Technical Journal, University of Engineering and Technology (UET) Taxila, Pakistan Vol. 28 No. 3-2023
ISSN:1813-1786 (Print) 2313-7770 (Online)

Mathematical and Statistical Analysis of Monthly Mean Temperature Trend in Pakistan

K. Abbas[1], M. Siddique[2], M. Altaf [3], T. Kanwal[4], F. Shahzad[5], N. Ahmed[6], M. I. Marwat[7], S. Alam[8]

[1,2,4]*Department of Statistics, King Abdullah Campus Chatter Kalas, The University of Azad Jammu & Kashmir, Pakistan*
[3,5]*Department of Basic Sciences, University of Engineering and Technology, Taxila, Pakistan*
[6]*Department of Physics, King Abdullah Campus Chatter Kalas, The University of Azad Jammu & Kashmir, Pakistan*
[7]*Department of Community Medicine Khyber Medical College, Peshawar, Pakistan*
[8]*Department of Mathematics & Statistics, Faculty of Basic and Applied Sciences, International Islamic University, Islamabad, Pakistan*

[1]kamran.abbas@ajku.edu.pk

Abstract- Due to the temperature rises global warming has become very popular. Increasing of global temperature will disturb the agronomic sector, intensification some of the contagious diseases that may lead to high mortality rates in humans, high demand for electricity, water and food which ultimately affecting the economy of Pakistan. The current research aims to study the best fitted probability distribution that describes the monthly mean temperature (MMT) of four sites in Pakistan are Islamabad, Lahore, Muzaffarabad and Karachi based on secondary data sets which was collected from Pakistan Meteorological department Lahore for the period 1991 to 2020. The Frechet, Weibull and Log-Logistic distributions are applied and the parameters of these distributions are estimated by maximum likelihood and Bayesian estimation methods. Additionally, Log-normal and Generalized Extreme value distribution are considered using the maximum likelihood estimation method to estimate the parameters. Moreover, the graphs of probability density functions also constructed for comparison purposes. The goodness of fit test and model selection criteria such as Kolmogorov-Smirnov test, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are used to measure the accuracy of the predicted data using theoretical probability distributions. The results show that four sites favor the candidate distributions based on the P-values of Kolmogorov-Smirnov test at 5 percent level of significance. However, Log-Logistic distribution is the best fitted as compare to other candidate probability distributions based on AIC and BIC values. Furthermore, quantiles are also calculated using maximum likelihood and Bayesian estimation methods and concluded that quantile estimates are closed to the observed MMT series on the basis of Log-Logistic distribution.

Keywords- Log-normal distribution, Generalized Extreme value distribution, Frechet distribution, Weibull distribution, Log-Logistic distribution, Akaike Information Criterion, Bayesian Information Criterion, Kolmogorov-Smirnov test, Quantile estimation, Time series analysis.

I. INTRODUCTION

The issues of global warming are becoming more common due to the high temperature. Climate change got strong attention on a global scale during last decades. Gradually rising temperatures and change in rainfall pattern are evident in many regions around the world. Although the changes in mean temperature are small, it is actually due to the large changes in the frequency. According to Intergovernmental Panel Climate Change (IPCC), based on observations of increases in global average air temperatures, the warming of the climate system is now "unequivocal" and the surface temperatures is predicted to rise over the 21st and the heat waves are expected to occur more frequently and last longer. This phenomenon due to the temperature rises eventually will increase the variability of climate and also the occurrence of natural disasters.

Pakistan is divided into cold and warm regions according to prevailing weather conditions. Temperature is one of the basic determinants of weather change. Temperature is the degree of heat present in the substance or object, especially as expressed according to comparative scale or shown by a thermometer or perceived by touch. Temperature is important in all fields of natural sciences including physics, chemistry, earth science, astronomy, medicine, biology, ecology, material science, metallurgy, mechanical engineering and geography as well as most aspects of our daily life. The current study focuses the comprehensive statistical trend analysis on the MMT for the four major sites in Pakistan namely Karachi, Lahore, Islamabad and Muzaffarabad. The MMT of the selected stations differ from each other and detail of MMT of the selected station is as under.

