Technical Journal, University of Engineering and Technology (UET) Taxila, Pakistan *Vol. 27 No. 1-2022*
ISSN:1813-1786 (Print) 2313-7770 (Online)

Statistical Analysis of Rainfall Trends in Balochistan and Sindh

A. Aurangzeb[1], K. Abbas[2], A. Iqbal[3], M. Altaf [4], M. T. Nadeem[5], F. Shahzad[6]

[1,2,3] *Department of Statistics, King Abdullah Campus, Chatter KALAS, The University of Azad Jammu and Kashmir, Pakistan*
[4,6] *Department of Basic Sciences, University of Engineering and Technology, Taxila, Pakistan*
[5] *Department of Science and Humanities, National University of Computer and Emerging Sciences, Islamabad, Pakistan*

Email: dm.altaf@uettaxila.edu.pk

Abstract- Frequency analysis of regional rainfall is very important in the design of the civil structure as well as the nonstructural problem. In the present study an attempt has been made to find out the best fitted distribution(s) to enlighten the rainfall trends in Balochistan and Sindh. The secondary data is obtained from Pakistan metrological department for the period 1980 to 2015. Firstly, we have checked the basic assumption such as randomness and identically of distribution of the observed data series by using Run test and Mann Whitney test. Further, three parameter distributions named as Generalized extreme value, Weibull and Frechet three parameters distributions, and compared with two parameters Weibull, Frechet and Log logistic distributions. The parameters of the three and two parameters candidate distributions were obtained by using maximum likelihood and probability weighted moment methods. The performances of the candidate distributions were checked out by using Kolmogorov Smirnove test at 5 percent level of significance. Moreover, probability density functions graphs were used to indorse the behavior and the theoretical framework of the rainfall distributions of selected sites. Furthermore, quantiles were estimated for future prediction. The resultant estimates depicted that two parameters distributions and three parameters distributions were the most appropriate for annual maximum rainfall series.

Keywords- Generalized extreme value distribution; Frechet distribution; Weibull distribution; Log-logistic distribution; Maximum likelihood method; Probability weighted moment method.

I. INTRODUCTION

The problem of climate variation has occurred over the last two decades on a global scale, given its expected impacts on the environment of week states. The dominant components of climatic variations were spatial changes in rainfall patterns, related with variations in the overall circulation of air in the region (Rodo and Comin 2003). Rain is the main source in the growing of production of food and crops, both in the fruit development stage and germination. But through a variation in world's climate, temperatures will rise and rainfall will decrease in some places. Rain acts as a good source of water as well as snow and glaciers. Therefore, rainfall is important for water source organization. During these ages it has been detected that all societies are living on the banks of rivers and where the abundance of available rainfall for domestic and agriculture use. Rain also has a lifelong impact on health. Ground water quality depends largely on available rainfall. Although other factors are involved, rain plays a main role in refreshing water reservoirs and channels.

Pakistan is situated from south to 61^0 –76^0 east latitude and northwest at 23^0 –37^0 north latitude. The summer monsoon is very important for agricultural and social needs economic in subcontinent. The monsoon of Pakistan gives nearly 65% to 75% of the overall annual rainfall. Maximum area of the country have monsoon season (Jameel *et al*., 2005) from July to September. Pakistan has four provinces and all have different climatic conditions. Here we discuss rainfall trends for two provinces that are Balochistan and Sindh.

1.1. Rainfall in Balochistan

Balochistan is situated at 66°59' 47.2272"E and 30°10' 59.7720" N. Total area of Balochistan is 347,190 km which is about forty four percent of the whole land area of Pakistan. The summers are warm and dry and winters of the lowlands vary from extremely cold. The plane areas are also very warm in summer with temperatures reaching 50 degrees C (120 degrees F). The winters are mild in the plains with temperatures and not ever fall under the freezing point.

Regular annual rainfall in Balochistan varies. According to the Pakistan Meteorological Department (PMD), the maximum rainfall in the northeast is 200 to 500 mm (8 to 20 inches) with an average annual rainfall. The evaporation rate is much higher than the rainfall and usually costs 72 to 76 inches (1830 1930 mm) per year.

1.2. Rainfall in Sindh

Sindh is located at 24 ° 56 '24.33240' N and 68 ° 13 '51.1716' E southwest corner, which borders the Iranian Set in the west. Sindh is the third largest province, with an area of 140,915 km in the Pakistani border. Sindh is warm in summer and cold in winter. According to PMD, the regular rainfall in Sindh is only fifteen to eighteen centimeter in a year.

Daily rainfall modeling has been applied all over the world, using different mathematical models to develop a prediction model for rainfall patterns and their characteristics. Different methods and techniques have been planned in the literature for modeling rainfall amount. Some popular distributions are useful for extreme values like Generalized Pareto (GP), Generalized Exponential, Gamma, Kappa, Generalized Extreme Value distribution (GEVD) and Weibull distributions.

Objectives of the study

The current study is being conducted to

- test the basic assumption of annual maximum rainfall data such as randomness and identically of distribution by using Run test and Mann Whitney test respectively.
- perform regional rainfall frequency analysis using GEVD, Weibull 3 parametric distribution (W3D), Frechet 3 parametric distribution (F3D) and two parameters Weibull distribution (WD), Frechet distribution (FD) and Log-logistic distribution (LLD) based on Maximum Likelihood (ML) and Probability weighted Moment (PWM) methods.
- identifying the suitable distribution(s) for modeling the rainfall frequency analysis of all sites with both (ML and PWM) methods based on Kolmogorov Smirnove (KS) test.

Including this introduction section, the rest of the paper unfolds as follows. Section 2 comprises the literature review and description of study is presented in Section 3. Section 4 is dedicated for verification of basic assumptions and statistical analysis is conducted in Section 5.

II. LITERATURE REVIEW

Various probability distributions with different methods of estimations using regional as well as at-site approach have been applied in the literature for modeling rainfall data all over the world including Pakistan. Some of the published international and indigenous studies are summarized below.

2.1. International studies

Alam (2018) determined the best probability distributions of extreme monthly rainfall using thirty years data (1984 to 2013) from thirty-five sites in Bangladesh. Log Pearson type-III, GEVD and Pearson type-III distributions presented the best fist results. Bari (2016) investigated 50 years (from 1964 to 2013) rainfall trend and fluctuations over time in Northern Bangladesh. They found that pre monsoon and post monsoon rainfall was increasing in maximum of the locations they selected. They applied autocorrelation and nonparametric Mann Kendall test. The sequential Mann Kendall test identified numerous nonsignificant points of change for seasonal and yearly rainfall at maximum of the locations. They also identified the projected fluctuations. Chikobvu and Chifurira (2015) described the annual rainfall for the data from 1901 to 2009 in Zimbabwe. To estimate the probabilities of rainfall, GEVD and Weibull distribution were used. Mayooran and Laheetharan (2014) used regular rainfall data of Colombo, Sri Lanka for the period 1900 to 2009 and found that Burr (4P) and Pearson 3 were appropriate probability model. GEVD was also observed for monsoon remaining seasons. Sherif (2014) analyzed the temporal and spatial characteristics of rainfall in the United Arab Emirates. The Gumbel, Log Pearson, Weibull GEVD, GN and Wake by probability distributions were tested. Both Weibull and Gumbel distributions were found appropriate. Similarly, Khudri and Sadia (2013) determined the distribution of the best fit probability of the annual maximum rainfall data of twenty-two stations in Bangladesh using different statistical analyzes. GEVD, Normal, Pearson, Gamma, Weibull, lognormal and different forms of these distributions were estimated. Singh *et al.*, (2012) used the daily rainfall data from 1973 to 2011 to determine the annual maximum rainfall of Jhalarapatan area of Rajasthan, India. Expected values were estimated by four well known distribution functions viz., Log Normal, Normal, Gumbel and Log Pearson type-III and observed values were estimated by Weibull's plotting position. The estimated results showed that the Log Pearson type-III distribution was the best fit probability distribution to prediction the maximum annual rainfall of daily for changed return periods.

