \mathcal{H}_2 = (L, \mathcal{V}_1 \cap \mathcal{V}_2, \max(N_1, N_2))\ \forall\, v_k \in \mathcal{V}_1 \cap \mathcal{V}_2$ and $\mathfrak{b}_j \in \mathfrak{B}$, and $\tau(v_k)$ is presented by

$$\tau(v_k) = \begin{cases} \rho_1(v_k) & if\ v_k \in \mathcal{V}_1 - \mathcal{V}_2 \\ \rho_2(v_k) & if\ v_k \in \mathcal{V}_2 - \mathcal{V}_1 \\ \left(\left(\mathfrak{b}_j, \mathfrak{o}_{q_k}\right), x, y\right) & \left(\begin{array}{c} such\ that\ \mathfrak{o}_{v_k} = \max(\mathfrak{o}^1_{v_k}, \mathfrak{o}^2_{v_k})\ and\ x = \max\left(\mu_1^M\left(\mathfrak{b}_j, \mathfrak{o}^1_{q_k}\right), \mu_2^M\left(\mathfrak{b}_j, \mathfrak{o}^2_{q_k}\right)\right) \\ = \left(\max(\gamma_1^M, \gamma_2^M)\, e^{i2\pi\left(\max\left(\omega_{\gamma_1^M}, \omega_{\gamma_2^M}\right)\right)}\right), y = \min\left(\mu_1^N\left(\mathfrak{b}_j, \mathfrak{o}^1_{q_k}\right), \mu_2^N\left(\mathfrak{b}_j, \mathfrak{o}^2_{q_k}\right)\right) \\ = \left(\min(\gamma_1^N, \gamma_2^N)\, e^{i2\pi\left(\min\left(\omega_{\gamma_1^N}, \omega_{\gamma_2^N}\right)\right)}\right), where \\ \left(\left(\mathfrak{b}_j, \mathfrak{o}^1_{q_k}\right), \mu_1^M\left(\mathfrak{b}_j, \mathfrak{o}^1_{q_k}\right), \mu_1^N\left(\mathfrak{b}_j, \mathfrak{o}^1_{q_k}\right)\right) \in \rho_1(v_k)\ and \\ \left(\left(\mathfrak{b}_j, \mathfrak{o}^2_{q_k}\right), \mu_2^M\left(\mathfrak{b}_j, \mathfrak{o}^2_{q_k}\right), \mu_2^N\left(\mathfrak{b}_j, \mathfrak{o}^2_{q_k}\right)\right) \in \rho_2(v_k) \end{array}\right) \end{cases}$$

Example 9: Let$(\rho_1, \mathcal{H}_1)$ be CIF4-SS shown in table (7) and $(\rho_2, \mathcal{H}_2)$ be CIF5-SS shown in table (8). Then their extended union $(\rho_1, \mathcal{H}_1) \cup_{\mathcal{E}} (\rho_2, \mathcal{H}_2)$ is shown in table (12).

Table 11. The extended union of CIF4-SS displayed in table 7 and CIF5-SS displayed in table 8.

$(\boldsymbol{\tau}, (\boldsymbol{S}, \boldsymbol{\mathcal{V}_1} \cup_{\mathcal{E}} \boldsymbol{\mathcal{V}_2}, \mathbf{5}))$	$\boldsymbol{v_1}$	$\boldsymbol{v_2}$	$\boldsymbol{v_3}$	$\boldsymbol{v_4}$
$\mathfrak{b}_1$	$4, \begin{pmatrix} 0.85e^{i2\pi(0.88)}, \\ 0.1e^{i2\pi(0.1)} \end{pmatrix}$	$1, \begin{pmatrix} 0.3e^{i2\pi(0.38)}, \\ 0.6e^{i2\pi(0.5)} \end{pmatrix}$	$4, \begin{pmatrix} 0.95e^{i2\pi(0.8)}, \\ 0.04e^{i2\pi(0.15)} \end{pmatrix}$	$4, \begin{pmatrix} 0.96e^{i2\pi(0.96)}, \\ 0.03e^{i2\pi(0.03)} \end{pmatrix}$
$\mathfrak{b}_2$	$3, \begin{pmatrix} 0.85e^{i2\pi(0.85)}, \\ 0.1e^{i2\pi(0.12)} \end{pmatrix}$	$2, \begin{pmatrix} 0.67e^{i2\pi(0.65)}, \\ 0.2e^{i2\pi(0.3)} \end{pmatrix}$	$2, \begin{pmatrix} 0.62e^{i2\pi(0.6)}, \\ 0.3e^{i2\pi(0.25)} \end{pmatrix}$	$3, \begin{pmatrix} 0.7e^{i2\pi(0.73)}, \\ 0.1e^{i2\pi(0.2)} \end{pmatrix}$
$\mathfrak{b}_3$	$3, \begin{pmatrix} 0.7e^{i2\pi(0.75)}, \\ 0.2e^{i2\pi(0.15)} \end{pmatrix}$	$3, \begin{pmatrix} 0.8e^{i2\pi(0.9)}, \\ 0.1e^{i2\pi(0.05)} \end{pmatrix}$	$3, \begin{pmatrix} 0.87e^{i2\pi(0.9)}, \\ 0.1e^{i2\pi(0.05)} \end{pmatrix}$	$1, \begin{pmatrix} 0.3e^{i2\pi(0.29)}, \\ 0.1e^{i2\pi(0.15)} \end{pmatrix}$

Now we can relate CIFSSs with CIFN-SSs in different forms;

Definition 32: Let $(\rho, \mathcal{H})$ be a CIFN-SS over $\mathfrak{B}$, where $\mathcal{H} = (J, \mathcal{V}, N)$ is an N-SS. Let $0 < \beta < N$ be a threshold. The CIFSS over $\mathfrak{B}$ linked with $(\rho, \mathcal{H})$ and β is $(\rho^\beta, \mathcal{V})$ given by: for each $v \in \mathcal{V}$,

$$\rho^{\beta}(v_k) = \begin{cases} \left(\mathfrak{b}_j, \left(1.0e^{i2\pi(1.0)}, 0.0e^{i2\pi(0.0)}\right)\right) & if\ \mathfrak{o}_{v_k} \geq \beta \\ \left(\mathfrak{b}_j, \left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)\right) & otherwise \end{cases}$$

Particularly, $\rho^{1}(v_k)$ be the bottom CIFSS linked with $(\rho\ ,\mathcal{H})$ and $\rho^{N-1}(v_k)$ be the top CIFSS linked with $(\rho\ ,\mathcal{H})$.

Remark 2:

- if we let the phase term $\omega = 0$ then from definition (32) we get intuitionistic fuzzy-SS.
- If we let the NMG is equal to zero then from definition (32) we get complex fuzzy-SS.
- if we let the phase term $\omega = 0$ and NMG is equal to zero then from definition (32) we get SS.

Example 10: Let CIF6-SS presented in example (1) and $0 < \beta < 6$ be the threshold. Then the possible CIFSSs linked with thresholds 1, 2, 3, 4, and 5 are given from tables (13) to (17).

Table 12. The possible CIFSS linked with $(\rho, \mathcal{H})$ with threshold 1.

$(\boldsymbol{\rho}^{1}, \boldsymbol{\mathcal{V}})$	$\boldsymbol{v}_1$	$\boldsymbol{v}_2$	$\boldsymbol{v}_3$	$\boldsymbol{v}_4$
$\mathfrak{b}_1$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$
$\mathfrak{b}_2$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$
$\mathfrak{b}_3$	$\begin{pmatrix}0.0e^{i2\pi(0.0)},\\ 1.0e^{i2\pi(1.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$
$\mathfrak{b}_4$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$
$\mathfrak{b}_5$	$\begin{pmatrix}0.0e^{i2\pi(0.0)},\\ 1.0e^{i2\pi(1.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}0.0e^{i2\pi(0.0)},\\ 1.0e^{i2\pi(1.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$
$\mathfrak{b}_6$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$

Table 13. The possible CIFSS linked with $(\rho, \mathcal{H})$ with threshold 2.

$(\boldsymbol{\rho}^{2}, \boldsymbol{\mathcal{V}})$	$\boldsymbol{v}_1$	$\boldsymbol{v}_2$	$\boldsymbol{v}_3$	$\boldsymbol{v}_4$
$\mathfrak{b}_1$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}0.0e^{i2\pi(0.0)},\\ 1.0e^{i2\pi(1.0)}\end{pmatrix}$	$\begin{pmatrix}0.0e^{i2\pi(0.0)},\\ 1.0e^{i2\pi(1.0)}\end{pmatrix}$
$\mathfrak{b}_2$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$
$\mathfrak{b}_3$	$\begin{pmatrix}0.0e^{i2\pi(0.0)},\\ 1.0e^{i2\pi(1.0)}\end{pmatrix}$	$\begin{pmatrix}0.0e^{i2\pi(0.0)},\\ 1.0e^{i2\pi(1.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$
$\mathfrak{b}_4$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$
$\mathfrak{b}_5$	$\begin{pmatrix}0.0e^{i2\pi(0.0)},\\ 1.0e^{i2\pi(1.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}0.0e^{i2\pi(0.0)},\\ 1.0e^{i2\pi(1.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$
$\mathfrak{b}_6$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}0.0e^{i2\pi(0.0)},\\ 1.0e^{i2\pi(1.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$

Table 14. The possible CIFSS linked with $(\rho, \mathcal{H})$ with threshold 3.