Karachi (KHI) has pleasant weather for the greater part of the year. May and June are the hottest months, when the mean maximum temperature is about 93°F (34°C). The coolest months are January and February, during which the mean minimum temperature remains about 56°F (13°C). (Khan, Z, Ahmed 2021. Karachi). Lahore (LHR) structures a climate with five seasons. June considered the hottest month, where average highs routinely exceed 40 °C (104.0°F). The coolest month is January with dense fog. The city's peak maximum temperature was 48.3 °C (118.94 °F) recorded on May 30, 1944 and 48 °C (118 °F) recorded on June 9, 2007. The lowest temperature in LHR is –2.2 °C recorded on 17 January 1935 (PMD Lahore, 2007). Muzaffarabad (MZD) district is the capital of Azad Jammu and Kashmir, The wind blows from west to east during the day time, while at night it blows from south east to north. Wind velocity is higher during afternoon as compared to early morning. The snow line in winter remains around 1200 m while in summer, it rises to 3300 m (Qasim 2010a; 2010b; Anon., 2015). Islamabad (ISB) is a moist subtropical climate with four seasons. The weather ranges from a minimum of −3.9 °C (25.0 °F) in January to a maximum of 46.1 °C (115.0 °F) in June. The average low is 2 °C (35.6 °F) in January, while the average high is 38.1 °C (100.6 °F) in June. The highest temperature recorded was 46.5 °C (115.7 °F) in June, while the lowest temperature was −3.9 °C (25.0 °F) in January. ("Extremes of Islamabad". Meteorological Department of Pakistan, 2015).

It has been shown in many parts of the world that extreme temperatures can have adverse effects on environment, public health and economy. A number of researchers studied the trend detection in temperature. For instance, Sivakumar, (2012) studied meteorological parameters including average temperature, relative humidity, wind speed and rainfall for the period of 2001 to 2007 in northeastern zone of Tamilnadu (India) using cluster analysis, and found that all the parameters varied with year and the maximum changes were observed during 2005 and 2004 followed by 2007. Jain and Kumar (2012) reviewed several studies on trend analysis of temperature, rainfall and rainy days in India and observed that significant trend was determined frequently by Mann-Kendall test and its magnitude by Sen’s non-parametric slope estimator. Jakob and Walland (2016) introduced a number of probability density functions to temperature data analysis. Further, Vivekanandan (2018), Lee (2018), Lazoglou *(*2019), and Dehghan *(*2019), Torsen (2015) and Vivekanandan (2018) discussed different probability distributions related to temperature data sets. Parey (2010), Cannon (2010), Wang (2013), Ouarda and Charron (2018), Nemukula and Sigauke (2018), and Rydén (2019) recommended that generalized extreme value distribution (GEV) distribution was extensively used in studies for the assessment of changes (nonstationarity) in temperature analysis. However, Hossain (2016) and Vivekanandan (2018) found the GEV may not always be able to represent every temperature regime. Further, Karmeshu (2012) applied Mann-Kendall test statistic for analysis and detection of trends in meteorological data and precipitation of nine states in the USA from 1900 to 2011. Further, Singh *et al, (2013)* introduced the best fit models and their variables and that analyses could be used as a baseline study and as an input for the upcoming planning and projects. Pedrosa & Gama (2004) applied different probability distributions on average monthly temperature series in the municipality of Mossoró, northeastern Brazil. Hasan (2013), Sharma and Singh, (2010), Reiss and Thomas (2007) studied some applications of extreme value theory. Schaefer and Domroes (2009) analyzed mean daily temperature of many stations in Japan. Moreover, Yue and Hashino (2003) mentioned that the quantum change of different climatic variables, including temperature and precipitation, were not globally uniform, and large scale spatial and temporal variation may exist in climatically diverse regions. Brohan et al (2006), Lean and Rind (2008) analyzed the geographic distribution of the temperature response. Additionally, Sajjad (2009) used linear regression for identifying trends in climate data. Lado (2007) assessed the influence of geographical factors such as nominal altitude, latitude and longitude of the meteorological stations to the air temperature in State of São Paulo (Brazil) using multiple regression analysis. Peng (2011) interpolated the monthly and annual near surface air temperature (NSAT) in the Jiangsu province, China. Zhao (2005) estimated the near surface air temperature in the southern Qilian mountains of China, in which the weather stations are sparse, and the result indicated that the accuracy of the multiple linear regression model was higher than that of spatial interpolation models. Afsar (2013) analyzed temperature and rainfall volatility in Gilgit Baltistan area.