2.2. Case studies of Pakistan

Hussain (2016) presented the results of regional frequency analysis of Annual Maximum Monthly Rainfall Totals (AMMRT) of seven sites of Sindh, Pakistan. The results of L moment have shown that the three distributions: Pearson type-III, Generalized Normal (GN) and GP were appropriate candidates for regional distribution. Amin (2016) found the probability distribution of best adjustment of the maximum annual rainfall, based on a twenty four hour sample in the northern regions of Pakistan using four distributions Log Pearson type III, Log Normal, Gumbel maximum and Normal. Results showed that the distribution of Log Pearson type-III is better adjustment in the rest of the rain measuring sites. Hussain and Pasha (2016) conducted a frequency analysis of seven regions of Pakistan to identify suitable distribution for those regions. Which were based on Z statistic and L moments ratio, they recognized Generalized Normal, Pareto and GEV distributions were appropriate for that area. Ahmad (2016) carried out at site frequency analysis of annual maximum daily rainfall (AMDR) series using linear moments (L moments), trimmed linear moments (TL moments) and higher order linear moments (LH moments) by detecting 28 meteorological observatories across Pakistan. They used five distributions as more suitable namely: Pearson type III, GEV, three parameter Lognormal, GP and Generalized Logistic. They determined that the method of L moments was good for eight locations, TL moments method good fit for six locations and LH moments method good fit for fourteen locations.
Shahzadi (2013) and Shahzadi (2013) performed L-moments based regional frequency analysis using annual maximum rainfall series of 23 stations of Pakistan. The study concluded that Generalized Normal, Generalized Extreme Value and Generalized Logistic distributions are suitable to model the observed data series for the regions under study. Moreover, Abbas (2012) explained the annual maximum of daily rainfall form the years 1954 to 2005 of four sites in Pakistan. For that purpose GEVD, Gamma distribution and GP distribution were fitted for each station to annual maximum of daily rainfall data. Parameters of distributions were estimated using the ML and PWM methods.

Description Demography of Study Area

This study consist of regional frequency analysis by using annual maximum rainfall of nine sites in Balochistan viz; Barkan, Dalbandin, Jiwani, Kalat, Nokkundi, Panjur, Pansi, Sibbi, Quetta and also three sites of Sindh viz; Karachi, Badin and Hyderabad from 1980 to 2015. We have selected the largest amount of rainfall for each year. Sites characteristics and map of all sites are discussed below.

Table 1. Sites characteristics of Balochistan and Sindh

Sites Name	Latitude	Longitude	Elevation
Barkhan	29.53	69.43	1097
Dalbandin	28.53	64.24	848
Jiwani	25.04	61.48	56
Kalat	29.02	66.35	2007
Nokkundi	28.49	62.45	0682
Panjur	26.58	64.06	968
Badin	24.38	68.54	09
Hyderabad	25.23	68.25	28
Karachi	24.54	66.56	22
Pasni	25.16	63.29	11
Quetta	30.11	66.57	1626
Sibbi	29.33	67.53	133

The characteristic of twelve sites are shown in Table-1 and Figure 1. It is observed that Kalat has got tremendous highest elevation of sea level 2007 meter (m), Quetta is considered a second highest site i.e., 1626m elevation above sea level. Badin and Pasni have got almost same elevation (9m and 11m) rather than sea level. Remaining sites Hyderabad, Karachi, Barkhan, Dalbadin, Jiwani, Nokkundi, Panjur pasni and Sibbi possessed are 28m, 22m, 1097m, 848m, 56m, 682m, 968m and 133m respectively elevation than the sea level.

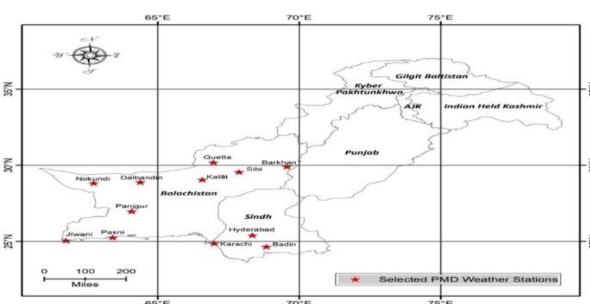

Figure 1: Map of twelve sites of Balochistan and Sindh

Verification of Basic Assumptions

Before selecting the candidate distribution namely GEVD, W3D, F3D, WD, FD and LLD it is necessary to check the basic assumptions identically of distributions. To check the identically of distribution Mann Whitney (W) test is used and result is presented in Table 2.

Table 2: Mann Whitney statistics values and P- values

S.No.	Sites Name	W	P-value
1	Badin	103	0.0636
2	Hyderabad	132	0.3550
3	Karachi	149	0.6925
4	Barkhan	60	0.0008*
5	Dalbandin	169	0.8391
6	Jiwani	190	0.3888
7	Quetta	165	0.9378
8	Nokkundi	136	0.4245
9	Panjur	163	0.9875
10	Panjur	174	0.7193
11	Sibbi	123	0.2262
12	Kalat	163	0.9875

Table 2 shows the results of Mann Whitney statistics (W), first we divide the data of each site into two groups (18, 18). The corresponding P-value of eleven sites except Barkhan (P-value<0.05) provides evidence about the acceptance of hypothesis of identical distribution at 5 percent level of significance.

Statistical Analysis

Table 3 shows the basic statistics of twelve sites of Balochistan and Sindh. It is observed that Kalat site receives highest rainfall as 15.99 mm whereas Karachi lowest rainfall as 12.24mm on average. Coefficient of skewness ranges 0.61 to 3.76 indicates that distribution is asymmetric. Similarly, Coefficient of Variation (CV) ranges from 35.08 to 102.65 also shows that there is wide variation of rainfall data. The kurtosis ranges from 2.5338 to 19.6088 showing that there are maximum peak flows in the observed dataset.

Table 3: Basic statistics of twelve sites

Sites Name	Mean	Var.	Min.	Max.	Skewness	CV	Kurtosis
Barkhan	12.75	20.032	7.067	28.12	1.38	35.08	5.1229
Dalbandin	12.70	62.008	4.570	32.04	1.17	61.97	3.1467
Jiwani	14.66	134.89	1.433	45.788	1.37	79.23	3.7706
Kalat	15.99	140.40	5.373	61.297	2.19	74.09	8.2435
Nokundi	12.59	77.490	1.070	33.48	0.61	69.92	2.5338
Panjur	13.92	123.77	0.781	60.633	2.11	79.90	9.5735
Pasni	14.32	104.79	1.280	37.80	1.11	71.510	3.1936
Sibbi	13.59	92.180	1.930	56.70	2.75	70.66	12.5690
Quetta	15.08	37.860	4.740	28.52	0.77	40.81	2.7518
Badin	13.93	129.58	0.644	67.26	3.15	81.74	14.9901
Hyderabad	13.50	109.04	2.080	51.12	2.03	77.38	7.1523
Karachi	12.24	157.89	0.884	75.96	3.76	102.65	19.6088

Note: Min. = Minimum, Max. = Maximum, CV = Coefficient of Variation.

Estimation of parmaters of Candidate Distributions

ML and PWM methods are used to estimate the parameters of GEVD, W3D, F3D, FD, WD and LLD distributions.

GEVD

The GEVD (Jenkinson, 1955) is important for indicating extremes events, and it is importance in hydrology, since it was recommended by the Flood Studies Report (Natural Environment Research Council 1975) for modeling the distribution of annual maximum of everyday stream flows of British rivers. Probability Density Dunction (PDF) and Commulative Distribution Function (CDF) of GEVD are given below.

$$f(x) = \begin{cases} \frac{1}{\sigma} exp\left(-(1+\varepsilon z)^{-\frac{1}{\varepsilon}}(1+\varepsilon z)^{-1-\frac{1}{\varepsilon}}\right) & \varepsilon \neq 0 \\ \frac{1}{\sigma} exp(-z - exp(-z)) & \varepsilon = 0 \end{cases} \quad Where \quad z = \frac{x-\mu}{\sigma} \quad (1)$$

$$F(x) = \begin{cases} \frac{1}{\sigma} exp\left(-(1+\varepsilon z)^{-\frac{1}{\varepsilon}}\right) & \varepsilon \neq 0 \\ \frac{1}{\sigma} exp(-exp(-z)) & \varepsilon = 0 \end{cases} \quad -\infty < x, \mu < \infty, \ \sigma, \varepsilon > 0 \quad (2)$$

$\mu, \sigma \ and \ \varepsilon$ are location scale and shape parameters respectively. The estimates of the parameter along with p-values of KS test are presented in Table 4.