$(\boldsymbol{\rho}^{3}, \boldsymbol{\mathcal{V}})$	$\boldsymbol{v}_1$	$\boldsymbol{v}_2$	$\boldsymbol{v}_3$	$\boldsymbol{v}_4$
$\mathfrak{b}_1$	$\begin{pmatrix}0.0e^{i2\pi(0.0)},\\ 1.0e^{i2\pi(1.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}0.0e^{i2\pi(0.0)},\\ 1.0e^{i2\pi(1.0)}\end{pmatrix}$	$\begin{pmatrix}0.0e^{i2\pi(0.0)},\\ 1.0e^{i2\pi(1.0)}\end{pmatrix}$
$\mathfrak{b}_2$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}0.0e^{i2\pi(0.0)},\\ 1.0e^{i2\pi(1.0)}\end{pmatrix}$	$\begin{pmatrix}0.0e^{i2\pi(0.0)},\\ 1.0e^{i2\pi(1.0)}\end{pmatrix}$	$\begin{pmatrix}0.0e^{i2\pi(0.0)},\\ 1.0e^{i2\pi(1.0)}\end{pmatrix}$
$\mathfrak{b}_3$	$\begin{pmatrix}0.0e^{i2\pi(0.0)},\\ 1.0e^{i2\pi(1.0)}\end{pmatrix}$	$\begin{pmatrix}0.0e^{i2\pi(0.0)},\\ 1.0e^{i2\pi(1.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$
$\mathfrak{b}_4$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$	$\begin{pmatrix}1.0e^{i2\pi(1.0)},\\ 0.0e^{i2\pi(0.0)}\end{pmatrix}$

$\mathfrak{b}_5$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(1.0e^{i2\pi(1.0)}, 0.0e^{i2\pi(0.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$
$\mathfrak{b}_6$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(1.0e^{i2\pi(1.0)}, 0.0e^{i2\pi(0.0)}\right)$	$\left(1.0e^{i2\pi(1.0)}, 0.0e^{i2\pi(0.0)}\right)$

Table 15. The possible CIFSS linked with $(\rho, \mathcal{H})$ with threshold 4.

$(\boldsymbol{\rho}^4, \boldsymbol{\mathcal{V}})$	$\boldsymbol{v}_1$	$\boldsymbol{v}_2$	$\boldsymbol{v}_3$	$\boldsymbol{v}_4$
$\mathfrak{b}_1$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$
$\mathfrak{b}_2$	$\left(1.0e^{i2\pi(1.0)}, 0.0e^{i2\pi(0.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$
$\mathfrak{b}_3$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(1.0e^{i2\pi(1.0)}, 0.0e^{i2\pi(0.0)}\right)$
$\mathfrak{b}_4$	$\left(1.0e^{i2\pi(1.0)}, 0.0e^{i2\pi(0.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(1.0e^{i2\pi(1.0)}, 0.0e^{i2\pi(0.0)}\right)$	$\left(1.0e^{i2\pi(1.0)}, 0.0e^{i2\pi(0.0)}\right)$
$\mathfrak{b}_5$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(1.0e^{i2\pi(1.0)}, 0.0e^{i2\pi(0.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$
$\mathfrak{b}_6$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(1.0e^{i2\pi(1.0)}, 0.0e^{i2\pi(0.0)}\right)$

Table 16. The possible CIFSS linked with $(\rho, \mathcal{H})$ with threshold 5.

$(\boldsymbol{\rho}^5, \boldsymbol{\mathcal{V}})$	$\boldsymbol{v}_1$	$\boldsymbol{v}_2$	$\boldsymbol{v}_3$	$\boldsymbol{v}_4$
$\mathfrak{b}_1$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$
$\mathfrak{b}_2$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$
$\mathfrak{b}_3$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$
$\mathfrak{b}_4$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(1.0e^{i2\pi(1.0)}, 0.0e^{i2\pi(0.0)}\right)$
$\mathfrak{b}_5$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(1.0e^{i2\pi(1.0)}, 0.0e^{i2\pi(0.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$
$\mathfrak{b}_6$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$	$\left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right)$

Further, we interpret the definitions of null and whole CIFN-SS.

Definition 33: Let $(\rho, \mathcal{H})$ be a CIFNSS over $\mathfrak{B}$, where $\mathcal{H} = (J, \mathcal{V}, N)$ be an N-SS over $\mathfrak{B}$. The $(\rho, \mathcal{H})$ is called null CIFN-SS, designated by $(\Theta, \mathcal{H})$ if $\forall\, v_k \in \mathcal{V}$, $(0, \mathcal{V}, N)$ is an N-SS on $\mathfrak{B}$ with the null CIFS $0'$ of $\mathfrak{B}$, where $0'(\mathfrak{b}) = \left(0.0e^{i2\pi(0.0)}, 1.0e^{i2\pi(1.0)}\right) \forall\, \mathfrak{b} \in \mathfrak{B}$.

Definition 34: Let $(\rho, \mathcal{H})$ be a CIFNSS over $\mathfrak{B}$, where $\mathcal{H} = (J, \mathcal{V}, N)$ be an N-SS over $\mathfrak{B}$. The $(\rho, \mathcal{H})$ is called whole CIFN-SS, designated by $(\varpi, \mathcal{H})$ if $\forall\, v_k \in \mathcal{V}$, $(\mathcal{W}, \mathcal{V}, N)$ is an N-SS on $\mathfrak{B}$ with the whole CIFS Λ of $\mathfrak{B}$, where $\Lambda(\mathfrak{b}) = \left(1.0e^{i2\pi(1.0)}, 0.0e^{i2\pi(0.0)}\right) \forall\, \mathfrak{b} \in \mathfrak{B}$.

IV. APPLICATIONS

In this part of the manuscript, we interpret the DM method that handles the model we have described in section 3. Consequently, we interpret the algorithm for issues that are identified by CIFN-SS. To show its credibility and effectiveness, we apply it to real-life problems that are completely developed.

We defined the following algorithm of CIFN-SSs for DM.

Algorithm: The selection of an alternative in a CIFN-SS.

1. Input $\mathfrak{B} = \{\mathfrak{b}_1, \mathfrak{b}_2, \mathfrak{b}_3, \ldots, \mathfrak{b}_p\}$ as universe set
2. Input $\mathcal{V} = \{v_1, v_2, v_3, \ldots, v_q\} \subseteq \mathcal{U}$ as a set of attributes.
3. Compose a CIFN-SS in tabular representation.
4. Compose the tables of CMGs and CNMGs.

5. Calculate the comparison tables for both CMGs and CNMGs. (two complex MGs or NMGs will be compared by lexicographical order i.e. if $\mu_1^M = \gamma_1^M e^{i2\pi\left(\omega_{\gamma_1^M}\right)}, \mu_2^M = \gamma_2^M e^{i2\pi\left(\omega_{\gamma_2^M}\right)}$ be two complex MGs then $\gamma_1^M < \gamma_2^M$ then we say $\mu_1^M < \mu_2^M$. But if $\gamma_1^M = \gamma_2^M$ then we observe the phase term i.e. if $\omega_{\gamma_1^M} < \omega_{\gamma_2^M}$ then $\mu_1^M < \mu_2^M$. If both $\gamma_1^M = \gamma_2^M$ and $\omega_{\gamma_1^M} = \omega_{\gamma_2^M}$ then we say the $\mu_1^M = \mu_2^M$. Similarly, we compare CNMGs.)
6. Compose the score tables for both CMGs and CNMGs.
7. Calculate the final score by subtracting the CNMG score from CMG score for each alternative.
8. Note the highest score with maximum grades, if it is in $k-th$ row, then we will select the option $\mathfrak{b}_k$, $1 \leq k \leq n$.

4.1. Selection of best and economical bus

Public transport or public transportation is a system of transport, totally different from private transport, usually, managed by schedule, operated on confirmed routes, and charges a fixed fee for each trip. Governments provide different types of public transport such as city buses, trains, trams (light rail), etc. In the following example, we will see that the government wants to find out the best and most economical bus in 6 public transport buses.

Example 11. Reconsider the example (1) in which $\mathfrak{B} = \{\mathfrak{b}_1, \mathfrak{b}_2, \mathfrak{b}_3, \mathfrak{b}_4, \mathfrak{b}_5, \mathfrak{b}_6\}$ is a set of 6 public transport buses and $\mathcal{V} = \{v_1 = Income\ of\ bus, v_2 = number\ of\ passanger, v_3 = short\ route, v_4 = saving\ fuel\}$ is a set of parameters. The 6-SS is given in table (2) and CIF6-SS is presented in table (3).

Now we construct tables for CMGs and CNMGs. The CMGs are given in table (18) and CNMGs are given in table (19)

Table 17. The tabular form of CMGs interpreted in example 11.

μ^M	v_1	v_2	v_3	v_4
$\mathfrak{b}_1$	$0.4e^{i2\pi(0.45)}$	$0.6e^{i2\pi(0.7)}$	$0.2e^{i2\pi(0.31)}$	$0.18e^{i2\pi(0.29)}$
$\mathfrak{b}_2$	$0.8e^{i2\pi(0.75)}$	$0.37e^{i2\pi(0.45)}$	$0.42e^{i2\pi(0.5)}$	$0.38e^{i2\pi(0.65)}$
$\mathfrak{b}_3$	$0.05e^{i2\pi(0.1)}$	$0.2e^{i2\pi(0.28)}$	$0.8e^{i2\pi(0.6)}$	$0.77e^{i2\pi(0.85)}$
$\mathfrak{b}_4$	$0.75e^{i2\pi(0.88)}$	$0.9e^{i2\pi(0.4)}$	$0.95e^{i2\pi(0.8)}$	$0.96e^{i2\pi(0.96)}$
$\mathfrak{b}_5$	$0.1e^{i2\pi(0.14)}$	$0.9e^{i2\pi(0.92)}$	$0.07e^{i2\pi(0.1)}$	$0.35e^{i2\pi(0.53)}$
$\mathfrak{b}_6$	$0.48e^{i2\pi(0.5)}$	$0.23e^{i2\pi(0.33)}$	$0.6e^{i2\pi(0.4)}$	$0.82e^{i2\pi(0.79)}$

Table 18. The tabular form of CNMGs interpreted in example 11.