Pinheiro (2019) studied monthly average temperature in Mossoró, northeastern Brazil using different probability distributions. Similarly, Torsen (2015) and Vivekananda (2018) analyzed temperature trends.

Fitting probability distributions to climatic variables over time aims to understand the meteorological phenomena, clarify their occurrence patterns, and make probabilistic forecasts. Even when there is no considerable natural basis behind the choice of a particular theoretical distribution, empirically find that such distributions suit very well set of data. The simple construction of a frequency histogram or boxplot for the visualization of a sample of data becomes insufficient to choose which function best represent the data among the several probability density distributions. Some criteria and goodness of fit tests help to verify if a known function suits the distribution of a dataset (Pedrosa & Gama, 2004).

Hence present work aims to find the MMT series distribution by comparing several distributions and determine the probability distribution that describes the best for the MMT series in Pakistan based on AIC and BIC. Including this introduction section the remaining paper unfolds as follows. Section 2 introduces the methodology. Results and discussions are presented in Section 3 and finally conclusion is given in Section 4.

Data Description

The secondary data sets about MMT measured in degree Celsius (°C) over the period of January 1991 to December 2020 of four sites, namely, Islamabad (ISL), Lahore (LHR), Muzaffarabad (MZD) and Karachi (KHI) have been collected from Pakistan meteorological department Lahore. The monthly averages were calculated from the daily readings which are analyzed.

II. METHODOLOGY

Many probability distribution functions (PDF) have been proposed in recent past, but in present study Log-logistic distribution (LLD), Frechet distribution (FD), Weibull distribution (WD), Log-normal distribution (LND) and Generalized Extreme Value distribution (GEVD) are used to describe the behavior of MMT series. The PDF of these distributions are presented in Table 1. Maximum likelihood and Bayesian estimation methods are used to estimate the parameters of FD, WD and LLD. However, the parameters of LND and GEVD are estimated by only Maximum likelihood method.

Table 1. The PDF of five distributions and its parameter.

Distribution	PDF	Parameter
LLD	$f(x;\ \alpha,\beta) = \frac{\left(\frac{\beta}{\alpha}\right)\left(\frac{x}{\alpha}\right)^{\beta-1}}{\left\{1+\left(\frac{x}{\alpha}\right)^{\beta}\right\}^{2}},\quad x > 0,\ \alpha,\ \beta > 0,$	$\alpha = Scale$ $\beta = Shape$
FD	$f(x;\ \alpha,\beta) = \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{x}\right)^{\alpha+1} e^{-\left(\frac{\beta}{x}\right)^{\alpha}},\quad x > 0,\ \alpha,\beta > 0,$	$\alpha = Shape$ $\beta = Scale$
LND	$f(x;\ \mu,\sigma) = \left(\frac{1}{\sigma^2 2\pi}\right)^{\frac{1}{2}} exp\left\{\frac{-(x-\mu)^2}{2\sigma^2}\right\},\quad x > 0$	$\mu = Shape$ $\sigma^2 = Scale$
WD	$f(x;\ \alpha,\beta) = \left(\frac{\alpha}{\beta}\right)\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}},\ x > 0,\ \alpha,\beta > 0,$	$\alpha = Shape$ $\beta = Scale$
GEVD	$f(x;\ \theta,\alpha,\eta) = \frac{1}{\alpha}\left\{1 + \eta\frac{(x-\theta)}{\alpha}\right\}^{-\frac{1}{\eta}-1} exp\left[-\left\{1 + \eta\frac{(x-\theta)}{\alpha}\right\}^{-\frac{1}{\eta}}\right]$	$\eta = Shape$ $\alpha = Scale$ $\theta = Location$

Model Selection

The following goodness of fit tests are used for the selection of best fitted distribution for MMT series.

Kolmogrove Smirnov (KS) Test

KS (Kolmogrove, 1933) test was performed under the null hypothesis to check whether MMT samples originate from a hypothesized continuous distribution. The KS test statistic (D) can be expressed as

$$D = \max\left|F(x_i) - \frac{i-1}{n},\ \frac{i}{n} - F(x_i)\right|$$

Where $\boldsymbol{x_i}$ represents the MMT samples.

AIC and BIC

AIC and BIC are used to pick and endorse the most applicable distribution for describing the behavior of MMT series of four selected sites based on the minimum AIC and BIC values. The AIC and BIC values can be calculated as

$$AIC = 2p - 2ln(\hat{L})$$

$$BIC = Pln(n) - 2ln(\hat{L})$$

where $\hat{L}$ be the maximum value of the likelihood function and 'p' is the number of parameters estimated.