Table 4: Estimates of Parameters for GEVD.

Sites Name	Methods	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\varepsilon}$	KS	P-values
BARKHAN	ML	10.5567	2.8169	0.1792	0.0680	0.9922
	PWM	10.5584	2.9514	0.1457	0.0652	0.9954
DALBANDIN	ML	8.2130	3.5558	0.5264	0.1102	0.7326
	PWM	8.6260	4.3714	0.2679	0.1163	0.6713
JIWANI	ML	8.9116	6.2760	0.2707	0.1313	0.5215
	PWM	8.8731	6.1135	0.2752	0.1250	0.5834
KALAT	ML	9.9247	4.8734	0.5010	0.0955	0.8665
	PWM	10.2738	5.5706	0.3164	0.0758	0.9756
NOKKUNDI	ML	8.3526	6.7209	0.0481	0.0934	0.8829
	PWM	8.4815	7.3951	0.0223	0.0741	0.9805
PANJUR	ML	8.6361	6.3954	0.2167	0.0869	0.9264
	PWM	8.8029	6.9738	0.1381	0.0862	0.9307
PASNI	ML	9.0166	6.0382	0.2610	0.0774	0.9704
	PWM	9.1268	6.5110	0.1836	0.0816	0.9542
SIBBI	ML	9.5431	5.0797	0.1710	0.0727	0.9837
	PWM	9.2265	4.6027	0.2758	0.0891	0.9132
QUEETA	ML	12.2614	4.7120	0.0142	0.0839	0.9431
	PWM	12.1225	4.7380	0.0448	0.0779	0.9688
BADIN	ML	9.4729	6.0715	0.1252	0.1521	0.3403
	PWM	9.2001	5.2401	0.2501	0.1533	0.3315
HYDERABAD	ML	8.6932	5.4660	0.2386	0.1010	0.8201
	PWM	8.5707	5.3060	0.2651	0.1058	0.7761
KARACHI	ML	7.3707	5.4714	0.2199	0.1220	0.6573
	PWM	7.0907	5.0292	0.3151	0.1381	0.4574

On the basis of P- values of KS test at 5 percent level of significance it is concluded that ML and PWM are best fitted for GEVD. Therefore, we recommend that ML and PWM methods can be used to study the annual maximum rainfall for such sites.

Weibull 3 Parameters Distribution (W3D)

W3D is continuous probability distribution named after Sewdish mathematiciation W.Weibull who described in 1939. As an extreme value distribution the W3D has proven quite successful in predicting the accurance of extreeme phenomena like floods, rainfall, earth quack and high wind speed. The PDF and CDF of W3D are given below

$$f(x;\mu,\sigma,\varepsilon) = \frac{\mu}{\sigma}\left(\frac{x-\mu}{\sigma}\right)^{\mu-1} exp\left\{-\left(\frac{x-\varepsilon}{\sigma}\right)^{\mu}\right\}, \quad -\infty < \mu < +\infty, \quad \sigma > 0, \quad \varepsilon > 0 \quad (3)$$

$$F(x,\mu,\sigma,\varepsilon) = 1 - exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\varepsilon} \; x \geq \mu\right], \quad (4)$$

μ, σ *and* ε are location, scale and shape parameters respectively and the estimates of paramters for different sites are presented in Table 5.

Table 5: Estimates of Parameters for W3D.

Sites Name	Methods	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\varepsilon}$	KS	P-values
BARKHAN	ML	6.9639	6.2822	1.3213	0.0691	0.9907
	PWM	7.3233	5.9122	1.1979	0.0679	0.9924
DALBANDIN	ML	4.5717	11.3137	0.9999	0.20412	0.0859
	PWM	4.6816	7.7677	0.9477	0.0966	0.8577
JIWANI	ML	1.3305	14.0256	1.1608	0.1568	0.3056
	PWM	3.4186	10.7879	0.9349	0.1681	0.2331
KALAT	ML	5.3731	15.8495	0.9281	0.2145	0.0620
	PWM	5.6069	9.46375	0.8662	0.0762	0.9746
NOKKUNDI	ML	0.9569	12.2686	1.1906	0.0919	0.8936
	PWM	-2.4693	16.5452	1.7080	0.0691	0.9906
PANJUR	ML	1.0000	15.4727	1.4071	0.1174	0.6599
	PWM	1.0623	14.0701	1.2165	0.1327	0.5074
PASNI	ML	1.0636	14.4439	1.3390	0.1066	0.7677
	PWM	2.4179	12.5704	1.1123	0.0849	0.9380
SIBBI	ML	1.7110	13.0805	1.3970	0.1339	0.4966
	PWM	5.1235	8.1175	0.9339	0.1246	0.5874
QUEETA	ML	4.0229	12.4655	1.9031	0.10684	0.7663
	PWM	5.9404	10.3159	1.4731	0.08685	0.9271
BADIN	ML	1.0000	15.8424	1.4722	0.2436	0.0229
	PWM	4.3401	9.4729	0.9798	0.1901	0.1300
HYDERABAD	ML	2.0009	12.1507	1.1733	0.13976	0.4428
	PWM	3.7619	9.4543	0.9528	0.13055	0.5289
KARACHI	ML	1.0000	13.2927	1.3398	0.1709	0.2435
	PWM	2.8693	8.5535	0.8628	0.1931	0.1361

The estimation of the parameter of W3D by using ML and PWM methods are close to each other. On the basis of P values of KS test at 5 percent level of significance it is decided that ML and PWM are best fitted for W3D. We can recommend that ML and PWM methods can be used to study the annual maximum rainfall for such sites.

F3D

F3D was presented by Maurice Ferchet in 1927 for large extrems. In hydrology it is applied to extreme events such as annually maximum rainfall and river discharge. It is also known as inverse Weibull distribution and a special case of GEVD. It has PDF

$$f(x;\ \mu,\sigma,\varepsilon) = \frac{\varepsilon}{\sigma}\left(\frac{x-\mu}{\mu}\right)^{-(\varepsilon+1)} exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{-\varepsilon}\right], x > 0, \delta, \varepsilon > 0, -\infty < \mu + \infty \quad (5)$$

And the CDF

$$F(x;\mu,\sigma,\varepsilon) = exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{-\varepsilon}\right], \quad x > 0, \delta, \varepsilon > 0, -\infty < \mu + \infty \quad (6)$$

μ, σ *and* ε are location scale and shape parameters respectively and their estimates are presented in Table 6.

Table 6: Estimates of Parameters for F3D.