μ^N	v_1	v_2	v_3	v_4
$\mathfrak{b}_1$	$0.2e^{i2\pi(0.5)}$	$0.1e^{i2\pi(0.15)}$	$0.7e^{i2\pi(0.4)}$	$0.45e^{i2\pi(0.65)}$
$\mathfrak{b}_2$	$0.1e^{i2\pi(0.2)}$	$0.6e^{i2\pi(0.3)}$	$0.4e^{i2\pi(0.4)}$	$0.33e^{i2\pi(0.23)}$
$\mathfrak{b}_3$	$0.5e^{i2\pi(0.1)}$	$0.7e^{i2\pi(0.5)}$	$0.1e^{i2\pi(0.3)}$	$0.17e^{i2\pi(0.1)}$
$\mathfrak{b}_4$	$0.2e^{i2\pi(0.1)}$	$0.09e^{i2\pi(0.5)}$	$0.04e^{i2\pi(0.15)}$	$0.03e^{i2\pi(0.03)}$
$\mathfrak{b}_5$	$0.1e^{i2\pi(0.14)}$	$0.05e^{i2\pi(0.03)}$	$0.9e^{i2\pi(0.7)}$	$0.61e^{i2\pi(0.27)}$
$\mathfrak{b}_6$	$0.3e^{i2\pi(0.3)}$	$0.52e^{i2\pi(0.63)}$	$0.25e^{i2\pi(0.4)}$	$0.1e^{i2\pi(0.15)}$

Next, we will construct the comparison tables for CMGs and CNMGs which are given in tables (20) and (21) respectively.

Table 19. Comparison table for CMGs interpreted in example 11.

.	$\mathfrak{b}_1$	$\mathfrak{b}_2$	$\mathfrak{b}_3$	$\mathfrak{b}_4$	$\mathfrak{b}_5$	$\mathfrak{b}_6$
$\mathfrak{b}_1$	4	1	2	0	2	1
$\mathfrak{b}_2$	3	4	2	1	3	2
$\mathfrak{b}_3$	2	2	4	0	2	1
$\mathfrak{b}_4$	4	3	4	4	3	4
$\mathfrak{b}_5$	2	1	2	1	4	1
$\mathfrak{b}_6$	3	2	3	0	3	4

Table 20. Comparison table for CNMGs interpreted in example 11.

.	$\mathfrak{b}_1$	$\mathfrak{b}_2$	$\mathfrak{b}_3$	$\mathfrak{b}_4$	$\mathfrak{b}_5$	$\mathfrak{b}_6$
$\mathfrak{b}_1$	4	3	2	3	2	2
$\mathfrak{b}_2$	1	4	2	3	2	3
$\mathfrak{b}_3$	2	2	4	4	2	2
$\mathfrak{b}_4$	0	1	0	4	2	0
$\mathfrak{b}_5$	2	2	2	2	4	2
$\mathfrak{b}_6$	2	1	1	4	2	4

Now we will calculate the complex membership (CM) and complex non-membership (CNM) scores. For finding both scores we will subtract the column sum from the row sum of the table (20) and (21). Both scores are given in tables (22) and (23) respectively.

Table 21. CM score table for example 11.

	Grade sum $\left(\sum_{i=1}^{4} \mathfrak{o}_{v_i}\right)$	Row sum ($\mathfrak{R}s_1$)	Column sum ($\mathfrak{C}s_1$)	$\Omega_1 = \mathfrak{R}s_1 - \mathfrak{C}s_2$
$\mathfrak{b}_1$	7	10	18	8
$\mathfrak{b}_2$	10	15	13	2
$\mathfrak{b}_3$	8	11	17	−6
$\mathfrak{b}_4$	16	22	6	16
$\mathfrak{b}_5$	7	11	17	−6
$\mathfrak{b}_6$	10	15	13	2

Table 22. CNM score table for example 11.

	Grade sum $\left(\sum_{i=1}^{4} \mathfrak{o}_{v_i}\right)$	Row sum ($\mathfrak{R}s_2$)	Column sum ($\mathfrak{C}s_2$)	$\Omega_2 = \mathfrak{R}s_2 - \mathfrak{C}s_2$)
$\mathfrak{b}_1$	7	16	11	5
$\mathfrak{b}_2$	10	15	13	2
$\mathfrak{b}_3$	8	16	11	5
$\mathfrak{b}_4$	16	7	20	−13
$\mathfrak{b}_5$	7	14	14	0
$\mathfrak{b}_6$	10	14	13	1

The final score for each alternative is calculated by subtracting the CNMG score (Ω_2) from CMG score (Ω_1) as given in table (24).

Table 23. Final score along with the grades linked with CIF6-SS for example 11.

	Grade sum $\left(\sum_{i=1}^{4} \mathfrak{o}_{v_i}\right)$	Ω_1	Ω_2	Final Score $\Omega_2 - \Omega_1$
$\mathfrak{b}_1$	7	8	5	3
$\mathfrak{b}_2$	10	2	2	0
$\mathfrak{b}_3$	8	−6	5	−11
$\mathfrak{b}_4$	16	16	−13	29
$\mathfrak{b}_5$	7	−6	0	−6
$\mathfrak{b}_6$	10	2	1	1

It is clear from the table (24) that the highest score is 29, which is got by bus $\mathfrak{b}_4$. So the bus $\mathfrak{b}_4$ is the best and most economical bus of the 6 public transport buses.

4.2. Selection of appropriate teaching method

Teaching is the way toward going to people's requirements, experiences and emotions, and mediating so they learn specific things and go past the given. Teaching is the world's biggest profession and the toughest job to do. Every teacher wants to deliver maximum knowledge to the class but it is not possible without a proper teaching method. There are a lot of teaching methods all around the world. We will show in the following example how CIFN-SS help teachers in the selection of appropriate teaching method.

Example 12: Let $\mathfrak{T} = \{t_1, t_2, t_3, t_4, t_5\}$ be set of 5 short listed teaching methods and $\mathcal{V} = \{v_1, v_2, v_3, v_4\}$ be set of parameters, and on the basis of these parameters teaching an expert allocate grading to the teaching methods. The information obtained from real data is given in table (25).

Table 24. information obtained from real data interpreted in example 12.

$\mathfrak{T}$	v_1	v_2	v_3	v_4
t_1	××××	×××	×××	××××
t_2	∘	××	×	×××
t_3	××	∘	××	××××
t_4	×	×××	××	×××
t_5	∘	×××	∘	××

where
Four cross marks represent 'Excellent',
Three cross marks represent 'Very Good',
Two cross marks represent 'Good',
One cross mark represents 'Normal',
Hole represents 'Poor',
The set $\mathfrak{O} = \{0, 1, 2,3,4\}$ can undoubtedly link with the cross marks presented in table (25), where
0 denotes "∘",
1 denotes "×",
2 denotes "××",
3 denotes "×××",
4 denotes "××××",
The tabular representation of 5-SS is described in table (26).

Table 25. The tabular representation of 5-SS interpreted in example 12.

$\mathfrak{T}$	v_1	v_2	v_3	v_4
t_1	4	3	3	4
t_2	0	2	1	3
t_3	2	0	2	4
t_4	1	3	2	3
t_5	0	3	0	2

The grading criteria for CMG and CNMG of elements of the set $\mathfrak{T}$ are presented below.

$$0.0 \le \Delta\mu^M(t) < 0.2 \text{ when } \mathfrak{o} = 0;$$
$$0.2 \le \Delta\mu^M(t) < 0.4 \text{ when } \mathfrak{o} = 1;$$
$$0.4 \le \Delta\mu^M(t) < 0.6 \text{ when } \mathfrak{o} = 2;$$
$$0.6 \le \Delta\mu^M(t) < 0.8 \text{ when } \mathfrak{o} = 3;$$
$$0.8 \le \Delta\mu^M(t) \le 1.0 \text{ when } \mathfrak{o} = 4.$$

Where $\Delta\mu^M(\mathrm{t}) = \frac{\gamma^M(\mathrm{t})+\omega_{\gamma^M}(\mathrm{t})}{2}$, and $0 \leq \mu^M + \mu^N \leq 1$.
The tabular representation of CIF5-SS is given below in table (27).

Table 26. The tabular representation of CIF5-S interpreted in example 12.