III. RESULTS AND DISCUSSION

The descriptive statistics provided in Table 2 are useful to understand the general trends and tendencies of MMT at given sites. The maximum and the minimum temperature varies from 31.80 ℃ to 8.100℃ for Islamabad site and for Muzaffarabad maximum is 31.40 and the minimum temperature is 6.500. It is witnessed that for Islamabad and

Muzaffarabad sites have higher MMT for the month of June as compared to remaining months. The maximum temperature of Lahore 35.3 ℃ and the minimum is 10.90 ℃. Observed data set indicates that MMT of Lahore site for the month of May is greater than the remaining months. Similarly, the maximum and minimum temperature of Karachi were recorded in 36℃ to 11.90 ℃ respectively. It is perceived that MMT of Karachi site for October is higher than the remaining months. Coefficient of variation (CV) has been calculated as a relative measure of dispersion and consistent behavior in MMT can be visualized via CV values. The values of CV show that the least consistency exists in MMT of Karachi, followed by Lahore, Islamabad and Muzaffarabad. Moreover, Figure 1 shows boxplots at given sites corresponding to each month. Boxplot displays the 5-number summary of complete dataset. Starting from bottom the three horizontal lines represent 1st, 2nd and 3rd quartiles of corresponding of observed datasets, respectively. It can be observed that 1st (Q_1) and 3rd (Q_3) quartiles were not equidistant from mean indicate that the distributions associated to the observed data series at these sites are non-normal. Therefore, it would be appropriate to select positively skewed distribution(s) as candidate(s) for the observed data series at given sites.

Table 2: Descriptive Statistics of MMT of four Sites.

Month	Stations	Mean	Median	Min.	Q_1	Q_3	CV
Jan.	ISL	10.22	10.35	8.100	9.625	10.775	0.0885
	LHR	13.05	13.20	10.90	12.62	13.70	0.0726
	KHI	18.87	19.05	11.90	18.43	19.88	0.0881
	MZD	9.49	9.800	6.500	8.900	10.300	0.1484
Feb.	ISL	13.31	13.20	10.60	12.10	13.97	0.1663
	LHR	16.50	16.55	14.10	15.40	17.38	0.0857
	KHI	21.70	21.65	19.00	20.85	22.68	0.0576
	MZD	11.84	11.80	8.80	11.10	12.70	0.1159
Mar.	ISL	17.40	17.25	14.50	16.27	18.25	0.0905
	LHR	21.58	21.50	19/00	20.30	22.27	0.0761
	KHI	25.91	25.90	23.30	25.30	26.57	0.0424
	MZD	16.24	15.95	12.90	15.20	17.43	0.1296
Apr.	ISL	22.50	22.65	19.30	21.50	23.45	0.0710
	LHR	27.71	27.55	24.50	26.62	29.10	0.0629
	KHI	29.40	29.65	25.40	29.10	30.00	0.0377
	MZD	21.13	20.90	18.10	20.00	22.20	0.0862
May	ISL	27.36	27.45	24.30	25.90	28.40	0.0656
	LHR	32.29	32.60	29.30	31.50	33.08	0.0385
	KHI	31.39	31.30	29.90	30.93	31.88	0.0211
	MZD	25.92	26.05	22.70	24.12	27.00	0.0824
Jun.	ISL	30.28	30.35	28.70	29.40	31.18	0.0309
	LHR	33.20	33.00	31.30	32,20	34.27	0.0365
	KHI	31.18	32.10	31.20	31.90	32.40	0.0190
	MZD	28.93	29.25	25.70	28.12	30.05	0.0499
Jul.	ISL	29.37	29.40	24.70	29.00	29.98	0.0401
	LHR	31.33	31.20	29.80	30.62	31.88	0.0278
	KHI	30.84	30.90	29.10	30.60	31.18	0.0235
	MZD	28.81	28.90	27.10	28.12	29.40	0.0318
Aug.	ISL	28.55	28.55	27.70	28.32	28.80	0.0150
	LHR	30.28	30.95	28.90	30.32	31.30	0.0294
	KHI	29.59	29.55	28.00	29,25	30.10	0.0239
	MZD	28.34	28.30	27.10	27.93	28.65	0.0249
Sep.	ISL	26.78	26.85	25.30	26.40	27.30	0.0283
	LHR	29.87	29.80	28.30	29.50	30.30	0.0245
	KHI	29.70	29.65	27.90	29.12	30.38	0.0303
	MZD	26.36	26.50	24.60	25.80	26.90	0.0345
Oct.	ISL	22.19	22.35	20.10	21.32	23.20	0.0487
	LHR	26.25	26.45	22.90	25.93	26.90	0.0384
	KHI	29.42	29.30	27.20	28.65	29.90	0.0513
	MZD	21.75	22.00	19.20	20.70	22.77	0.0594
Nov.	ISL	16.53	16.40	15.00	16.10	17.07	0.0447
	LHR	20.41	20.50	18.70	19.90	21.07	0.0412
	KHI	25.17	25.20	23.50	24.62	25.95	0.0362
	MZD	15.70	15.90	12.80	14.97	16.65	0.0829
Dec.	ISL	12.05	12.10	10.30	11.50	12.55	0.0631
	LHR	15.12	15.25	12.60	14.62	15.88	0.0721
	KHI	20.88	20.75	19.60	20.32	21.27	0.0441
	MZD	11.24	11.25	9.30	10.50	11.78	0.1031

