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Parametric Graph Theory and Matrix Approach Model for Assessing the Surface Water Quality

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Abstract- In this paper a systematic parametric approach for assessing the water quality for given water resource by using the concepts of graph theory and matrix algebra is presented. A new, novel, generic and more sensitive water quality index (WQI) named as "Baluch" index is proposed that can assess and ranks the water quality for a given water resource. The water quality parameters listed in FAO-29 guidelines (belong to resource under investigation) and their relative importances are utilized to create a water quality assessment graph (WQG) and adjacency matrix. As a case study, the water quality of eighteen stations of upper basin of Indus River, Pakistan is assessed. Eight physico-chemical quality parameters including pH, TDS, Total hardness, Ca Hardness, Mg hardness, SO4-2, Cl-1, and EC are used for constructing WQA. This WQA is then used to figure out "Baluch" index for each monitoring station, and it is found that "Baluch" index for Darband station has lowest value i.e. 0.85 and "Baluch" index highlight the situation that all other stations have relatively higher values of pH, which is concern able problem. Furthermore, this new index indicate the lowest quality water belongs to Jijal stations due relatively higher values of pH (i.e. 8.25) and Cl- ion, above average values of TDS, Hardness, SO42- ion concentrations.

Keywords- Water Quality; Water Quality Assessment Graph; Adjacency Matrix; "Baluch" Index; Indus River; Upper Indus Basin

I. INTRODUCTION

The fresh water springs are streams, water ways and lakes. Studies show that the main water pollutants are agricultural wastes, inhabitation and Industries[1-2]. In addition to this, fast populace development, expanded urbanization, local, provincial and municipal wastes into fresh water bodies worsen the quality of water[3-4]. Research shows that annually 80 percent of municipal and industrial wastes are thrown in water untreated. United Nations (UN) reported that around $1500 km^3$ Every year waste water is created that is several times the amount of water available in all water bodies in the world[5]. Besides, releasing harmful wastes into water bodies without any treatment have been proved toxic for aquatic life, according to its specific chemical properties, bio accessibility and aquatic species[6]. Moreover, global warming and climatic vulnerabilities are the prime factors that deplete the main water sources globally[7].

Pakistan is among those countries where fresh water bodies are reducing in terms of quality and quantity, because of low precipitation / higher dissipation brought about by environmental change and heavy incursion of contaminants[8]. In Pakistan researches on the quality of water shows that uncontrolled and untreated disposal of municipal contaminants into fresh water bodies are degrading the quality of water to alarming degree[9]. Another study reported that heavy incursion of human excreta in Indus River is contaminating it and causing many deadly diseases[10-11]. Similarly, a research on the water quality of Kallar Kahar Lake in the province of Punjab, Pakistan reported about the deteriorating quality of lake water above threshold value due of accumulation of toxic and untreated waste material [12]. Another study about the water bodies of southern region of Khyber Pakhtunkhwa demonstrated the presences of high amount of cations and traces of metals at the bottom of rivers and streams [13]. Researchers evaluated the quality of ground water in Bahawalpur and found toxins in it more than threshold limit [14].The reported results concluded that there are heavy contaminants above threshold value in the ground water. In a later report, it is indicated that water bodies of southern areas of KPK contain heavy incursion of toxic effluents and hence it is not suitable for irrigation and agricultural activities [15].

In Pakistan, water contamination is the major issue that needs legitimate observing and appraisal thinks about. A huge proportion of the population is living in the country side areas of Pakistan. The population utilizes natural water sources for their domestic needs and livestock. Immediate and indirect use of dirtied water spreads fatal diseases such as stomach ulcers, skin maladies, eye infection, hair loss and so forth [16]. It is therefore, this particular chapter focused on the assessing the water quality while framing new index named as "Baluch" Index. The ranking of 18 water samples (as far as their quality is concerned) collected from stations from upper indus basin (UIB) is performed with the help of graph theory and matrix approach.

Graph theory could be a coherent and systematical approach. The progressed hypothesis of graphs and its applications are exceptionally well archived. Graph/digraph representations have demonstrated to be valuable for modeling and analyzing different sorts of systems and issues in various areas of science and innovation[17, 18]. The matrix method is important for the graph / digraph template analysis to quickly deduce the system function and the system file to achieve the objectives. For the construction of the Water Quality Assessment (WQA) graph, a graph theory and matrix approach are implanted in order to assess the water quality while utilizing quality parameters measured in terms of value. The details are given in the section 2.4

II. Material and Method

The regular approaches and procedures which were adapted for the sampling, data recording, and water sample analysis as well as development of GIS maps under Water Quality Monitoring, Surveillance and Development of database for UIB. The details of these methods are given below:

2.1. Planning for Sample Collection from Field

The meetings were conducted with the relevant officials of the Government and officials of Pakistan Council of Research in Water Resources (PCRWR) concerned to the water quality monitoring of major water resources in Pakistan. The briefing was provided regarding the objectives of the research as well as the future implications. After getting the positive consent and willingness of support for research, the proposal of testing parameters and sites was made. List of eighteen stations that were selected for collection of water and sediment samples is given in the table 1 below. These sites are selected with consultation of PCRWR in order to access the Indus river Water quality from its origin. It is reported in the literature [11] that the quality of water of Indus River passing through these sites is polluted with toxic heavy metals including Arsenic and Lead.