$(\boldsymbol{\rho}, (\boldsymbol{J}, \boldsymbol{\mathcal{V}}, \mathbf{5}))$	$\boldsymbol{v}_1$	$\boldsymbol{v}_2$	$\boldsymbol{v}_3$	$\boldsymbol{v}_4$
$\mathbf{t_1}$	$4, \begin{pmatrix} 0.9e^{i2\pi(0.85)}, \\ 0.08e^{i2\pi(0.1)} \end{pmatrix}$	$3, \begin{pmatrix} 0.65e^{i2\pi(0.75)}, \\ 0.2e^{i2\pi(0.1)} \end{pmatrix}$	$3, \begin{pmatrix} 0.7e^{i2\pi(0.6)}, \\ 0.1e^{i2\pi(0.3)} \end{pmatrix}$	$4, \begin{pmatrix} 0.98e^{i2\pi(0.9)}, \\ 0.01e^{i2\pi(0.05)} \end{pmatrix}$
$\mathbf{t_2}$	$0, \begin{pmatrix} 0.15e^{i2\pi(0.1)}, \\ 0.7e^{i2\pi(0.8)} \end{pmatrix}$	$2, \begin{pmatrix} 0.47e^{i2\pi(0.45)}, \\ 0.5e^{i2\pi(0.4)} \end{pmatrix}$	$1, \begin{pmatrix} 0.22e^{i2\pi(0.3)}, \\ 0.5e^{i2\pi(0.5)} \end{pmatrix}$	$3, \begin{pmatrix} 0.68e^{i2\pi(0.78)}, \\ 0.2e^{i2\pi(0.15)} \end{pmatrix}$
$\mathbf{t_3}$	$2, \begin{pmatrix} 0.5e^{i2\pi(0.4)}, \\ 0.4e^{i2\pi(0.5)} \end{pmatrix}$	$0, \begin{pmatrix} 0.1e^{i2\pi(0.14)}, \\ 0.7e^{i2\pi(0.5)} \end{pmatrix}$	$2, \begin{pmatrix} 0.55e^{i2\pi(0.46)}, \\ 0.3e^{i2\pi(0.5)} \end{pmatrix}$	$4, \begin{pmatrix} 0.7e^{i2\pi(0.9)}, \\ 0.2e^{i2\pi(0.05)} \end{pmatrix}$
$\mathbf{t_4}$	$1, \begin{pmatrix} 0.3e^{i2\pi(0.38)}, \\ 0.6e^{i2\pi(0.33)} \end{pmatrix}$	$3, \begin{pmatrix} 0.76e^{i2\pi(0.68)}, \\ 0.2e^{i2\pi(0.3)} \end{pmatrix}$	$2, \begin{pmatrix} 0.43e^{i2\pi(0.5)}, \\ 0.43e^{i2\pi(0.4)} \end{pmatrix}$	$3, \begin{pmatrix} 0.69e^{i2\pi(0.71)}, \\ 0.3e^{i2\pi(0.2)} \end{pmatrix}$
$\mathbf{t_5}$	$0, \begin{pmatrix} 0.13e^{i2\pi(0.1)}, \\ 0.36e^{i2\pi(0.46)} \end{pmatrix}$	$3, \begin{pmatrix} 0.74e^{i2\pi(0.75)}, \\ 0.1e^{i2\pi(0.15)} \end{pmatrix}$	$0, \begin{pmatrix} 0.07e^{i2\pi(0.1)}, \\ 0.9e^{i2\pi(0.7)} \end{pmatrix}$	$2, \begin{pmatrix} 0.45e^{i2\pi(0.53)}, \\ 0.51e^{i2\pi(0.27)} \end{pmatrix}$

Now we construct tables for CMGs and CNMGs. The CMGs are given in table (28) and CNMGs are given in table (29).

Table 27. The tabular form of CM interpreted in example 12.

$\boldsymbol{\mu^M}$	$\boldsymbol{v}_1$	$\boldsymbol{v}_2$	$\boldsymbol{v}_3$	$\boldsymbol{v}_4$
$\mathbf{t_1}$	$0.9e^{i2\pi(0.85)}$	$0.65e^{i2\pi(0.75)}$	$0.7e^{i2\pi(0.6)}$	$0.98e^{i2\pi(0.9)}$
$\mathbf{t_2}$	$0.15e^{i2\pi(0.1)}$	$0.47e^{i2\pi(0.45)}$	$0.22e^{i2\pi(0.3)}$	$0.68e^{i2\pi(0.78)}$
$\mathbf{t_3}$	$0.5e^{i2\pi(0.4)}$	$0.1e^{i2\pi(0.14)}$	$0.55e^{i2\pi(0.46)}$	$0.7e^{i2\pi(0.9)}$
$\mathbf{t_4}$	$0.3e^{i2\pi(0.38)}$	$0.76e^{i2\pi(0.68)}$	$0.43e^{i2\pi(0.5)}$	$0.69e^{i2\pi(0.71)}$
$\mathbf{t_5}$	$0.13e^{i2\pi(0.1)}$	$0.74e^{i2\pi(0.75)}$	$0.07e^{i2\pi(0.1)}$	$0.45e^{i2\pi(0.53)}$

Table 28. The tabular form of CNMGs interpreted in example 12.

$\boldsymbol{\mu^N}$	$\boldsymbol{v}_1$	$\boldsymbol{v}_2$	$\boldsymbol{v}_3$	$\boldsymbol{v}_4$
$\mathbf{t_1}$	$0.08e^{i2\pi(0.1)}$	$0.2e^{i2\pi(0.1)}$	$0.1e^{i2\pi(0.3)}$	$0.01e^{i2\pi(0.05)}$
$\mathbf{t_2}$	$0.7e^{i2\pi(0.8)}$	$0.5e^{i2\pi(0.4)}$	$0.5e^{i2\pi(0.5)}$	$0.2e^{i2\pi(0.15)}$
$\mathbf{t_3}$	$0.4e^{i2\pi(0.5)}$	$0.7e^{i2\pi(0.6)}$	$0.3e^{i2\pi(0.5)}$	$0.2e^{i2\pi(0.05)}$
$\mathbf{t_4}$	$0.6e^{i2\pi(0.33)}$	$0.2e^{i2\pi(0.3)}$	$0.43e^{i2\pi(0.4)}$	$0.3e^{i2\pi(0.2)}$
$\mathbf{t_5}$	$0.36e^{i2\pi(0.46)}$	$0.1e^{i2\pi(0.15)}$	$0.9e^{i2\pi(0.7)}$	$0.51e^{i2\pi(0.27)}$

Next, we will construct the comparison tables for CMGs and CNMGs which are given in table (30) and (31) respectively.

Table 29. Comparison table for CMGs interpreted in example 12.

.	$\mathbf{t_1}$	$\mathbf{t_2}$	$\mathbf{t_3}$	$\mathbf{t_4}$	$\mathbf{t_5}$
$\mathbf{t_1}$	4	4	4	3	3
$\mathbf{t_2}$	0	4	1	0	3
$\mathbf{t_3}$	0	3	4	3	3
$\mathbf{t_4}$	1	4	1	4	4
$\mathbf{t_5}$	1	1	1	0	4

Table 30. comparison table for CNMGs interpreted in example 12 .

.	$\mathbf{t_1}$	$\mathbf{t_2}$	$\mathbf{t_3}$	$\mathbf{t_4}$	$\mathbf{t_5}$
$\mathbf{t_1}$	4	0	0	0	1
$\mathbf{t_2}$	4	4	3	2	2
$\mathbf{t_3}$	4	1	4	1	2
$\mathbf{t_4}$	4	2	3	4	2
$\mathbf{t_5}$	3	2	2	2	4

Now we will calculate the CM and CNM scores. For finding both scores we will subtract the column sum from the row sum of the table (30) and (31). Both scores are given in tables (32) and (33) respectively.

Table 31. CM score table for example 12.

	Grade sum $\left(\sum_{i=1}^{4} \mathfrak{o}_{\mathcal{v}_i}\right)$	Row sum $(\mathfrak{R}s_1)$	Column sum $(\mathfrak{C}s_1)$	$\Omega_1 = \mathfrak{R}s_1 - \mathfrak{C}s_2$
$\mathfrak{t}_1$	14	18	6	12
$\mathfrak{t}_2$	6	8	16	−8
$\mathfrak{t}_3$	8	13	11	2
$\mathfrak{t}_4$	9	14	10	4
$\mathfrak{t}_5$	5	7	17	−10

Table 32. CNM score table interpreted in example 12.

	Grade sum $\left(\sum_{i=1}^{4} \mathfrak{o}_{\mathcal{v}_i}\right)$	Row sum $(\mathfrak{R}s_2)$	Column sum $(\mathfrak{C}s_2)$	$\Omega_2 = \mathfrak{R}s_2 - \mathfrak{C}s_2)$
$\mathfrak{t}_1$	14	5	19	−14
$\mathfrak{t}_2$	6	15	9	6
$\mathfrak{t}_3$	8	12	12	0
$\mathfrak{t}_4$	9	15	9	6
$\mathfrak{t}_5$	5	13	11	2

The final score for each alternative is calculated by subtracting the CNM score (Ω_2) from CM score (Ω_1) as given in table (34).

Table 33. Final score along with the grades linked with CIF5-SS interpreted in example 12.

	Grade sum $\left(\sum_{i=1}^{4} \mathfrak{o}_{\mathcal{v}_i}\right)$	Ω_1	Ω_2	Final Score $\Omega_2 - \Omega_1$
$\mathfrak{t}_1$	14	12	−14	26
$\mathfrak{t}_2$	6	−8	6	−14
$\mathfrak{t}_3$	8	2	0	2
$\mathfrak{t}_4$	9	4	6	−2
$\mathfrak{t}_5$	5	−10	2	−12

It is clear from the table (34) that the highest score is 26, which is got by the teaching method $\mathfrak{t}_1$. So the teaching method $\mathfrak{t}_1$ is the appropriate teaching method in short listed 5 teaching methods.

4.3. Selection of a mask in COVID-19

To keep ourselves safe in this pandemic we have to keep a social distance from each other and keep wearing a mask whenever we go outside. How we will select the best mask in too many masks. Here we will use CIFN-SS to select the best mask for ourselves. Let's see the following example

Example 13: Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2, \mathfrak{m}_3, \mathfrak{m}_4\}$ be set of 4 masks and $\mathcal{V} = \{\mathcal{v}_1, \mathcal{v}_2, \mathcal{v}_3\}$ be set of parameters, and on the basis of these parameters, an expert allocates grading to the masks. The information obtained from real data is given in table (35).

Table 34. the information obtained from real data interpreted in example 13.