Sites Name	Methods	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\varepsilon}$	KS	P-values
BARKHAN	ML	-5.1557	15.7230	5.5806	0.0680	0.9922
	PWM	-9.6910	20.2494	6.8608	0.0652	0.9954
DALBANDIN	ML	1.4576	6.7549	1.8998	0.1102	0.7326
	PWM	-7.6880	16.3141	3.7319	0.1163	0.6713
JIWANI	ML	-14.2630	23.1750	3.6920	0.1313	0.5216
	PWM	-13.3362	22.2094	3.6327	0.1250	0.5833
KALAT	ML	0.1977	9.7254	1.9956	0.0955	0.8665
	PWM	-7.3303	17.6042	3.1602	0.0758	0.9756
NOKKUNDI	ML	-131.8700	140.2300	20.8610	0.0912	0.8901
	PWM	*	*	*	*	*
PANJUR	ML	-20.8480	29.4840	4.6111	0.0869	0.9264
	PWM	-41.6611	50.4640	7.2361	0.0862	0.9307
PASNI	ML	-14.1150	23.1320	3.8301	0.0774	0.9704
	PWM	-26.3278	35.4546	5.4452	0.0816	0.9542

SIBBI	ML	-20.1460	29.6890	5.8451	0.0727	0.9837
	PWM	-7.4592	16.6857	3.6251	0.0891	0.9132
QUEETA	ML	-320.3200	332.5800	70.5730	0.0830	0.9431
	PWM	-93.4822	105.6047	22.2882	0.0779	0.9688
BADIN	ML	-38.9200	48.3920	7.9734	0.1521	0.3403
	PWM	-11.7456	20.9460	3.9972	0.1533	0.3315
HYDERABAD	ML	-14.2120	22.9050	4.1900	0.1010	0.8200
	PWM	-11.4441	20.0149	3.7721	0.1058	0.7761
KARACHI	ML	-17.4990	24.8690	4.5450	0.1220	0.6138
	PWM	-8.8661	15.9569	3.1728	0.1381	0.4980

On the basis of p-values of KS test it is concluded that two methods of estimation except PWM for Nokkundi as for PWM it does not give any value. Therefore, we cannot recommend that PWM methods can be used study the annual maximum rainfall for all sites. Therefore F3D is not suitable with both methods for all sites.

We applied the three parametric distributions namely (GEVD, W3D and F3D) on rainfall data by using two methods of estimation (MLE and PWM) which based on result. So we can say that the GEVD and W3D are quite good for (all sites) but F3D is not suitable for all sites of rainfall data at 5 percent level of significance.

Further, we applied WD, FD and LLD to check the appropriation of two parameters distributions to observed data series which is presented in Tables 8-10. Actually we want to find best fitted three parameters distribution(s) and also most appropriate from two parameters candidate distribution(s) with both methods (ML and PWM) for all sites at 5 percent level of significance.

WD

When location parameter becomes zero in W3D then it is also called WD. The PDF of WD is

$$f(x;\ \alpha,\beta)\ =\ \frac{\sigma}{\mu}\left(\frac{\sigma}{\mu}\right)^{\mu-1} e^{-\left(\frac{x}{\mu}\right)^{\sigma}}, x>0, \qquad \mu,\sigma>0 \quad (7)$$

and the corresponding CDF of WD is

$$F(x)\ =\ 1-e^{-\left(\frac{x}{\mu}\right)^{\sigma}}, \quad (8)$$

Table 7: Estimates of Parameters for WD.

Sites Name	Methods	$\hat{\mu}$	$\hat{\sigma}$	KS	P-value
BARKHAN	ML	2.9369	14.2690	0.1451	0.3960
	PWM	3.3465	14.2092	0.1476	0.3754
DALBANDIN	ML	1.7842	14.4156	0.1741	0.2002
	PWM	1.7464	14.2643	0.1629	0.2645
JIWANI	ML	1.3794	16.1537	0.1870	0.1415
	PWM	1.3504	15.9879	0.1742	0.1997
KALAT	ML	1.5501	17.9790	0.1425	0.4183
	PWM	1.5659	17.8005	0.1510	0.3486
NOKKUNDI	ML	1.4274	13.8437	0.0932	0.8841
	PWM	1.3613	13.7498	0.0851	0.9366
PANJUR	ML	1.3701	15.2905	0.0998	0.8311
	PWM	1.3510	15.1859	0.0960	0.8628
PASNI	ML	1.5074	15.9542	0.1251	0.5825
	PWM	1.4764	15.6577	0.1133	0.7018
SIBBI	ML	1.6145	15.3129	0.1342	0.4932
	PWM	1.5855	15.0391	0.1454	0.3941
QUEETA	ML	2.6530	17.0003	0.1382	0.4569
	PWM	2.6040	16.8780	0.1290	0.5437
BADIN	ML	1.4121	15.3855	0.1981	0.1029
	PWM	1.3866	15.0696	0.1887	0.1348
HYDERABAD	ML	1.4660	15.0383	0.1660	0.2454
	PWM	1.4377	14.7508	0.1565	0.3076
KARACHI	ML	1.2088	13.1285	0.1593	0.3200
	PWM	1.1857	12.7788	0.1730	0.2312

Table 7 showed estimated parameters (ML, PWM) of WD. On the basis of P values of KS test at 5 percent level of significance it is concluded that ML and PWM are best fitted for WD. Therefore, we recommend that ML and PWM methods can be used study the annual maximum rainfall for such sites.

FD

FD to assess the extreme events. For instance, Flood, Maximum Rainfall, Earthquake etc. When location parameter becomes zero in F3D then it is also called FD.

The PDF of FD is

$$\mathrm{f}(\mathrm{x};\ \alpha,\beta)\ =\ \frac{\mu}{\sigma}\left(\frac{\sigma}{x}\right)^{\mu+1} e^{-\left(\frac{\sigma}{x}\right)^{\mu}}, \quad x>0, \mu, \sigma>0 \quad (9)$$

and the conforming CDF of FD is

$$F(x)=exp\left(-\left(\frac{\sigma}{x}\right)^{\mu}\right), \qquad x>0 \quad (10)$$

Table 8: Estimates of Parameters for FD.

Sites Name	Methods	$\hat{\mu}$	$\hat{\sigma}$	KS	P-value
BARKHAN	ML	3.7976	10.4451	0.0678	0.9924
	PWM	3.7409	10.4416	0.0652	0.9953
DALBANDIN	ML	2.2788	8.3703	0.1183	0.6515
	PWM	2.2457	8.3474	0.1155	0.6795
JIWANI	ML	1.1462	7.2559	0.2152	0.0606
	PWM	1.1374	7.1244	0.2225	0.0478**
KALAT	ML	2.0343	9.9476	0.0948	0.8718
	PWM	2.0060	9.9103	0.0937	0.8809
NOKKUNDI	ML	1.0424	5.6659	0.1715	0.2141
	PWM	1.0341	5.5391	0.1792	0.1749
PANJU	ML	1.0188	6.5974	0.1920	0.1228

R	PWM	1.0126	6.4385	0.1889	0.1342
PASNI	ML	1.2188	7.5957	0.1319	0.5159
	PWM	1.2097	7.4763	0.1313	0.5212
SIBBI	ML	1.5129	8.4317	0.1534	0.3304
	PWM	1.5006	8.3520	0.1593	0.2887
QUEETA	ML	2.3921	11.3618	0.1047	0.7862
	PWM	2.3671	11.3301	0.1063	0.7707
BADIN	ML	0.9312	6.8485	0.2497	0.0182**
	PWM	0.9266	6.6419	0.2595	0.0125**
HYDERABAD	ML	1.4260	7.5192	0.1418	0.4241
	PWM	1.4126	7.4382	0.1477	0.3744
KARACHI	ML	0.9765	5.4064	0.2454	0.0261**
	PWM	0.9702	5.2652	0.2542	0.0190**

Table showed that value of KS for Badin and Karachi are less than 0.05 and PWM for Jiwani also less than 0.05. On the basis of P values of KS test at 5 percent level of significance it is decided that ML and PWM are not best fitted for all twelve sites.

Log Logistic Distribution (LLD)

The LLD is a probability distribution of continuous random variable ranges positively. The LLD has been used in hydrology for demonstrating stream flow rates and rainfall. For instance, river discharge per month or annual maximum rainfall and maximum one day rainfall etc.

The PDF of LLD is,

$$f(x,\mu,\sigma)=\frac{\left(\frac{\sigma}{\mu}\right)\left(\frac{x}{\mu}\right)^{\sigma-1}}{\left[1+\left(\frac{x}{\mu}\right)^{\sigma}\right]^{2}},\quad x>0,\mu,\sigma>0 \qquad (11)$$

The corresponding CDF of LLD is

$$F(x)=\frac{1}{1+\left(\frac{x}{\mu}\right)^{-\sigma}},\qquad x>0,\mu,\sigma>0 \qquad (12)$$

For LLD, the estimates of shape parameter (μ) and scale parameter ($\boldsymbol{\sigma}$) by using two methods of estimation are presented in Table 10.