Table 1: List of sampling stations with latitude and longitude

Identification Code	Station	Latitude & Longitude
1	Dandai	34.807258, 72.948489
2	Bilani	34.96178, 72948489
3	Gunar	35.407676, 74.366798
4	Jalipur	35.406443, 74.320492
5	Chillas	35.422948, 74.092776
6	Sazin	35.531445, 73.506914
7	Komaila	35.260077, 73.219172
8	Kotgal	35.444418, 73.207888
9	Pattan	35.114184, 73.005350
10	Thakot	34.788129, 72.929310
11	Jijal	35.040797, 72.922762
12	Dubair	35.047692, 72.895675
13	Torghar mud	34.584397, 72.858536
14	Darband	34.364808, 72.857895
15	Bisham	34.938704, 72.830405
16	Kamach	34.563392, 72.812068
17	Judbah	34.913017, 72.787853
18	Shang	34.895073, 72.757577
19	Serai	34.171228, 72.404612

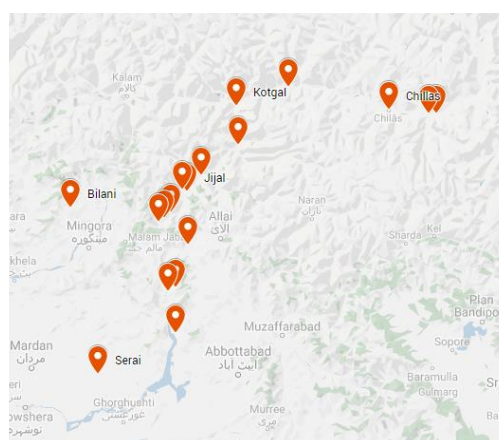

Figure 1 Sample Collection point highlighted on Google Map

2.2. Samples Collection and Preservation

Three to five samples of water and sediments were collected from each station mentioned above. For the purpose of the physio chemical analysis, 1.5-liter plastic bottles were used to collect the water samples. The bottles were washed before the collection and they were also rinsed thoroughly multiple times with the distilled water subsequent to washing them fresh water. After washing, each bottle was dried and sterilized by placing them in oven at 50 ˚C for one week. Subsequent to sterilization procedure, sealing of each bottle was done in inert atmosphere which was created in the lab locally with the help of 20 kg plastic bag. From the river the water samples was collected directly in disinfected testing bottles. The bottles were made clean, sterile as per required for the testing of the microbiological contamination as well. There was an extra care taken to avoid the unintentional pollution interference during the process of sampling. Most of the quality parameters were assessed on site to minimize the effect of delay in transportation from site to laboratory. Only those tests were performed in laboratory whose execution at site is not possible. Each sample was labeled with identification code as given in the table 1.

2.3. Testing

2.3.1. Testing on site

Portable measuring devices at sample testing site are used to measure physical and chemical parameters of sample such as pH, electrical conductivity (EC), total dissolved solids (TDS), dissolved oxygen (DO), and biological dissolved oxygen (BOD).

2.3.2. Testing in Laboratory

Standard methods are used to determine other parameters i.e. total solids (TS), total suspended solids (TSS), total hardness (TH), chemical dissolved oxygen (COD), calcium (Ca^{2+}), and magnesium (Mg^{2+}), anions chloride (Cl^-), bicarbonate (HCO_3^-), nitrate (NO_3^-), phosphate (PO_4^{2-}), and sulfate (SO_4^{2-}). Water tests were evaluated for all out coliforms and Escherichia coli (E. coli) inside 24 h of test gathering. The samples were handled in a laminar stream hood utilizing cleaned culture media. The bacterial burden in 100 mL water tests was estimated by most likely number (MPN) procedure.
The parameters tested on site as well as in Lab. There are no significant differences found in the values tested on site and in lab for these parameters. For details of standard methods readers are referred to literature

2.4. Graph Theory and Matrix Algebra Base Model Development

Although there are number of quality indices for assessing the water quality but there is still need of further improvement in the methodology for assessing the water quality of because all them do not include the effect of allowed minimum and maximum value of each quality parameter i.e. permissible prescribed limits. It is therefore, novel methodology based on the Graph Theory and Matrix Algebra (GTMA) has been developed during this study. The details of GTMA model are given below