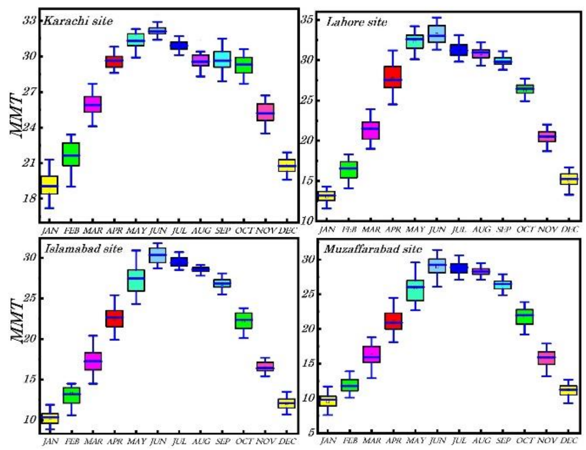


Figure 1. Box plots of MMT of four sites.

Parameters Estimates and Goodness of Fit Tests for FD, WD and LLD

ML and Bayesian estimation methods are used to estimate the parameters of FD, WD and LLD and results are presented in Tables 3-4. ML and the Bayesian estimates are almost same of all the candidate distributions. Moreover, AIC, BIC and KS test are used to confirm the suitability of selected distribution and results are presented in Tables 5-6 for comparison purposes.

Table 3: ML estimates for FD, WD and LLD.

Month	Sites	FD		WD		LLD	
		$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$
Jan.	ISL	10.6331	9.7330	12.3986	10.6315	10.2278	19.9142
	LHR	12.8434	12.5336	15.9060	13.4760	13.0789	24.2583
	KHI	6.0290	17.6601	17.2116	19.4719	19.0347	24.1259
	MZD	5.8031	8.6356	8.4107	10.0748	9.5591	11.5303
Feb.	ISL	9.4898	12.3881	4.9403	14.2762	13.0100	14.3815
	LHR	13.0909	15.7828	11.0904	17.1547	16.4394	21.1363
	KHI	17.2790	21.0460	18.4003	22.2798	21.6981	30.5944
	MZD	7.7246	11.0590	9.7553	12.4360	11.8171	15.3730
Mar.	ISL	12.5580	16.6099	11.2569	18.1343	17.2795	19.7230
	LHR	16.0958	20.7830	12.8531	22.3624	21.4342	23.8897
	KHI	22.5008	25.3312	26.8073	26.4178	25.9212	41.7829
	MZD	8.8258	15.1575	7.8668	17.1790	16.0498	14.0158
Apr.	ISL	14.4006	21.6498	16.1643	23.2244	22.4858	23.9342
	LHR	16.9188	26.8052	17.4829	28.5191	27.6655	27.3291
	KHI	18.3390	28.7330	39.0392	29.8443	29.5240	53.0777
	MZD	12.848	20.2033	12.0940	21.9766	21.0154	20.6157
May	ISL	17.4762	26.4639	15.6409	28.2121	27.2542	26.8113
	LHR	23.6158	31.6094	33.2810	32.8386	32.3894	45.8622