Table 9: Estimates of Parameters for LLD.

Sites Name	Methods	$\hat{\mu}$	$\hat{\sigma}$	KS	P-value
BARKHAN	ML	5.5790	11.8731	0.0861	0.9314
	PWM	5.3452	12.0332	0.0733	0.9825
DALBANDIN	ML	3.1070	10.3241	0.1326	0.5092
	PWM	3.0525	10.5784	0.1207	0.6268
JIWANI	ML	2.3206	11.1575	0.1224	0.6091
	PWM	2.4909	11.0709	0.1149	0.6857
KALAT	ML	2.9180	12.7598	0.0793	0.9636
	PWM	2.7959	12.8336	0.0805	0.9587
NOKKUNDI	ML	1.9144	9.9333	0.1086	0.7485
	PWM	2.5063	9.5425	0.1510	0.3488
PANJUR	ML	2.1316	10.8601	0.1077	0.7574
	PWM	2.4917	10.5169	0.1097	0.7384
PASNI	ML	2.3749	11.2722	0.0751	0.9778
	PWM	2.5946	11.0653	0.0782	0.9680
SIBBI	ML	3.1801	11.3860	0.0776	0.9700
	PWM	3.0639	11.3292	0.0740	0.9808
QUEETA	ML	4.3344	13.8772	0.0812	0.9559
	PWM	4.3966	13.8262	0.0818	0.9530
BADIN	ML	2.6644	11.5904	0.1468	0.3823
	PWM	2.8649	11.2989	0.1425	0.4185
HYDERABAD	ML	2.6375	10.8103	0.0977	0.8485
	PWM	2.6825	10.6151	0.1108	0.7266
KARACHI	ML	2.1938	9.3126	0.1262	0.6142
	PWM	2.3752	8.9712	0.1321	0.5560

P values of KS test indicate that LLD is also most appropriate distribution for all sites at 5 percent level of significance. Therefore, we can also recommend that LLD is also good fit for all sites.

III. DISCUSSION

We have used three (three parametric) distributions GEVD, W3D and F3D and also (two parametric distributions) FD, WD and LLD. The results of three parameter distributions GEVD and W3D are good fitted with both methods while F3D is not significant for all sites with both methods.

Similarly, LLD and WD give significant results (at 5 percent level of significance) for all sites with both methods while FD is not good fitted with both methods.

From the above results of estimated parameters for three and two parametric distributions we have concluded that we can use both three and two parametric distributions but with three parametric distributions (with different method) is laborious and time consuming work. Therefore, we recommend that instead of using three parametric distributions the most suitable is to apply two parameters distributions for these sites to avoid the laborious work.

Plots of Probability Density Function (PDF) Graphs

The PDF graphs are used only for best fitted distributions for comparison purpose of estimation methods from six distributions. The graphs showed that only GEVD, W3D, WD and LLD are best fit for the rainfall data of these given sites. Where color less graphs are for three parameter and grey color graphs for two parameters distributions.