$\mathfrak{M}$	$\mathcal{v}_1$	$\mathcal{v}_2$	$\mathcal{v}_3$
$\mathfrak{m}_1$	××	∘	××
$\mathfrak{m}_2$	×	××	∘
$\mathfrak{m}_3$	×××	××××	××××
$\mathfrak{m}_4$	×	××	×××

where
Four cross marks represent 'Excellent',
Three cross marks represent 'Very Good',
Two cross marks represent 'Good',
One cross mark represents 'Normal',
Hole represents 'Poor',
The set $\mathfrak{O} = \{0, 1, 2,3,4\}$ can undoubtedly link with the cross marks presented in table (35), where
0 denotes "∘",
1 denotes "×",
2 denotes "××",
3 denotes "×××",
4 denotes "××××",
The tabular representation of 5-SS is described in table (36).

Table 35. The tabular representation of 5-SS interpreted in example 13.

$\mathfrak{M}$	$\mathcal{v}_1$	$\mathcal{v}_2$	$\mathcal{v}_3$
$\mathfrak{m}_1$	2	0	2
$\mathfrak{m}_2$	1	2	0
$\mathfrak{m}_3$	3	4	4
$\mathfrak{m}_4$	1	2	3

The grading criteria for CMG and CNMG of elements of the set $\mathfrak{M}$ is presented below.

$$0.0 \leq \Delta\mu^M(\mathfrak{m}) < 0.2 \text{ when } \mathfrak{o} = 0;$$
$$0.2 \leq \Delta\mu^M(\mathfrak{m}) < 0.4 \text{ when } \mathfrak{o} = 1;$$
$$0.4 \leq \Delta\mu^M(\mathfrak{m}) < 0.6 \text{ when } \mathfrak{o} = 2;$$
$$0.6 \leq \Delta\mu^M(\mathfrak{m}) < 0.8 \text{ when } \mathfrak{o} = 3;$$
$$0.8 \leq \Delta\mu^M(\mathfrak{m}) \leq 1.0 \text{ when } \mathfrak{o} = 4.$$

Where $\Delta\mu^M(\mathfrak{m}) = \frac{\gamma^M(\mathfrak{m}) + \omega_{\gamma^M}(\mathfrak{m})}{2}$, and $0 \leq \mu^M + \mu^N \leq 1$. The tabular representation of CIF5-SS is given below in table (37).

Table 36. The tabular representation of CIF5-SS interpreted in example 13.

$(\rho,(J,\mathcal{V},5))$	v_1	v_2	v_3
$\mathfrak{m}_1$	$2,\begin{pmatrix}0.55e^{i2\pi(0.46)},\\0.3e^{i2\pi(0.5)}\end{pmatrix}$	$0,\begin{pmatrix}0.15e^{i2\pi(0.05)},\\0.5e^{i2\pi(0.8)}\end{pmatrix}$	$2,\begin{pmatrix}0.45e^{i2\pi(0.56)},\\0.3e^{i2\pi(0.3)}\end{pmatrix}$
$\mathfrak{m}_2$	$1,\begin{pmatrix}0.25e^{i2\pi(0.2)},\\0.7e^{i2\pi(0.6)}\end{pmatrix}$	$2,\begin{pmatrix}0.47e^{i2\pi(0.45)},\\0.5e^{i2\pi(0.4)}\end{pmatrix}$	$0,\begin{pmatrix}0.1e^{i2\pi(0.13)},\\0.8e^{i2\pi(0.6)}\end{pmatrix}$
$\mathfrak{m}_3$	$3,\begin{pmatrix}0.79e^{i2\pi(0.7)},\\0.2e^{i2\pi(0.2)}\end{pmatrix}$	$4,\begin{pmatrix}0.95e^{i2\pi(0.85)},\\0.04e^{i2\pi(0.1)}\end{pmatrix}$	$4,\begin{pmatrix}0.93e^{i2\pi(0.88)},\\0.05e^{i2\pi(0.1)}\end{pmatrix}$
$\mathfrak{m}_4$	$1,\begin{pmatrix}0.3e^{i2\pi(0.38)},\\0.6e^{i2\pi(0.33)}\end{pmatrix}$	$2,\begin{pmatrix}0.5e^{i2\pi(0.58)},\\0.3e^{i2\pi(0.4)}\end{pmatrix}$	$3,\begin{pmatrix}0.65e^{i2\pi(0.75)},\\0.33e^{i2\pi(0.2)}\end{pmatrix}$

Now we construct tables for CMGs and CNMGs. The CMGs are given in table (38) and CNMGs are given in table (39)

Table 37. The tabular form of CMGs interpreted in example 13.

μ^M	v_1	v_2	v_3
$\mathfrak{m}_1$	$0.55e^{i2\pi(0.46)}$	$0.15e^{i2\pi(0.05)}$	$0.45e^{i2\pi(0.56)}$
$\mathfrak{m}_2$	$0.25e^{i2\pi(0.2)}$	$0.47e^{i2\pi(0.45)}$	$0.1e^{i2\pi(0.13)}$
$\mathfrak{m}_3$	$0.79e^{i2\pi(0.7)}$	$0.95e^{i2\pi(0.85)}$	$0.93e^{i2\pi(0.88)}$
$\mathfrak{m}_4$	$0.3e^{i2\pi(0.38)}$	$0.5e^{i2\pi(0.58)}$	$0.65e^{i2\pi(0.75)}$

Table 38. The tabular form of CNMGs interpreted in example 13.

μ^N	v_1	v_2	v_3
$\mathfrak{m}_1$	$0.3e^{i2\pi(0.5)}$	$0.5e^{i2\pi(0.8)}$	$0.3e^{i2\pi(0.3)}$
$\mathfrak{m}_2$	$0.7e^{i2\pi(0.6)}$	$0.5e^{i2\pi(0.4)}$	$0.8e^{i2\pi(0.6)}$
$\mathfrak{m}_3$	$0.2e^{i2\pi(0.2)}$	$0.04e^{i2\pi(0.1)}$	$0.05e^{i2\pi(0.1)}$
$\mathfrak{m}_4$	$0.6e^{i2\pi(0.33)}$	$0.3e^{i2\pi(0.4)}$	$0.33e^{i2\pi(0.2)}$

Next, we will construct the comparison tables for CMGs and CNMGs which are given in tables (40) and (41) respectively.

Table 39. Comparison table of CMGs interpreted in example 13.

.	$\mathfrak{m}_1$	$\mathfrak{m}_2$	$\mathfrak{m}_3$	$\mathfrak{m}_4$
$\mathfrak{m}_1$	3	2	0	1
$\mathfrak{m}_2$	1	3	0	0
$\mathfrak{m}_3$	3	3	3	3
$\mathfrak{m}_4$	2	3	0	3

Table 40. Comparison table of CNMGs interpreted in example 13.

.	$\mathfrak{m}_1$	$\mathfrak{m}_2$	$\mathfrak{m}_3$	$\mathfrak{m}_4$
$\mathfrak{m}_1$	3	1	3	1
$\mathfrak{m}_2$	2	3	3	3
$\mathfrak{m}_3$	0	0	3	0
$\mathfrak{m}_4$	2	1	3	3

Now we will calculate the CM and CNM scores. For finding both scores we will subtract the column sum from the row sum of the table (40) and (41). Both scores are given in tables (42) and (43) respectively.

Table 41. CM score table for example 13.

	Grade sum $\left(\sum_{i=1}^{4} \mathfrak{o}_{v_i}\right)$	Row sum $(\mathfrak{R}s_1)$	Column sum $(\mathfrak{C}s_1)$	$\Omega_1 = \mathfrak{R}s_1 - \mathfrak{C}s_2$
$\mathfrak{m}_1$	4	6	9	3
$\mathfrak{m}_2$	3	4	11	−7
$\mathfrak{m}_3$	11	12	3	9
$\mathfrak{m}_4$	6	8	7	1

Table 42. CNM score table for example 13.

	Grade sum $\left(\sum_{i=1}^{4} \mathfrak{o}_{v_i}\right)$	Row sum $(\mathfrak{R}s_2)$	Column sum $(\mathfrak{C}s_2)$	$\Omega_2 = \mathfrak{R}s_2 - \mathfrak{C}s_2)$
$\mathfrak{m}_1$	4	8	7	1
$\mathfrak{m}_2$	3	11	5	6
$\mathfrak{m}_3$	11	3	12	−9
$\mathfrak{m}_4$	6	9	7	2

The final score for each alternative is calculated by subtracting the CNM score (Ω_2) from CM score (Ω_1) as given in table (44).

Table 43. Final score long with the grades linked with CIF5-SS for example 13.

	Grade sum $\left(\sum_{i=1}^{4} \mathfrak{o}_{\mathcal{v}_i}\right)$	Ω_1	Ω_2	Final Score $\Omega_2 - \Omega_1$
$\mathfrak{m}_1$	4	3	1	2
$\mathfrak{m}_2$	3	−7	6	−13
$\mathfrak{m}_3$	11	9	−9	18
$\mathfrak{m}_4$	6	1	2	−1

It is clear from the table (44) that the highest score is 18, which is got by the mas $\mathfrak{m}_3$. So the mask $\mathfrak{m}_3$ is the best mask to use in this pandemic.

V. COMPARISON

In this Section, we do a comparison of our novel model called CIFN-SS with some existing work done by Akram et al. [41]. Here $\gamma = (\gamma^M, \gamma^N)$ where γ^M be MG and γ^N be NMG in IFN-SS.

Example 14: A family wants to go on a trip, for which a family has to select the best place. A family has the option of 4 places which are $\mathfrak{P} = \{\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_3, \mathfrak{p}_4\}$ and $\mathcal{V} = \{\mathcal{v}_1 = Economcial, \mathcal{v}_2 = Mountain, \mathcal{v}_3 = water\}$ be set of parameters, on the basis of these parameters a team A of experts give rating and raking to these places. The information obtained from real data is given in table (45).