2.5. Water Quality Assessment (WQA) Graph

Water Quality Assessment (WQA) Graph models the quality parameters and their interrelationship for assessing the good water quality. This graph comprises of a set of nodes V = vi, with i = 1, 2, .. ., N and a set of coordinated edges D = {dij} . A node ni speaks to i-th quality parameter and edges speak to the relative significance among the parameter. The number of nodes N, considered is rise to the number of quality parameter considered. On the off chance that a node 'i' is having relative significance over another node 'j' within the proposed problem, at that point a coordinated edge or bolt is drawn from node i to node j i.e. dij. In the event that 'j' is having relative significance over 'i', at that point a coordinated edge or 'i' is drawn from node j to node 'i' i.e. dji. In order to explain the WQA Graph, number of quality parameters need to be defined first. Here in this case the quality parameters for the construction of WQA graph are:

- pH
- TDS
- Total Hardness
- Ca Hardness
- Sulphate
- CI-
- Mg Hardness
- EC

WQA Graph constructed with the help above mentioned quality parameters is shown in figure 2. As eight water quality parameters are considered, there are eight nodes within the WQA graph with nodes 1, 2, 3, 4, 5, 6, 7 and 8 for assessing the water quality at 12 different stations of UIB. From the abovementioned eight quality parameters, five parameters including pH, TDS, total hardness, Cl- and EC are used to rank the quality of water by defining new index named as "Baluch" Index with help of extracting the adjancy matrix from the graph shown in figure 2. The figure 2 gives a graphical representation of the components and their relative significance for fast visual evaluation. As the number of nodes and their interrelations increments, the graph gets to be complex. In such a case, the visual examination of the chart becomes troublesome and complex. In order to overcome this issue, WQA graph is represented in matrix form that is used for extracting the "Baluch" index value at each sample collection point. The details about the matrix representation of are given in the section below.

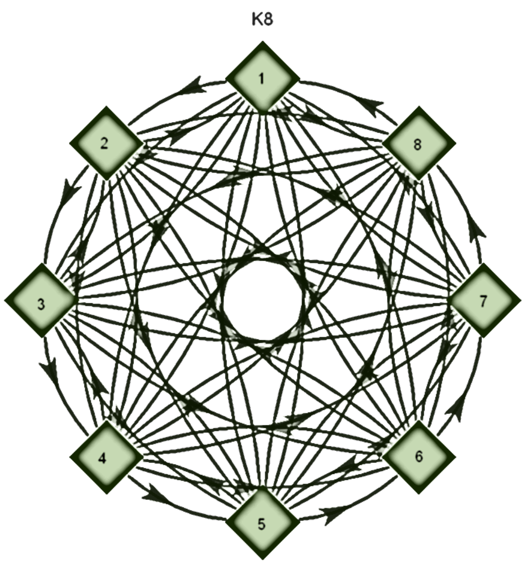

Figure 2: WQA graph

2.6. Matrix representation of WQA graph

Matrix representation of the WQA graph gives one to one representation of water quality parameter. Typically an N x N matrix which consists of all of the quality parameters (i.e. Ri) and their relative significance (i.e. rij). The matrix A corresponds to the WQA graph shown in figure 2 is given below:

Table 2 adjacency matrix for water quality assessment

A =

Parameter	pH	TDS	TH	Ca	SO^{2-}	Cl^{-1}	Mg^{+}	EC
pH	$R1$	$r12$	$r13$	$r14$	$r15$	$r16$	$r17$	$r18$
TDS	$r21$	$R2$	$r23$	$r24$	$r25$	$r26$	$r27$	$r28$
TH	$r31$	$r32$	$R3$	$r34$	$r35$	$r36$	$r37$	$r38$
Ca^{+}	$r41$	$r42$	$r43$	$R4$	$r45$	$r46$	$r47$	$r48$
SO^{2-}	$r51$	$r52$	$r53$	$r54$	$R5$	$r56$	$r57$	$r58$
Cl^{-1}	$r61$	$r62$	$r63$	$r64$	$r65$	$R6$	$r67$	$r68$
Mg^{+}	$r71$	$r72$	$r73$	$r74$	$r75$	$r76$	$R7$	$r78$
EC	$r81$	$r82$	$r83$	$r84$	$r85$	$r86$	$r87$	$R8$

Where Ri is representing the node ni, and rij is the relative significance of the i-th parameter over the j-th parameter and represented by the edge dij. The permanent of this matrix A, i.e. per (A), is characterized as the quality parameter's function. The permanent may be a standard matrix and is utilized in combinatorial arithmetic [17-18]. Application of permanent concept will lead to better appreciation and utilization of water quality parameter. Furthermore, utilizing this instead of determinant of matrix A remove the negative sign and consequently no data will be missed in final index calculation. The permanent function is only the determinant of a matrix, but all the defining terms are considered positive. For the WQA map adjustability matrix, it is written as:

Expression 1 given above is the total expression of the quality parameter as considered since it takes account of the proximity between the parameters and all of their imaginable relative importance. The words are the sequence of similar diagonal elements and circles of off-diagonal components (e.g. rijrji, rijrjkrki, etc.) of different sizes. Expression 1 performance parameters include terminology organized into (6 + 1), and these groupings apply to the measurements of parameters and to the relative significance circles. The second grouping is missing, as there is no self-circle within the chart. The third grouping contains 2-factor relative significance circles and measures of (6-2) parameters. A set of 3-parameter relative significance circles, or match and dimensions of (6-3) factors speaks to each term of the fourth group. There are two subgroups in the fifth group. The terms of the primary sub collection are two relatively important circles of 2 parameters and the components (6-4) are measured. Each term of sub-grouping may consist of a set of four-component or its combined relative significance circle and measurements of (6-4) variables. The sixth group comprises two subgroups. The terms of the primary sub-grouping could be a set of 3-component relative significance circle or its combine and 2-component relative significance circle and the measures of (6-5) variables. Each term of the second sub-grouping could be a set of 5-component relative significance circle or its combine and the measures of (6-5) variables. So also, other terms of the Expression are characterized. Hence, the WQA Graph characterizes the considered water sample categorizing issue because it contains all conceivable auxiliary parameters of the sample and their relative significance. The above term in equation 2 is simply the determinant of a matrix of six in 6 but takes all terms positively into consideration. In order to calculate the Per(A), Mathematica is used during this study.

2.7. Calculation of "Baluch" Index

"Baluch" index categorically lists the water samples according their raking on the basis of measured values of aforementioned quality parameters. The expression 1 contains measures values of parameters and their relative significance, which are then utilized for WQA. The numerical value of Per (A) serves as to rank the water sample and named here as “Baluch” index. As the nature of the problem is such that lowest value of quality parameters is required for good quality water. It is therefore, lower values of Ri and rij will result in lower value of “Baluch” index i.e. good quality water. In order to obtain this, the normalized values are calculated from this ratio vj/vi. In this case, vj is the value for the j-th parameter which is having lower value among the quality parameters under investigation. It is worth to mention here that in this particular area, each parameter’s lower measures are desirable. Therefore, lower value of “Baluch” index is desirable for good quality water.

$$
\begin{aligned}
&PER(A)\\
&=\prod_{i=1}^{N} Ri\\
&+\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\dots\\
&\times\sum_{N=t+1}^{N}(r_{ij}r_{ji})R_kR_lR_mR_nR_o\dots.R_tR_N\\
&+\sum_{i=1}^{N-2}\sum_{j=i+1}^{N-1}\sum_{k=j+1}^{N}\dots\times\sum_{N=t+1}^{N}(r_{ij}r_{jk}r_{ki}+r_{ik}r_{kj}r_{ji})\times R_kR_lR_mR_nR_o\dots.R_tR_N\\
&+\Bigg(\sum_{i=1}^{N-3}\sum_{j=i+1}^{N}\sum_{k=i+1}^{N-1}\sum_{l=i+2}^{N}\dots\times\sum_{N=t+1}^{N}(r_{ij}r_{jk}r_{kl}r_{li}+r_{il}r_{lk}r_{kj}r_{ji})\\
&\times R_kR_lR_mR_nR_o\dots.R_tR_N\Bigg)\\
&+\Bigg(\sum_{i=1}^{N-2}\sum_{j=i+1}^{N-1}\sum_{k=j+1}^{N}\sum_{l=1}^{N-1}\sum_{m=l+1}^{N}\dots.\sum_{N=t+1}^{N}(r_{ij}r_{jk}r_{ki}+r_{ik}r_{kj}r_{ji})(r_{lm}r_{ml})\times R_nR_o\dots.R_tR_N\\
&+\sum_{i=1}^{N-4}\sum_{j=i+1}^{N-1}\sum_{k=i+1}^{N}\sum_{l=i+1}^{N}\sum_{m=j+1}^{N}\dots.\times\sum_{N=t+1}^{N}(r_{ij}r_{jk}r_{kl}r_{lm}r_{li}+r_{im}r_{ml}r_{lk}r_{kj}r_{ji})\\
&\times R_nR_o\dots.R_tR_N\Bigg)\\
&+\Bigg(\sum_{i=1}^{N-3}\sum_{j=i+1}^{N-1}\sum_{k=j+1}^{N}\sum_{l=j+1}^{N}\sum_{m=1}^{N-1}\sum_{n=m+1}^{N}\dots.\times\sum_{N=t+1}^{N}(r_{ij}r_{jk}r_{kl}r_{li}+r_{il}r_{lk}r_{kj}r_{ji})(r_{mn}r_{nm})\\
&\times R_o\dots.R_tR_N+\sum_{i=l}^{N-5}\sum_{j=i+1}^{N-1}\sum_{k=j+1}^{N}\sum_{l=1}^{N-2}\sum_{m=l+1}^{N-1}\sum_{n=m+1}^{N}\dots.\sum_{N=t+1}^{N}(r_{ij}r_{jk}r_{ki}\\
&+r_{ik}r_{kj}r_{ji})(r_{lm}r_{mn}r_{nl}+r_{ln}r_{nm}r_{ml})\times R_o\dots.R_tR_N\\
&+\sum_{i=1}^{N-5}\sum_{j=i+1}^{N}\sum_{k=i+1}^{N-3}\sum_{l=i+2}^{N}\sum_{m=k+1}^{N-1}\sum_{n=k+2}^{N}\dots.\sum_{N=t+1}^{N}(r_{ij}r_{ji})(r_{kl}r_{lk})(r_{mn}r_{nm})R_o\dots.R_tR_N\\
&+\sum_{i=1}^{N-5}\sum_{j=i+1}^{N-1}\sum_{k=i+1}^{N}\sum_{l=i+1}^{N}\sum_{m=i+1}^{N}\sum_{n=j+1}^{N}\dots\dots\sum_{N=t+1}^{N}(r_{ij}r_{jk}r_{kl}r_{lm}r_{mn}r_{nl}
\end{aligned}
$$