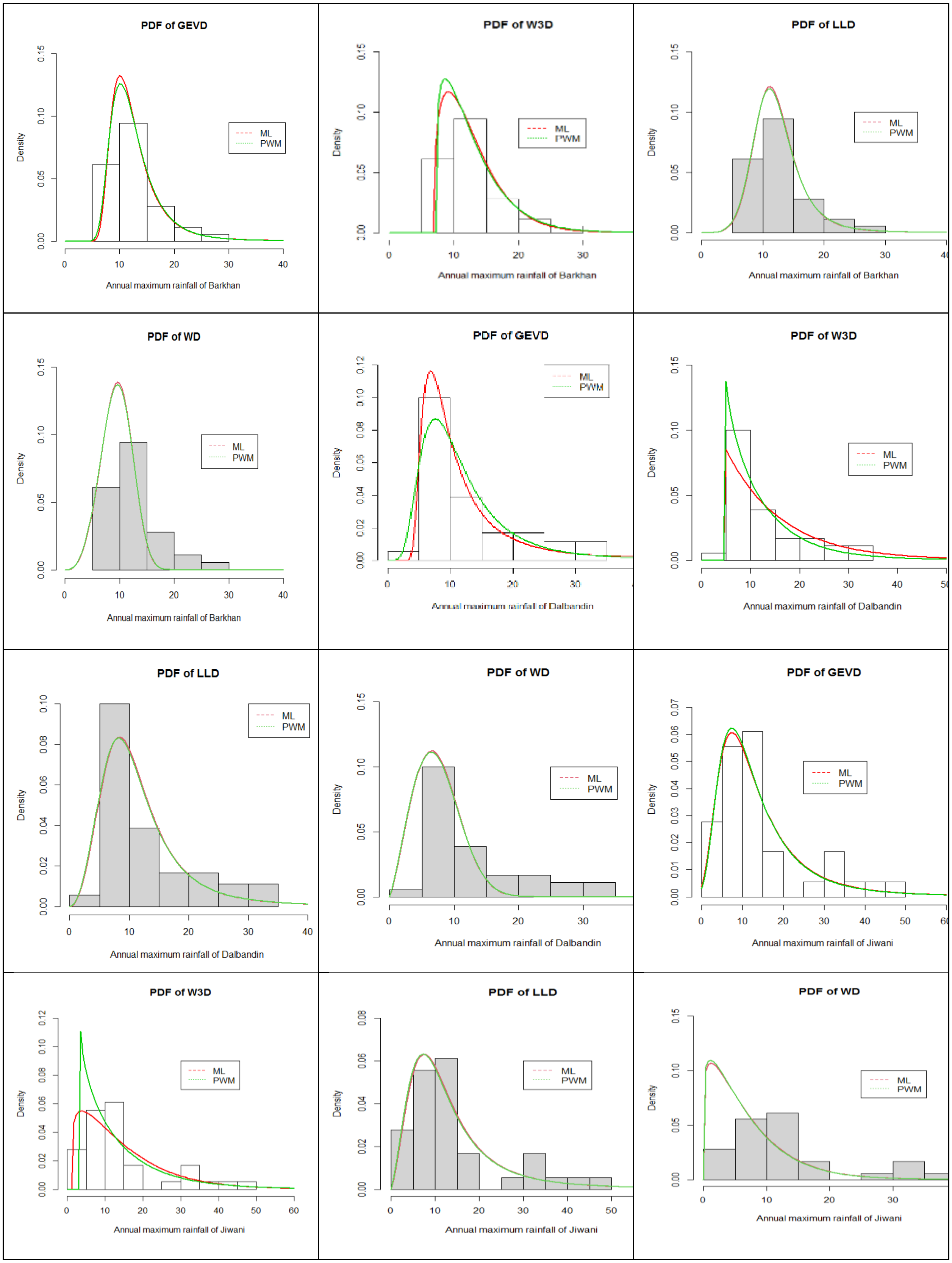

Figure 2: GEVD, W3D, WD and LLD for Barkhan, Dalbandin and Jiwani

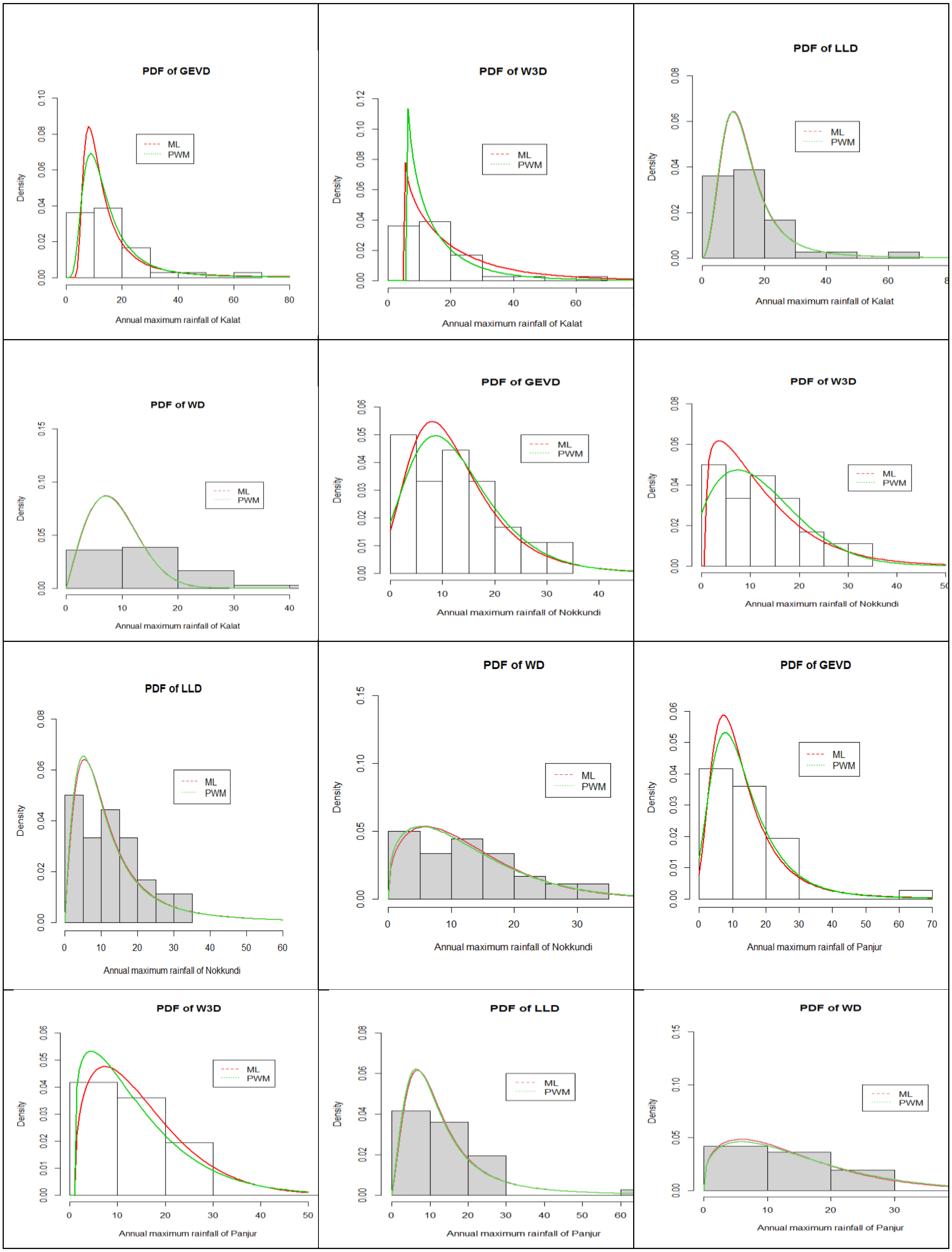

Figure 3: GEVD, W3D, WD and LLD for Kalat, Nokkundi and Panjur

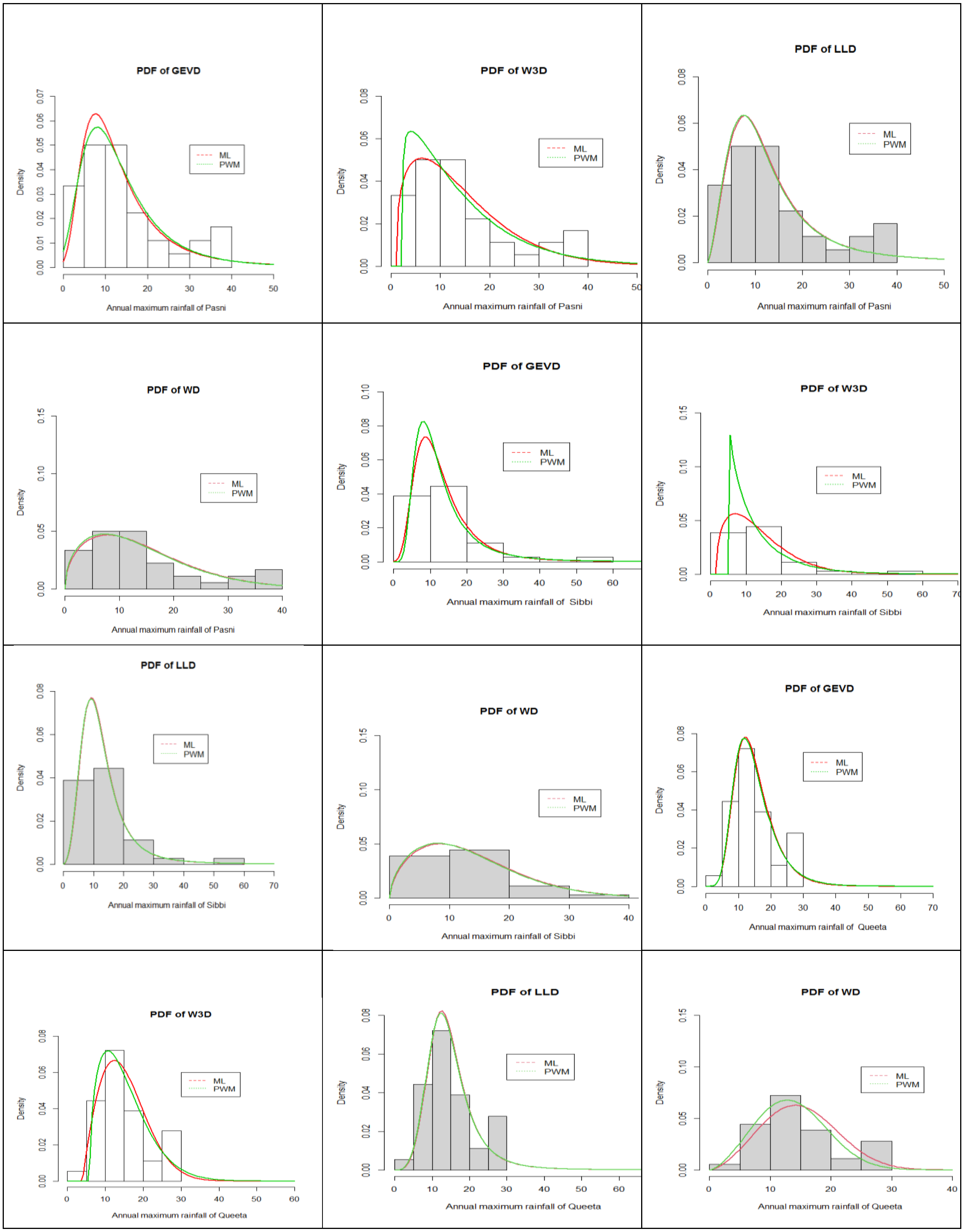

Figure 4: GEVD, W3D, WD and LLD for Pasni, Sibbi and Queeta

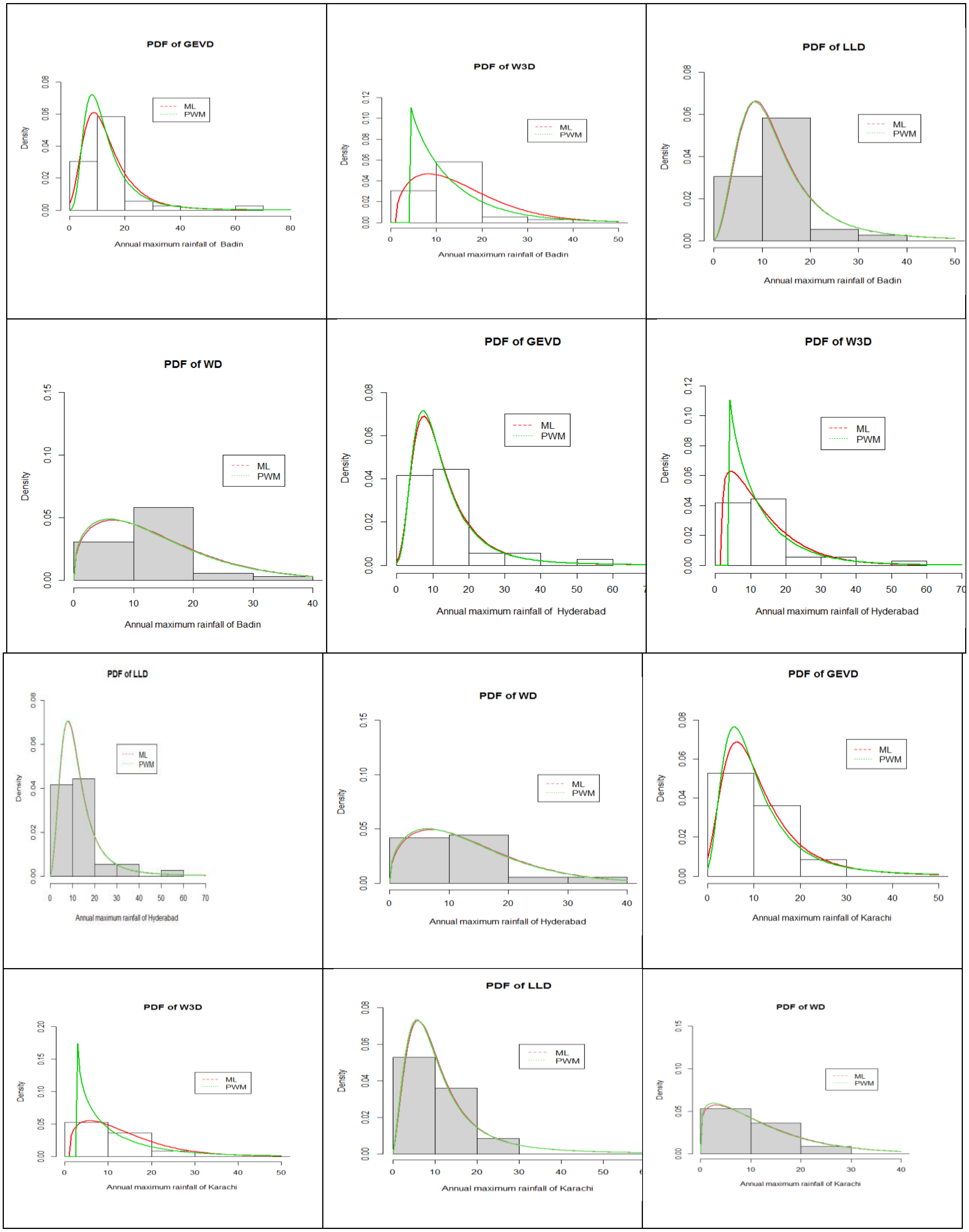

Figure 5: GEVD, W3D, WD and LLD for Badin, Hyderabad and Karachi

ISSN:1813-1786 (Print) 2313-7770 (Online)

QUANTILES

Quantiles are used to know the future predication, other feature of distribution for risk management and regulatory reporting purpose etc. Using the quantile functions of mentioned probability distributions, we have estimated the quantiles for various periods of time. These quantiles (From Table 10 to Table 13) have been calculated using ML and PWM methods for best fitted distributions known as GEVD, W3D, WD and LLD. It is observed that the estimated quantiles for F=0.9 (return period = 10 years) and upcoming values are larger than the average value of the annual maximum rainfall series at selected sites.

Table 10: Quantiles Estimation for GEVD Distribution.

Quantile estimates (mm) with non exceedance probability (years)					
Sites Name	**Methods**	**0.1 (1)**	**0.5 (2)**	**0.9 (10)**	**0.99 (100)**
Barkhan	ML	6.63	12.59	18.95	24.02
	PWM	6.50	12.49	18.92	24.05
Dalbandin	ML	4.08	11.73	23.00	33.92
	PWM	6.52	12.51	18.96	24.10
Jiwani	ML	3.16	12.38	29.57	48.87
	PWM	2.98	12.04	29.28	48.91
Kalat	ML	4.22	14.19	30.79	48.15
	PWM	4.02	13.87	30.55	48.19
Nokkundi	ML	2.86	10.70	24.83	40.35
	PWM	2.69	10.42	24.66	40.54
Panjur	ML	2.95	11.70	28.10	46.61
	PWM	2.79	11.38	27.85	46.69