Table 44. The information is obtained from real data disaplyed in example 14.

$\mathfrak{P}$	$\mathcal{v}_1$	$\mathcal{v}_2$	$\mathcal{v}_3$
$\mathfrak{p}_1$	×××	∘	××
$\mathfrak{p}_2$	××××	×××	××××
$\mathfrak{p}_3$	∘	××	×
$\mathfrak{p}_4$	××	××	××

where
Four cross marks represent ‘Excellent’,
Three cross marks represent ‘Very Good’,
Two cross marks represent ‘Good’,
One cross mark represents ‘Normal’,
Hole represents ‘Poor’,
The set $\mathfrak{O} = \{0, 1, 2,3,4\}$ can undoubtedly link with the cross marks presented in table (45), where
0 denotes “∘”,
1 denotes “×”,
2 denotes “××”,
3 denotes “×××”,
4 denotes “××××”,
The tabular representation of 5-SS is described in table (46).

Table 45. The tabular form of 5-SS disaplyed in example 14.

$\mathfrak{P}$	$\mathcal{v}_1$	$\mathcal{v}_2$	$\mathcal{v}_3$
$\mathfrak{p}_1$	3	0	2
$\mathfrak{p}_2$	4	3	4
$\mathfrak{p}_3$	0	2	1
$\mathfrak{p}_4$	2	2	2

The grading criteria for MG and NMG of elements of the set $\mathfrak{P}$ is presented below as defined by Akram [41].
$0.0 \leq \gamma^M(\mathfrak{p}) < 0.2$ when $\mathfrak{o} = 0$;
$0.2 \leq \gamma^M(\mathfrak{p}) < 0.4$ when $\mathfrak{o} = 1$;
$0.4 \leq \gamma^M(\mathfrak{p}) < 0.6$ when $\mathfrak{o} = 2$;
$0.6 \leq \gamma^M(\mathfrak{p}) < 0.8$ when $\mathfrak{o} = 3$;
$0.8 \leq \gamma^M(\mathfrak{p}) \leq 1.0$ when $\mathfrak{o} = 4$.
$0 \leq \gamma^M + \gamma^N \leq 1$. The tabular representation of IF5-SS is given below in table (47).

Table 46. The tabular form of IF5-SS disaplyed in example 14.

$(\gamma, (J, \mathcal{V}, 5))$	$\mathcal{v}_1$	$\mathcal{v}_2$	$\mathcal{v}_3$
$\mathfrak{p}_1$	3, (0.7,0.2)	0, (0.1, 0.8)	2, (0.5, 0.4)
$\mathfrak{p}_2$	4, (0.95, 0.03)	3, (0.75, 0.1)	4, (0.93, 0.04)
$\mathfrak{p}_3$	0, (0.15, 0.8)	2, (0.45, 0.3)	1, (0.3, 0.6)
$\mathfrak{p}_4$	2, (0.55, 0.2)	2, (0.44, 0.5)	2, (0.5, 0.4)

Now we have data in the form of IFN-SS. We can use both algorithms defined by Akram [41] and our proposed algorithm in section (4). Both algorithms will give the same result which we will see below.
Compose the MG and NMG tables

Table 47. The tabular form of MGs disaplyed in example 14.

γ^M	$\mathcal{v}_1$	$\mathcal{v}_2$	$\mathcal{v}_3$
$\mathfrak{p}_1$	0.7	0.1	0.5
$\mathfrak{p}_2$	0.95	0.75	0.73
$\mathfrak{p}_3$	0.15	0.45	0.3
$\mathfrak{p}_4$	0.55	0.44	0.5

Table 48. The tabular form of NMGs disaplyed in example 14.

γ^N	$\mathcal{v}_1$	$\mathcal{v}_2$	$\mathcal{v}_3$
$\mathfrak{p}_1$	0.2	0.8	0.4
$\mathfrak{p}_2$	0.03	0.1	0.04
$\mathfrak{p}_3$	0.8	0.3	0.6
$\mathfrak{p}_4$	0.2	0.5	0.4

We can get CMG and CNMG tables in CIFN-SS by letting $1 = e^{i2\pi(0.0)}$ which are given in tables (50) and (51).

Table 49. The tabular form of CMGs disaplyed in example 14.

γ^M	$\boldsymbol{\upsilon_1}$	$\boldsymbol{\upsilon_2}$	$\boldsymbol{\upsilon_3}$
$\mathfrak{p}_1$	$0.7e^{i2\pi(0.0)}$	$0.1e^{i2\pi(0.0)}$	$0.5e^{i2\pi(0.0)}$
$\mathfrak{p}_2$	$0.95e^{i2\pi(0.0)}$	$0.75e^{i2\pi(0.0)}$	$0.73e^{i2\pi(0.0)}$
$\mathfrak{p}_3$	$0.15e^{i2\pi(0.0)}$	$0.45e^{i2\pi(0.0)}$	$0.3e^{i2\pi(0.0)}$
$\mathfrak{p}_4$	$0.55e^{i2\pi(0.0)}$	$0.44e^{i2\pi(0.0)}$	$0.5e^{i2\pi(0.0)}$

Table 50. The tabular form of CNMGs disaplyed in example 14.

γ^N	$\boldsymbol{\upsilon_1}$	$\boldsymbol{\upsilon_2}$	$\boldsymbol{\upsilon_3}$
$\mathfrak{p}_1$	$0.2e^{i2\pi(0.0)}$	$0.8e^{i2\pi(0.0)}$	$0.4e^{i2\pi(0.0)}$
$\mathfrak{p}_2$	$0.03e^{i2\pi(0.0)}$	$0.1e^{i2\pi(0.0)}$	$0.04e^{i2\pi(0.0)}$
$\mathfrak{p}_3$	$0.8e^{i2\pi(0.0)}$	$0.3e^{i2\pi(0.0)}$	$0.6e^{i2\pi(0.0)}$
$\mathfrak{p}_4$	$0.2e^{i2\pi(0.0)}$	$0.5e^{i2\pi(0.0)}$	$0.4e^{i2\pi(0.0)}$

Next, we will construct the comparison tables for MGs, NMGs, CMGs, and CNMGs which are given from tables (48) to (51). Note that the comparison tables by both algorithms defined by Akram [41] and our proposed algorithm will same so we will write one comparison table for both MG and CMG and one for NMG and CNMG.

Table 51. Comparison table for both MG and CMG disaplyed in example 14.

.	$\mathfrak{p}_1$	$\mathfrak{p}_2$	$\mathfrak{p}_3$	$\mathfrak{p}_4$
$\mathfrak{p}_1$	3	0	2	2
$\mathfrak{p}_2$	3	3	3	3
$\mathfrak{p}_3$	1	0	3	1
$\mathfrak{p}_4$	2	0	2	3

Table 52. Comparison table for both NMG and CNMG.

.	$\mathfrak{p}_1$	$\mathfrak{p}_2$	$\mathfrak{p}_3$	$\mathfrak{p}_4$
$\mathfrak{p}_1$	3	3	1	3
$\mathfrak{p}_2$	0	3	0	0
$\mathfrak{p}_3$	2	3	3	2
$\mathfrak{p}_4$	2	3	1	3

Now we will calculate the membership and non-membership scores. For finding both scores we will subtract the column sum from the row sum of tables (52) and (53). Both scores are given in the table (54) and (55) respectively.

Table 53. membership score table for example 14.

	Grade sum $\left(\sum_{i=1}^{4} \mathfrak{d}_{\upsilon_i}\right)$	Row sum $(\mathfrak{R}\mathcal{s}_1)$	Column sum $(\mathfrak{C}\mathcal{s}_1)$	$\Omega_1 = \mathfrak{R}\mathcal{s}_1 - \mathfrak{C}\mathcal{s}_2$
$\mathfrak{p}_1$	5	7	9	-2
$\mathfrak{p}_2$	11	12	3	9
$\mathfrak{p}_3$	3	5	10	-5
$\mathfrak{p}_4$	6	7	9	-2

Table 54. non-membership score table for example 14.

	Grade sum $\left(\sum_{i=1}^{4} \mathfrak{d}_{\upsilon_i}\right)$	Row sum $(\mathfrak{R}\mathcal{s}_2)$	Column sum $(\mathfrak{C}\mathcal{s}_2)$	$\Omega_2 = \mathfrak{R}\mathcal{s}_2 - \mathfrak{C}\mathcal{s}_2)$
$\mathfrak{p}_1$	5	10	7	3
$\mathfrak{p}_2$	11	3	12	-9
$\mathfrak{p}_3$	3	10	5	5
$\mathfrak{p}_4$	6	9	8	1

The final score for each alternative is calculated by subtracting the non-membership score (Ω_2) from membership score (Ω_1) as given in table (56).

Table 55. Final score along with the grades linked with IF5-SS for example 14.

	Grade sum $\left(\sum_{i=1}^{4} \mathfrak{d}_{\upsilon_i}\right)$	Ω_1	Ω_2	Final Score $\Omega_2 - \Omega_1$
$\mathfrak{p}_1$	5	-2	3	-5
$\mathfrak{p}_2$	11	9	-9	18
$\mathfrak{p}_3$	3	-5	5	-10
$\mathfrak{p}_4$	6	-2	1	-3

It is clear from tables (56) that the highest score is 18, which is got by the place $\mathfrak{p}_2$. So the place $\mathfrak{p}_2$is the best place where a family will go for a trip.

As one can note that we get the result for IFN-SS through our proposed algorithm. We can easily transform IFN-SS to CIFN-SS by letting $1 = e^{i2\pi(0.0)}$ and then we can apply an algorithm to get a solution in DM. what will happen, if a family also wants to know the view of another team of experts which is team B about these 4 places. The IFN-SS can't provide them any type of information about team B of experts but our proposed novel model can help them in this. We can provide them with this additional information To show how our model can provide them with this information and show the supremacy and superiority of our novel model we reconsider the example (14) below.