Table 3 Related significance (11 point scale) of the material selection factors [19-20]

Class description	**Relative importance (rij)**
One attribute is exceptionally less important over the other	0.045
One attribute is extremely less important over the other	0.135
One attribute is very less important over the other	0.255
One attribute is less important over the other	0.335
One attribute is slightly less important over the other	0.410
Two attributes are equally important over the other	0.500
One attribute is slightly more important over the	0.590
One attribute is more important over the other	0.665
One attribute is highly important over the other	0.745
One attribute is extremely more important over the other	0.865
One attribute is exceptionally more important over the other	0.955

The relative significance between two properties (i.e. rij) is assigned value on a fuzzy change scale, comparative to the one depicted for allotting the esteem of the subjective property, and is organized into eleven classes. The relative significance infers that an attribute ‘i’ is compared with another attribute ‘j’ in terms of relative significance for the given problem. Taking after the numerical guess system proposed in the literature [19-20], the relative significance is communicated in eleven classes that lead to minimization of subjectivity to a large degree whereas choosing the relative importance between two properties. Table 3 is proposed which helps in allotting rij values based on the expert opinion and can be varied from problem to problem. The steps for calculation of “Baluch” index for each collected sample are given below:

Step-I

Distinguish the quality parameters for the assessing the water quality and shortlist the monitoring stations for a particular case. It is worth to mention here that a threshold limiting quantitative or qualitative value may also be allotted to each quality parameter.

Step-II

1. Find the relative significance (rij) of each parameter and normalize component values (Ri) in the case of distinguishable choices after a short listing.
2. Build the WQA chart by taking into account the values and their relative importance of the parameter calculated. The number of nodes shall be the number of parameters in step 1 above. Importance between factors determines the magnitude and the direction of the edges.
3. Create a WQA map, with Ri diagonal elements and rij elements off diagonal. The matrix is a matrix of N versus N.
4. For given stations, calculate the "Baluch" index. The lowest "Baluch" index value is good quality water following the FDA-29 and WHO guidelines.
5. Arrange the stations with the meaning "Baluch" index in ascending order.

III. RESULTS AND DISCUSSION

The variation on pH values for the water samples collected from the eighteen stations listed in Table-1 is shown in figure 3. It is evident from the figure that pH values fluctuate from 8.1 to 8.4, which is above the recommended range 6-8 for good quality water. The average value for whole set of data is 8.25. Although according to allowed range as per National environmental quality standards (NEQ's), the reported values of pH are fine but still at all stations values are touching the upper allowed range (i.e. 8.5) that is concern able and this issue need to be addressed as far as assessing the water quality is concerned. This is because of the reason that these samples are from UIB where majority of water coming the snow covered peaks and glaciers, and still showing the pH values well above 7. The highest value of pH among all collected sample is 8.4, which is for the sample collected from Chillas, which is while the lowest among the collected ones is 7.9, which belong to sample collected from Shung. The plausible reasons for water being basic in nature at all collected station during this study are:

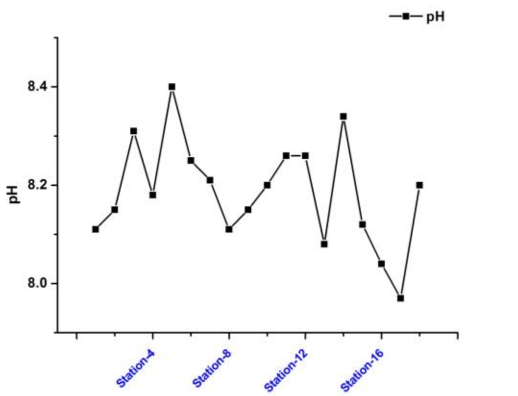

Figure 3 Vibration in pH values of water samples from UIB

1. Addition of carbonated rich minerals during the run off of river which are there in the soil sample
2. Agriculture run off, domestic and commercial waste

The higher value at Chillas is supporting our argument that in addition to addition of carbonated rich minerals during the, agriculture run off and domestic waste is also responsible for observed higher value of pH at this station.