Pasni	ML	3.58	12.51	27.74	43.94
	PWM	3.41	12.21	27.54	44.05
Sibbi	ML	3.79	12.20	25.66	39.43
	PWM	3.63	11.93	25.44	39.40
Queeta	ML	7.27	14.80	23.28	30.23
	PWM	7.11	14.66	23.24	30.34
Badin	ML	3.12	11.86	27.77	45.37
	PWM	2.97	11.56	27.49	45.33
Hyderabad	ML	3.23	11.71	26.56	42.62
	PWM	3.08	11.43	26.34	42.67
Karachi	ML	2.04	9.69	26.17	46.44
	PWM	1.91	9.38	25.82	46.32

Table 11: Quantiles Estimation for W3D Distribution

Quantile estimates (mm) with non exceedance probability (years)					
Sites Name	**Methods**	**0.1 (1)**	**0.5 (2)**	**0.9 (10)**	**0.99 (100)**
Barkhan	ML	8.12	11.72	18.77	26.92
	PWM	8.27	11.67	19.18	28.47
Dalbandin	ML	5.76	12.41	30.62	56.68
	PWM	5.40	9.95	23.40	43.59
Jiwani	ML	3.34	11.55	30.10	53.60
	PWM	4.39	10.70	29.74	58.67
Kalat	ML	6.77	16.05	44.30	87.52
	PWM	6.31	11.80	30.39	60.78
Nokkundi	ML	2.81	9.97	25.67	45.20
	PWM	1.96	10.88	24.49	37.98
Panjur	ML	4.12	12.92	28.98	46.80
	PWM	3.27	11.47	28.99	50.43
Pasni	ML	3.75	12.04	27.99	46.25
	PWM	4.08	11.45	29.02	52.03
Sibbi	ML	4.32	11.77	25.47	40.74
	PWM	5.85	10.60	24.95	46.77
Queeta	ML	7.84	14.30	23.34	31.83
	PWM	8.17	13.98	24.11	35.03
Badin	ML	4.43	13.35	28.91	45.70
	PWM	5.29	10.85	26.53	49.35
Hyderabad	ML	3.78	10.89	26.73	46.65
	PWM	4.65	10.19	26.44	50.72
Karachi	ML	3.47	11.11	25.77	42.55
	PWM	3.49	8.46	25.35	53.08

Two parameters distribution quantiles

Three distributions WD and LLD are candidate distributions for quantiles. These quantiles have been calculated using ML and PWM methods. It is observed that the estimated quantiles for F=0.9 (return period = 10 years) and upcoming values are larger than the average value of the annual maximum rainfall series at selected sites.