Example 15: Let the example (14) along with the additional information of the places provided by another team of experts. The grading criteria for CMG and CNMG of elements of the set $\mathfrak{P}$ is presented below as

$0.0 \leq \Delta\mu^M(\mathfrak{p}) < 0.2$ when $\mathfrak{o} = 0$;
$0.2 \leq \Delta\mu^M(\mathfrak{p}) < 0.4$ when $\mathfrak{o} = 1$;
$0.4 \leq \Delta\mu^M(\mathfrak{p}) < 0.6$ when $\mathfrak{o} = 2$;
$0.6 \leq \Delta\mu^M(\mathfrak{p}) < 0.8$ when $\mathfrak{o} = 3$;
$0.8 \leq \Delta\mu^M(\mathfrak{p}) \leq 1.0$ when $\mathfrak{o} = 4$.

Where $\Delta\mu^M(\mathfrak{p}) = \frac{\gamma^M(\mathfrak{p})+\omega_{\gamma^M}(\mathfrak{p})}{2}$, and $0 \leq \mu^M + \mu^N \leq 1$. The tabular representation of CIF5-SS is given below in table (57).

Table 56. The tabular representation of CIF5-SS displayed in example 15.

$(\rho, (\mathcal{J}, \mathcal{V}, 5))$	$\boldsymbol{v_1}$	$\boldsymbol{v_2}$	$\boldsymbol{v_3}$
$\mathfrak{p}_1$	$3, \begin{pmatrix} 0.7e^{i2\pi(0.65)}, \\ 0.2e^{i2\pi(0.15)} \end{pmatrix}$	$0, \begin{pmatrix} 0.1e^{i2\pi(0.15)}, \\ 0.8e^{i2\pi(0.7)} \end{pmatrix}$	$2, \begin{pmatrix} 0.5e^{i2\pi(0.5)}, \\ 0.4e^{i2\pi(0.25)} \end{pmatrix}$
$\mathfrak{p}_2$	$4, \begin{pmatrix} 0.95e^{i2\pi(0.9)}, \\ 0.03e^{i2\pi(0.05)} \end{pmatrix}$	$3, \begin{pmatrix} 0.75e^{i2\pi(0.7)}, \\ 0.1e^{i2\pi(0.2)} \end{pmatrix}$	$4, \begin{pmatrix} 0.93e^{i2\pi(0.9)}, \\ 0.04e^{i2\pi(0.09)} \end{pmatrix}$
$\mathfrak{p}_3$	$0, \begin{pmatrix} 0.15e^{i2\pi(0.1)}, \\ 0.8e^{i2\pi(0.75)} \end{pmatrix}$	$2, \begin{pmatrix} 0.45e^{i2\pi(0.5)}, \\ 0.3e^{i2\pi(0.4)} \end{pmatrix}$	$1, \begin{pmatrix} 0.3e^{i2\pi(0.35)}, \\ 0.6e^{i2\pi(0.4)} \end{pmatrix}$
$\mathfrak{p}_4$	$2, \begin{pmatrix} 0.55e^{i2\pi(0.45)}, \\ 0.2e^{i2\pi(0.4)} \end{pmatrix}$	$2, \begin{pmatrix} 0.44e^{i2\pi(0.55)}, \\ 0.5e^{i2\pi(0.3)} \end{pmatrix}$	$2, \begin{pmatrix} 0.5e^{i2\pi(0.45)}, \\ 0.4e^{i2\pi(0.4)} \end{pmatrix}$

To elaborate that how our novel model can carry the additional information, let $\begin{pmatrix} 0.5e^{i2\pi(0.45)}, \\ 0.4e^{i2\pi(0.4)} \end{pmatrix}$ in the bottom right cell in the table (57). One can note that 0.5 and 0.4 both carry the information about the view of team A of experts and 0.45 and 0.4 carry the information about the view of team B.
Now we construct tables for CMGs and CNMGs. The CMGs are given in table (58) and CNMGs are given in table (59).

Table 57. The tabular form of CMGs displayed in example 15.

μ^M	$\boldsymbol{v_1}$	$\boldsymbol{v_2}$	$\boldsymbol{v_3}$
$\mathfrak{p}_1$	$0.7e^{i2\pi(0.65)}$	$0.1e^{i2\pi(0.15)}$	$0.5e^{i2\pi(0.5)}$
$\mathfrak{p}_2$	$0.95e^{i2\pi(0.9)}$	$0.75e^{i2\pi(0.7)}$	$0.73e^{i2\pi(0.9)}$
$\mathfrak{p}_3$	$0.15e^{i2\pi(0.1)}$	$0.45e^{i2\pi(0.5)}$	$0.3e^{i2\pi(0.35)}$
$\mathfrak{p}_4$	$0.55e^{i2\pi(0.45)}$	$0.44e^{i2\pi(0.55)}$	$0.5e^{i2\pi(0.45)}$

Table 58. The tabular form of CNMGs displayed in example 15.

μ^N	$\boldsymbol{v_1}$	$\boldsymbol{v_2}$	$\boldsymbol{v_3}$
$\mathfrak{p}_1$	$0.2e^{i2\pi(0.15)}$	$0.8e^{i2\pi(0.7)}$	$0.4e^{i2\pi(0.25)}$
$\mathfrak{p}_2$	$0.03e^{i2\pi(0.05)}$	$0.1e^{i2\pi(0.2)}$	$0.04e^{i2\pi(0.09)}$
$\mathfrak{p}_3$	$0.8e^{i2\pi(0.75)}$	$0.3e^{i2\pi(0.44)}$	$0.6e^{i2\pi(0.4)}$
$\mathfrak{p}_4$	$0.2e^{i2\pi(0.4)}$	$0.5e^{i2\pi(0.3)}$	$0.4e^{i2\pi(0.4)}$

Next, we will construct the comparison tables for CMGs and CNMGs which are given in tables (60) and (61) respectively.

Table 59. Comparison table of CMG displayed in example 15.

.	$\mathfrak{p}_1$	$\mathfrak{p}_2$	$\mathfrak{p}_3$	$\mathfrak{p}_4$
$\mathfrak{p}_1$	3	0	2	2
$\mathfrak{p}_2$	3	3	3	3
$\mathfrak{p}_3$	1	0	3	1
$\mathfrak{p}_4$	1	0	2	3

Table 60. Comparison table of CNMG displayed in example 15.

.	$\mathfrak{p}_1$	$\mathfrak{p}_2$	$\mathfrak{p}_3$	$\mathfrak{p}_4$
$\mathfrak{p}_1$	3	3	1	1
$\mathfrak{p}_2$	0	3	0	0
$\mathfrak{p}_3$	2	3	3	2
$\mathfrak{p}_4$	2	3	1	3

Now we will calculate the CM and CNM scores. For finding both scores we will subtract the column sum from the row sum of the table (60) and (61). Both scores are given in tables (62) and (63) respectively.

Table 61. CM score table for example 15.

	Grade sum $\left(\sum_{i=1}^{4} \mathfrak{o}_{v_i}\right)$	Row sum $(\mathfrak{R}s_1)$	Column sum $(\mathfrak{C}s_1)$	$\Omega_1 = \mathfrak{R}s_1 - \mathfrak{C}s_2$
$\mathfrak{p}_1$	5	7	8	−1
$\mathfrak{p}_2$	11	12	3	9
$\mathfrak{p}_3$	3	5	10	−5
$\mathfrak{p}_4$	6	6	9	−3

Table 62. CNM score table for example 15.

	Grade sum $\left(\sum_{i=1}^{4} \mathfrak{d}_{\mathscr{v}_i}\right)$	Row sum $(\mathfrak{R}\mathscr{s}_2)$	Column sum $(\mathfrak{C}\mathscr{s}_2)$	$\Omega_2 = \mathfrak{R}\mathscr{s}_2 - \mathfrak{C}\mathscr{s}_2)$
$\mathfrak{p}_1$	5	8	7	1
$\mathfrak{p}_2$	11	3	12	-9
$\mathfrak{p}_3$	3	10	5	5
$\mathfrak{p}_4$	6	9	6	3

The final score for each alternative is calculated by subtracting the NM score (Ω_2) from membership score (Ω_1) as given in table (64).

Table 63. Final score along with the grades linked with CIF5-SS for example 15.

	Grade sum $\left(\sum_{i=1}^{4} \mathfrak{d}_{\mathscr{v}_i}\right)$	Ω_1	Ω_2	Final Score $\Omega_2 - \Omega_1$
$\mathfrak{p}_1$	5	-1	1	0
$\mathfrak{p}_2$	11	9	-9	18
$\mathfrak{p}_3$	3	-5	5	-10
$\mathfrak{p}_4$	6	-3	3	0

It is clear from table (64) that the highest score is 18, which is got by the place $\mathfrak{p}_2$. So the place $\mathfrak{p}_2$is the best place where a family will go for trip.

VI. Conclusion

The basic theme of this study was to diagnose a CIFN-SS which is the finest and richest structure to overcome the intricate and obstinate information which contains the parameters with grades in two-dimension. CIFN-SS is the fusion of N-SS and CIFSS which modified a few prevailing theories such as FS, SS, N-SS, CFS, CFSS, IFS, IFSS, etc. Moreover, this study contained the basic properties and operations of the diagnosed CIFN-SS along with an example to illustrate them. Further, this study developed the relationship of the novel model with prevailing models such as CIFSSs and SSs. After that, in this manuscript, the credibility and efficiency of the diagnosed work are shown with the assistance of DM numerical examples. For solving these examples this study contained a novel algorithm in the setting of CIFN-SS. Finally, this study showed the supremacy of the diagnosed model by comparing it with a prevailing model such as IFN-SS.