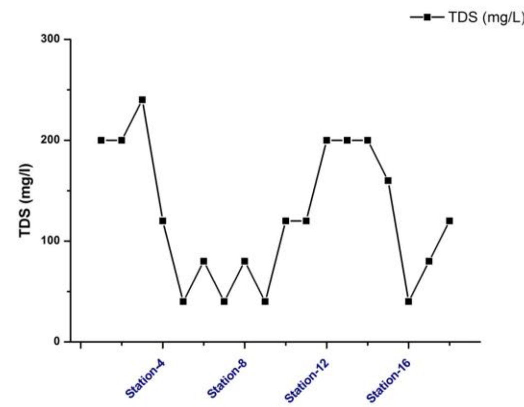

Figure 4 : Variation in TDS values of samples from UIB

Shown in figure 4 are the values of TDS in mg/l for all monitoring stations, which are under investigation in this study. It is evident from figure that the TDS values lies between 50 mg/l to 250 mg/l, which is in the range as far as prescribed National limits are concerned. However, the higher values i.e. 200 mg/l and >200 mg/l for the samples collected at Dandai, Bilani, and Gunar stations is of extreme concern because such higher amount of TDS at the start of Indus river indicate that TDS amount in main Indus river and at lower Indus Basin will be significantly higher. This is due to agriculture rich nature of Indus delta, and it is well established understanding that agriculture runoff is one of major cause of increase the amount of TDS. Soil erosion, heavy rain runoff might be responsible for slight higher amount of TDS at some stations.

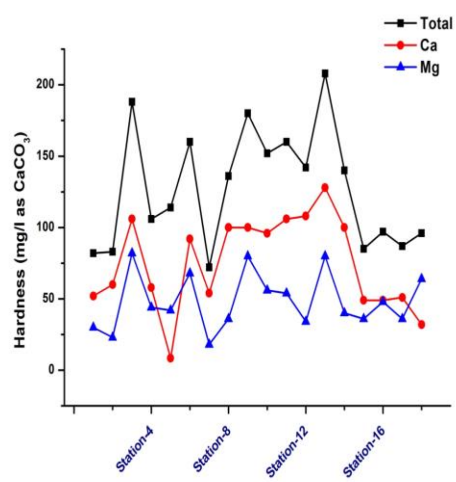

Figure 5: Variation in Hardness Total, Ca, Mg, of samples from UIB

Figure 5 reveals the Hardness (Total, Ca, and Mg Harness) in collected water sample in mg/l as CaCO3 from under investigation monitoring stations mentioned in table 1. The average value of total hardness is around 150 mg/l as CaCO3, which is due to higher values of Ca hardness. The average values of Ca hardness is 90 mg/l as as CaCO3 while the average values of Mg hardness is 50 mg/l as CaCO3 (see figure 5). The higher amount of Ca hardness as CaCO3 is due to addition of carbonated rich minerals during the soil erosion along the flow of river, and carbonated rich minerals are there in the soil samples. This factor is also responsible for increasing the pH of collected samples as shown in figure 3. Shown in figure 6 is the concentration of SO42- ions and Cl-1 ions in mg/l in each sample. As it is evident that the concentration of both ions is very low, except Chillas, Komaila, Kotgal where the concentration of SO42- ions is significantly higher and reaches to a value of 110 mg/l for sample collected from Komaila station. Sulfates are the second main bicarbonate in hard water tanks. Sulfates (SO42-) may occur naturally or as a result of municipal or industrial discharge. When naturally occurring, they often result from falling leaves in a river, water going through rocks or soil containing gypsum and other common minerals, or a deposition of the atmosphere. Main sources include wastewater processing plants and industrial waste disposal systems such as tanneries, pulp mills and fabrics. The rush of fertilized land also helps to sulphate. The reason for higher concentrations of SO42- ions in aforementioned 3 stations might be the Agriculture run off, domestic and commercial waste.