Table 12: Quantiles Estimation for WD Distribution

Quantile estimates (mm) with non exceedance probability (years)					
Sites Name	**Methods**	**0.1 (1)**	**0.5 (2)**	**0.9 (10)**	**0.99 (100)**
Barkhan	ML	6.63	12.59	18.95	24.02
	PWM	6.50	12.49	18.92	24.05
Dalbandin	ML	4.08	11.73	23.00	33.92
	PWM	6.52	12.51	18.96	24.10
Jiwani	ML	3.16	12.38	29.57	48.87
	PWM	2.98	12.04	29.28	48.91
Kalat	ML	4.22	14.19	30.79	48.15
	PWM	4.02	13.87	30.55	48.19
Nokkundi	ML	2.86	10.70	24.83	40.35
	PWM	2.69	10.42	24.66	40.54
Panjur	ML	2.95	11.70	28.10	46.61
	PWM	2.79	11.38	27.85	46.69
Pasni	ML	3.58	12.51	27.74	43.94
	PWM	3.41	12.21	27.54	44.05
Sibbi	ML	3.79	12.20	25.66	39.43
	PWM	3.63	11.93	25.44	39.40
Queeta	ML	7.27	14.80	23.28	30.23
	PWM	7.11	14.66	23.24	30.34
Badin	ML	3.12	11.86	27.77	45.37
	PWM	2.97	11.56	27.49	45.33
Hyderabad	ML	3.23	11.71	26.56	42.62
	PWM	3.08	11.43	26.34	42.67
Karachi	ML	2.04	9.69	26.17	46.44
	PWM	1.91	9.38	25.82	46.32

Table 13: Quantiles Estimation for LLD Distribution

Quantile estimates (mm) with non exceedance probability (years)					
Sites Name	**Methods**	**0.1 (1)**	**0.5 (2)**	**0.9 (10)**	**0.99 (100)**
Barkhan	ML	8.03	11.87	17.59	27.04

	PWM	7.92	11.84	17.69	27.41
Dalbandin	ML	5.09	10.32	20.94	45.31
	PWM	4.98	10.24	21.04	46.19
Jiwani	ML	4.32	11.15	28.74	80.76
	PWM	4.17	10.98	28.89	83.02
Kalat	ML	4.58	11.07	26.74	70.04
	PWM	5.86	12.63	27.21	62.89
Nokkundi	ML	3.15	9.93	31.29	109.50
	PWM	2.99	9.67	31.25	112.35
Panjur	ML	3.87	10.86	30.44	93.76
	PWM	3.71	10.64	30.48	96.13
Pasni	ML	4.46	11.27	28.43	78.03
	PWM	4.32	11.12	28.52	79.87
Sibbi	ML	5.70	11.38	22.72	48.29
	PWM	5.58	11.29	22.84	49.29
Queeta	ML	8.35	13.87	23.03	40.06
	PWM	8.23	13.81	23.15	40.68
Badin	ML	5.08	11.59	26.43	65.02
	PWM	4.92	11.45	26.63	66.89
Hyderabad	ML	4.69	10.81	24.86	61.72
	PWM	4.67	10.61	24.07	63.24
Karachi	ML	3.42	9.31	25.35	75.63
	PWM	3.28	9.14	25.47	77.88

Tables 10 to 13 depict the quantiles results for GEVD, W3D, WD and LLD by ML and PWM methods. It can be seen that quantiles of all two parameters distributions are suitable for all sites. Also for any period (return period=0.01 to period=0.99) does not reached 100mm.

IV. CONCLUSIONS

The study illustrates application of six atypical probability distributions with two different methods of estimation of parameters to model the behaviour of annual maximum rainfall at twelve sites of Balochistan and Sindh, Pakistan.

Some of the major conclusions are:

i. Summary statistics have shown that there exists variations in the observed data series and the distribution related to the observed data series at given sites is positively skewed.
ii. The observed data series at given sites is identically distributed by Man Whitney test.
iii. In general, all the distributions parameters were estimated with suggested methods (ML, PWM). Only four distributions GEVD, W3D, WD and LLD have passed the goodness of fit criteria of KS test and the p-values have suggested that fours distributions with both methods are appropriate for all sites.
iv. The study suggest that all the probability distributions using MLE and PWM methods are preferred choice for modelling the extremes of rainfall of the study area. These results are encouraging to adopt relatively atypical probability distributions for the estimation of extreme events, like floods, rainfall, etc. This will bring diversity and flexibility in the choice of selection of a probability distribution for the modelling of extremes of geophysical events.
v. For comparison purpose probability density function (PDF) graphs of the candidate good fitted distributions of all sites are constructed and presented in Figure 03 to 06. The graphs show for (best fitted distributions) that all distributions are best fit for the rainfall data of these given sites.
vi. Quantiles are used to know the future predication, other feature of distribution for risk management and regulatory reporting purpose etc. Using the quantile functions of mentioned probability distributions, we have estimated the quantiles for various periods of time. These quantiles have been calculated using ML and PWM methods for best fitted distributions. The results are presented in tables 11 to 14. It is observed that the estimated quantiles for F=0.9 (return period = 10 years) and upcoming values are larger than the average value of the annual maximum rainfall series at selected sites. Also, for the return period = 100 years, the magnitude is greater than the maximum value of the calculated annual maximum rainfall series for all sites (for all four distributions).

The study provides useful guidelines for the concerned officials of study area, home-grown farmers, meteorologists studying precipitation and rainwater management planning. Moreover, the study provides illustration of some new probability distributions with different methods of parameter estimation for at site frequency analysis of extreme events. The procedure can be adopted in general to bring diversity and flexibility in the choice of probability distributions for modelling of extremes of events like floods, rainfall, etc.

REFERENCES

[1] Abbas, K., Khan, S. A., Khan, D. M., & Ali, A. (2012). Modeling the distribution of annual maximum rainfall in Pakistan. European Journal of Scientific Research, 79(3), 418-429.
[2] Ahmad, I., Abbas, A., Saghir, A., & Fawad, M. (2016). Finding probability distributions.
[3] Alam, M. A., Emura, K., Farnham, C., & Yuan, J. (2018). Best fit probability distributions and return periods for maximum monthly rainfall in Bangladesh. Climate, 6(1), 9.

[4] Amin, M. T., Rizwan, M., & Alazba, A. A. (2016). A best-fit probability distribution for the estimation of rainfall in northern regions of Pakistan. Open Life Sciences, 11(1), 432-440.

[5] Bari, S. H., Rahman, M. T. U., Hoque, M. A., & Hussain, M. M. (2016). Analysis of seasonal and annual rainfall trends in the northern region of Bangladesh. *Atmospheric Research*, *176*, 148-158.

[6] Chikobvu, D., & Chifurira, R. (2015). Modelling of extreme minimum rainfall using generalised extreme value distribution for Zimbabwe. South African Journal of Science, 111(9-10), 01-08.

[7] Fréchet, M. (1927). Sur la loi de probabilité de l'écart maximum. Ann. Soc. Math. Polon., 6, 93-116.

[8] Hussain, Z., Shahzad, M. N., & Abbas, K. (2016). Application of regional rainfall frequency analysis on seven sites of Sindh, Pakistan. Korean Society of Civil Engineers, 21(5), 1812-1819.

[9] Jameel, A., Mahmood, A., & Jaffri, S. A. A. (2005). Weather in Pakistan monsoon Season. Pakistan Journal of Meteorology, 2(3).

[10] Jenkinson, A. F. (1955). The frequency distribution of the annual maximum (or minimum) values of meteorological elements. Quarterly Journal of the Royal Meteorological Society, 81(348), 158-171.

[11] Mayooran, T., & Laheetharan, A. (2014). The statistical distribution of annual maximum rainfall in Colombo district. Sri Lankan Journal of Applied Statistics, 15(2), 1765-1784.

[12] Natural Environment Research Council (Great Britain). (1975). Flood Studies Report in Five Volumes. NERC.

[13] Rodo, X., & Comin, F. (2003). Global climate: current research and uncertainties in the climate system. Springer Science & Business Media.

[14] Shahzadi, A., Akhter, A. S., & Saf, B. (2013). Regional frequency analysis of annual maximum rainfall in monsoon region of Pakistan using L moments. Pakistan Journal of Statistics and Operation Research, 9(1), 111-136.

[15] Sherif, M., Almulla, M., Shetty, A., & Chowdhury, R. K. (2014). Analysis of rainfall, probable maximum precipitation and drought in the United Arab Emirates. International journal of climatology, 34(4), 1318-1328.

[16] Singh, B., Rajpurohit, D., Vasishth, A., & Singh, J. (2012). Probability analysis for estimation of annual one day maximum rainfall of Jhalarapatan area of Rajasthan, India. Plant Archives, 12(2), 1093-1100.

[17] Weibull, W. (1939). A statistical theory of strength of materials. IVB-Handl.