In the future, our aim is to review numerous literature like T-spherical FS (TSFS) [4], interval-valued TSFS [44], bipolar CFS [45], etc., and try to utilize it in the diagnosed work.

Data Availability:

The data used in this article are artificial and hypothetical, and anyone can use these data before prior permission by just citing this article.

Conflicts of Interest:

The authors declare that they have no conflicts of interest.

Abbreviations

For better understanding, the abbreviations and full names are displayed in table 65.

Table 64. The abbreviations and full names of various terminologies.

Abbreviations	Full Name
FS	Fuzzy set
IFS	Intuitionistic fuzzy set
SS	Soft Set
N-SS	N-soft set
IFSS	Intuitionistic fuzzy soft set
IFN-SS	Intuitionistic fuzzy N-soft set
CFS	Complex fuzzy set
CFSS	Complex fuzzy soft set
CIFN-SS	Complex intuitionistic fuzzy N-soft set
DM	Decision-making
CMG	Complex membership grade
CNMG	Complex non-membership grade
CM	Complex membership
CNM	Complex non-membership

References

[1] Zadeh, L. A. (1965). Fuzzy sets. Information and control, 8(3), 338-353.

[2] Atanassov, K. (2016). Intuitionistic fuzzy sets. International Journal Bioautomation, 20, 1.

[3] Mahmood, T., Ullah, K., Khan, Q., & Jan, N. (2019). An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets. Neural Computing and Applications, 31(11), 7041-7053.

[4] Ullah, K., Mahmood, T., & Jan, N. (2018). Similarity measures for T-spherical fuzzy sets with applications in pattern recognition. Symmetry, 10(6), 193.

[5] Ullah, K., Mahmood, T., Jan, N., & Ahmad, Z. (2020). Policy decision making based on some averaging aggregation operators of t-spherical fuzzy sets; a multi-attribute decision making approach. Annals of Optimization Theory and Practice, 3(3), 69-92.

[6] Liu, P., Munir, M., Mahmood, T., & Ullah, K. (2019). Some similarity measures for interval-

valued picture fuzzy sets and their applications in decision making. Information, 10(12), 369.

[7] Ullah, K. (2021). Picture fuzzy maclaurin symmetric mean operators and their applications in solving multiattribute decision-making problems. Mathematical Problems in Engineering, 2021.

[8] Ramot, D., Milo, R., Friedman, M., & Kandel, A. (2002). Complex fuzzy sets. IEEE Transactions on Fuzzy Systems, 10(2), 171-186.

[9] Mahmood, T., Ur Rehman, U., Ali, Z., & Mahmood, T. (2020). Hybrid vector similarity measures based on complex hesitant fuzzy sets and their applications to pattern recognition and medical diagnosis. Journal of Intelligent & Fuzzy Systems, (Preprint), 1-22.

[10] Mahmood, T., Ur Rehman, U., Ali, Z., & Chinram, R. (2020). Jaccard and Dice Similarity Measures Based on Novel Complex Dual Hesitant Fuzzy Sets and Their Applications. Mathematical Problems in Engineering, 2020.

[11] Atanassov, K. T. (1994). Operators over interval valued intuitionistic fuzzy sets. Fuzzy sets and systems, 64(2), 159-174.

[12] Gorzałczany, M. B. (1987). A method of inference in approximate reasoning based on interval-valued fuzzy sets. Fuzzy sets and systems, 21(1), 1-17.

[13] Pawlak, Z., Grzymala-Busse, J., Slowinski, R., & Ziarko, W. (1995). Rough sets. Communications of the ACM, 38(11), 88-95.

[14] Pawlak, Z. (2012). Rough sets: Theoretical aspects of reasoning about data (Vol. 9). Springer Science & Business Media.

[15] Pawlak, Z., & Skowron, A. (2007). Rudiments of rough sets. Information sciences, 177(1), 3-27.

[16] Pawlak, Z., & Skowron, A. (2007). Rough sets: some extensions. Information sciences, 177(1), 28-40.

[17] Pawlak, Z., & Skowron, A. (2007). Rough sets and Boolean reasoning. Information sciences, 177(1), 41-73.

[18] Molodtsov, D. (1999). Soft set theory—first results. Computers & Mathematics with Applications, 37(4-5), 19-31.

[19] Ali, M. I., Feng, F., Liu, X., Min, W. K., & Shabir, M. (2009). On some new operations in soft set theory. Computers & Mathematics with Applications, 57(9), 1547-1553.

[20] Ma, X., Zhan, J., Ali, M. I., & Mehmood, N. (2018). A survey of decision making methods based on two classes of hybrid soft set models. Artificial Intelligence Review, 49(4), 511-529.

[21] Herawan, T., Rose, A. N. M., & Deris, M. M. (2009, December). Soft set theoretic approach for dimensionality reduction. In International Conference on Database Theory and Application (pp. 171-178). Springer, Berlin, Heidelberg.

[22] Sun, B., Ma, W., & Li, X. (2017). Linguistic value soft set-based approach to multiple criteria group decision-making. Applied Soft Computing, 58, 285-296.

[23] Min, W. K. (2012). Similarity in soft set theory. Applied Mathematics Letters, 25(3), 310-314.

[24] ALSHAMI, T., & EL-SHAFEI, M. O. H. A. M. M. E. D. (2020). $ T $-soft equality relation. Turkish Journal of Mathematics, 44(4), 1427-1441.

[25] Manna, S., Basu, T. M., & Mondal, S. K. (2020). A soft set based VIKOR approach for some decision-making problems under complex neutrosophic environment. Engineering Applications of Artificial Intelligence, 89, 103432.

[26] Zulqarnain, R. M., Abdal, S., Maalik, A., Ali, B., Zafar, Z., Ahamad, M. I., ... & Dayan, F. (2020). Application of TOPSIS method in decision making via soft set. Biomed J Sci Tech Res, 24(3).

[27] Vijayabalaji, S., & Ramesh, A. (2019). Belief interval-valued soft set. Expert Systems with Applications, 119, 262-27

[28] Roy, A. R., & Maji, P. K. (2007). A fuzzy soft set theoretic approach to decision making problems. Journal of computational and Applied Mathematics, 203(2), 412-418.

[29] Maji, P. K., Biswas, R., & Roy, A. R. (2001). Intuitionistic fuzzy soft sets. Journal of fuzzy mathematics, 9(3), 677-692.

[30] Thirunavukarasu, P., Suresh, R., & Ashokkumar, V. (2017). Theory of complex fuzzy soft set and its applications. International Journal for Innovative Research in Science & Technology, 3(10), 13-18.

[31] Kumar, T., & Bajaj, R. K. (2014). On complex intuitionistic fuzzy soft sets with distance measures and entropies. Journal of Mathematics, 2014.

[32] Ma, X., Liu, Q., & Zhan, J. (2017). A survey of decision making methods based on certain hybrid soft set models. Artificial Intelligence Review, 47(4), 507-530.

[33] Zou, Y., & Xiao, Z. (2008). Data analysis approaches of soft sets under incomplete information. Knowledge-Based Systems, 21(8), 941-945.

[34] Alcantud, J. C. R., & Laruelle, A. (2014). Dis&approval voting: a characterization. Social Choice and Welfare, 43(1), 1-10.

[35] Herawan, T., & Deris, M. M. (2009, September). On multi-soft sets construction in information systems. In International Conference on Intelligent Computing (pp. 101-110). Springer, Berlin, Heidelberg.

[36] Fatimah, F., Rosadi, D., Hakim, R. F., & Alcantud, J. C. R. (2018). N-soft sets and their decision making algorithms. Soft Computing, 22(12), 3829-3842.

[37] Alcantud, J. C. R., Feng, F., & Yager, R. R. (2019). An $ N $-soft set approach to rough sets. IEEE Transactions on Fuzzy Systems, 28(11), 2996-3007.

[38] Riaz, M., Çağman, N., Zareef, I., & Aslam, M. (2019). N-soft topology and its applications to multi-criteria group decision making. Journal of Intelligent & Fuzzy Systems, 36(6), 6521-6536.

[39] Akram, M., Adeel, A., & Alcantud, J. C. R. (2018). Fuzzy N-soft sets: A novel model with applications. Journal of Intelligent & Fuzzy Systems, 35(4), 4757-4771.

[40] Fatimah, F., & Alcantud, J. C. R. (2021). The multi-fuzzy N-soft set and its applications to decision-making. Neural Computing and Applications, 33(17), 11437-11446.

[41] Akram, M., Ali, G., & Alcantud, J. C. R. (2019). New decision-making hybrid model: intuitionistic fuzzy N-soft rough sets. Soft Computing, 23(20), 9853-9868.

[42] Mahmood, T. Ur Rehman U., & Ali, Z. (2021). A novel complex fuzzy N-soft sets and their decision-making algorithm. Complex & Intelligent Systems, 7(5), 2255-2280.

[43] Alkouri, A. M. D. J. S., & Salleh, A. R. (2012, September). Complex intuitionistic fuzzy sets. In AIP Conference Proceedings (Vol. 1482, No. 1, pp. 464-470). American Institute of Physics.

[44] Hussain, A., Ullah, K., Wang, H., & Bari, M. (2022). Assessment of the Business Proposals Using Frank Aggregation Operators Based on Interval-Valued T-Spherical Fuzzy Information. Journal of Function Spaces, 2022.

[45] Mahmood, T., & Ur Rehman, U. (2022). A novel approach towards bipolar complex fuzzy sets and their applications in generalized similarity measures. International Journal of Intelligent Systems, 37(1), 535-567.