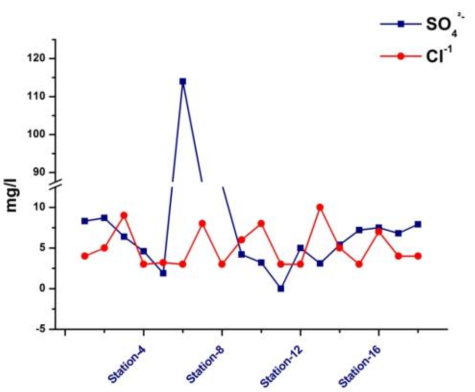

Figure 6: SO42- and Cl-1 ion fluctuation for collected samples from UIB

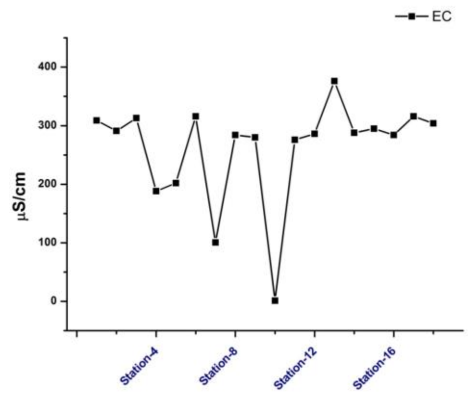

Figure: 7 EC in US/cm of water samples represented as a function of stations

Shown in figure 7 are the values of EC in μS/cm for each collected sample from different stations belonging to UIB. It is evident from figure that the average value of electrical conductivity is 300 μS/cm. The highest value of EC belongs to the sample collected from Bisham, which is approximately 400 μS/cm while the lowest value of EC belongs to the sample collected from Jijal. Following are the plausible reasons for the concentrations of higher values of EC:

1. TDS
2. Addition of minerals due to soil erosion
3. Agriculture run off
4. Water flow
5. Inclusion of minerals due to decomposition of leaves etc.

Salts and other inorganic chemicals are divided into small electrically charge ions when they dissolve into water. Ions increase the water's ability to provide electricity. Electrical current common ions of water include sodium, chloride, calcium and magnesium. The existence of Ca and Mg ions is reported above in all collected samples, which is due to the addition of carbonated rich minerals during the run off river, which are there in the soil sample. Although, separately each quality parameter reflects some information about the water quality at each station but as a whole, it is very difficult for policy makers to decide at which stations the quality of water is the best. In order to have conclusive saying about which stations is the best as far as its quality on the basis of investigating the quality parameters is concerned, a new quality index named as "Baluch" Index is framed while constructing the Water Quality Assessment (WQA) graph. The detail about the WQA graph and calculation of "Baluch" index is given in section 2. In the subsequent section the stepwise detail of calculating the “Baluch” index value is given

Case Study: Calculation “Baluch” Index for UIB

For assessing the water quality for the monitoring stations of UIB mentioned in table 1, different methodological steps are explained above, are carried out as described below:

Step-1

In this study, the parameters, which are used for calculating the “Baluch” Index are pH, TDS, Total Hardness, Ca Hardness, SO42-, CI-, Mg Hardness, EC. The quantitative data of the parameters for all 18 stations is given below in Table 4

Table 4: Quality parameters quantitative data

Stations	pH	TDS	Total Hardness	Ca Hardness	Sulphate	CI-	Mg Hardness
Dandai	8.11	200	82	52	8.3	4.004	30
Bilani	8.15	200	83	60	8.7	5.005	23
Gunar	8.31	240	188	106	6.4	9	82
Jalipur	8.18	120	106	58	4.6	3	44
Chillas	8.4	40	114	8.4	1.9	3.2	42
Komaila	8.25	80	160	92	114	3	68
Kotgal	8.21	40	72	54	27	8	18
Pattan	8.11	80	136	100	72	3	36
Thakot	8.15	40	180	100	4.2	6	80
Jijal	8.2	120	152	96	3.2	8	56
Dubair	8.26	120	160	106	0	3	54
Torghar mud	8.26	200	142	108	5	3	34
Darband	8.08	200	208	128	3.1	10	80
Bisham	8.34	200	140	100	5.4	5.005	40
Kamach	8.12	160	85	49	7.2	3.003	36
Judbah	8.04	40	97	49	7.5	7.007	48
Shang	7.97	80	87	51	6.8	4.004	36
Serai	8.2	120	96	32	7.9	4.004	64

Step-2

The Quantitative data given in table 4 is normalized while following the methodology explained above in section 2. The normalized data is given in table 5 below:

Table 5: Normalized data values

Stations	pH	TDS (mg/l)	Total Hardness	Ca Hardness	SO42- (mg/l)	CI- (mg/l)	Mg Hardness	EC (µS/cm)
Dandai	0.9827	0.2000	0.8780	0.6154	0.0602	0.7493	0.6000	0.0035
Bilani	0.9779	0.2000	0.8675	0.5333	0.0575	0.5994	0.7826	0.0037
Gunar	0.9591	0.1667	0.3830	0.3019	0.0781	0.3333	0.2195	0.0035
Jalipur	0.9743	0.3333	0.6792	0.5517	0.1087	1.0000	0.4091	0.0058
Chillas	0.9488	1.0000	0.6316	0.4444	0.2632	0.9375	0.4286	0.0054
Komaila	0.9661	0.5000	0.4500	0.3478	0.0044	1.0000	0.2647	0.0034
Kotgal	0.9708	1.0000	1.0000	0.5926	0.2632	0.3750	1.0000	0.0108
Pattan	0.9827	0.5000	0.5294	0.3200	0.0694	1.0000	0.5000	0.0038
Thakot	0.9779	1.0000	0.4000	0.3200	0.1190	0.5000	0.2250	0.0039
Jijal	0.9720	0.3333	0.4737	0.3333	0.1563	0.3750	0.3214	1.0000
Dubair	0.9649	0.3333	0.4500	0.3019	1.0000	1.0000	0.3333	0.0039
Torghar mud	0.9649	0.2000	0.5070	0.2963	0.1000	1.0000	0.5294	0.0038
Darband	0.9864	0.2000	0.3462	0.2500	0.1613	0.3000	0.2250	0.0029
Bisham	0.9556	0.2000	0.5143	0.3200	0.0926	0.5994	0.4500	0.0038
Kamach	0.9815	0.2500	0.8471	0.6531	0.0694	0.9990	0.5000	0.0037
Judbah	0.9913	1.0000	0.7423	0.6531	0.0667	0.4281	0.3750	0.0038
Shang	1.0000	0.5000	0.8276	0.6275	0.0735	0.7493	0.5000	0.0034
Serai	0.9720	0.3333	0.7500	1.0000	0.0633	0.7493	0.2813	0.0036

Step-3

Decision matrix is constructed while writing the relative importance of parameters i.e. rij as explained above. The values of relative importance are assigned based on expert survey. The decision matrix is given below

Table 6: Decision matrix while considering the relative importance (rij) of the parameter

	pH	**TDS (mg/l)**	**Total Hardness**	**Ca Hardness**	**SO42- (mg/l)**	**CI- (mg/l)**	**Mg Hardness**	**EC (µS/cm)**
pH	-	0.135	0.255	0.335	0.335	0.255	0.41	0.045
TDS (mg/l)	0.865	-	0.745	0.745	0.745	0.745	0.745	0.5
Total Hardness	0.745	0.255	-	0.5	0.5	0.5	0.5	0.045
Ca Hardness	0.665	0.255	0.5	-	0.5	0.5	0.5	0.045
SO42- (mg/l)	0.665	0.255	0.5	0.5	-	0.5	0.5	0.045
CI- (mg/l)	0.745	0.255	0.5	0.5	0.5	-	0.5	0.045
Mg Hardness	0.59	0.255	0.5	0.5	0.5	0.5	-	0.045
EC (µS/cm)	0.955	0.5	0.955	0.955	0.955	0.955	0.955	-

Step-4

Finally, "Baluch" index is calculated for each monitoring station. The index value is calculated while using the relation and given in table below

$$\text{"Baluch" Index} = Per\ (A)$$

Where "A" is the square matrix with diagonal elements belong to normalized values of quality parameter of each stations

Table 7 Calculated values of "Baluch" Index from the quality parameters of each stations

Sr. No	**Stations**	**"Baluch" Index**
1	Dandai	1.36267
2	Bilani	1.26055
3	Gunar	0.8707
4	Jalipur	1.42533
5	Chillas	1.53544
6	Komaila	1.30688
7	Kotgal	1.40435
8	Pattan	1.37349
9	Thakot	1.14205
10	Jijal	2.003
11	Dubair	1.26125
12	Torghar mud	1.26209
13	Darband	0.856165
14	Bisham	1.06432
15	Kamach	1.51355
16	Judbah	1.30032
17	Shang	1.43303
18	Serai	1.31982

It is evident from table above that the water collected from station number 13 i.e. Darband has lowest value of "Baluch" Index which is 0.856165, which is reflecting the fact that water collected from Darband is of best quality. From the experimental data shown above one can clearly sees that this stations has the lowest value of pH, which is the main concerned quality parameter as far as water quality of all stations is concerned (see figure of pH data).

IV. Conclusion

A new index named as "Baluch" Index is framed while using the concepts of graph theory and matrix approach, which made possible for assessing the quality of water based on experimental measured values of quality parameter. The proposed index not only considers quality parameter, their interrelations, their concentration, and expert opinion but also extremely sensitive to the recommended range for each quality parameter during the process of assessing the water quality at monitoring stations. The "Baluch" index is more generic in nature and quantitative and qualitative considerations water quality parameters at the same time, can be applied for any water reservoir and for any region. As a case study, it is applied for assessing the quality of water sample collected from 18 stations of UIB while utilizing the data of 9 quality parameters and "Baluch" index for Darband station has lowest value i.e. 0.85 and "Baluch" index highlight the situation that all stations have relatively higher values of pH, which is concern able problem. Furthermore, this new index indicate the lowest quality water belongs to Jijal stations due relatively higher values of pH (i.e. 8.25) and Cl- ion, above average values of TDS, Harness, SO42- ion concentrations. Finally, it is suggested that the results reported in this thesis and index framed during the course this research could be utilized for designing a comprehensive monitoring protocol for monitoring the Indus River thus saving this important resource of Pakistan.

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