	KHI	50.0620	31.0624	44.5308	31.7176	31.3622	86.0728
	MZD	14.4033	24.8427	12.6600	26.9092	25.7831	20.8123
Jun.	ISL	36.6791	29.8139	36.7284	30.7321	30.2690	53.3571
	LHR	31.6532	32.5968	30.1356	33.7825	33.1582	45.6416
	KHI	64.1409	31.8947	47.6451	32.4948	32.1340	97.1605
	MZD	18.3126	28.1359	25.0901	29.5702	29.0253	35.3010
Jul.	ISL	15.8953	28.6489	30.8505	29.8648	29.4285	53.8459
	LHR	43.5781	30.9164	4.1246	31.7673	31.2599	65.1771
	KHI	40.8604	30.4687	38.6704	31.1929	30.8511	83.6895
	MZD	33.3653	28.3448	34.2509	29.2458	28.8130	54.3581
Aug.	ISL	71.4251	28.3430	64.7926	28.7659	28.5457	121.6917
	LHR	35.2835	30.3686	33.2855	31.2628	30.8184	61.7379
	KHI	40.7088	29.2262	42.1364	29.9283	29.6011	77.0983
	MZD	45.7032	28.0013	39.9666	28.6931	28.3068	71.9723
Sep.	ISL	35.4879	26.3859	39.1018	27.1332	26.7942	62.4318
	LHR	43.3369	29.5080	41.1479	30.2277	29.8492	73.0614
	KHI	32.2686	29.2343	36.1020	30.1225	29.7009	58.4018
	MZD	30.4458	25.8985	32.4045	26.7931	26.3624	49.8387
Oct.	ISL	21.0104	21.6255	24.6581	22.6895	22.2153	34.5017
	LHR	19.3734	25.6654	36.0850	26.6729	26.3534	49.7470
	KHI	28.7688	28.8163	13.5066	30.2329	29.2800	46.0856
	MZD	16.8435	21.0538	20.4427	22.3297	21.7944	28.3886
Nov.	ISL	23.5386	16.1578	23.6402	16.8844	16.5159	38.9325
	LHR	23.1240	19.9616	29.2132	20.7935	20.4611	42.366
	KHI	28.6509	24.6968	31.9587	25.5960	25.1842	47.2234
	MZD	10.5975	14.9507	15.0526	16.2591	15.7735	21.3302
Dec.	ISL	15.4879	11.6494	16.4548	12.4007	12.0298	28.7348
	LHR	12.3123	14.4994	17.9164	15.5885	15.1937	24.2770
	KHI	28.9712	20.4537	20.4272	21.3429	20.7929	42.7925
	MZD	10.6027	10.6359	10.4779	11.7677	11.1796	16.8244

Table 4: Bayesian estimates for FD, WD and LLD.

Month	Sites	FD		WD		LLD	
		$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$
Jan.	ISL	10.4701	9.7377	12.1574	10.6212	10.2242	19.4490
	LHR	12.6352	12.5394	15.5976	13.4671	13.0753	23.6954
	KHI	5.9653	17.6639	16.8495	19.4620	19.0299	23.5303
	MZD	5.7121	8.6389	8.2164	10.0592	9.5475	11.2479
Feb.	ISL	9.3114	12.3942	4.8695	14.2299	13.0049	14.0380
	LHR	12.8538	15.7902	10.9025	17.1371	16.4348	20.6622
	KHI	16.9940	21.0537	18.0600	22.2653	21.6950	29.8979
	MZD	7.6075	11.0641	9.5664	12.4205	11.8119	15.0086
Mar.	ISL	12.3266	16.6178	11.0442	18.1147	17.2757	19.2772
	LHR	15.7785	20.7917	12.6305	22.3442	21.4308	23.3458
	KHI	22.1331	25.3385	26.2963	26.4098	25.9186	40.8203
	MZD	8.6674	15.1657	7.7181	17.1475	16.0431	13.6839
Apr.	ISL	14.1460	21.6588	15.8450	23.2109	22.4798	23.4121
	LHR	16.6121	26.8154	17.1308	28.5007	27.6619	26.7165
	KHI	18.1024	28.7417	38.1813	29.8361	33.3137	3.2535
	MZD	12.6180	20.2125	11.8760	21.9561	21.0108	20.1393
May	ISL	17.1489	26.4736	15.3475	28.1909	27.2516	26.2022
	LHR	23.2454	31.6187	32.5490	32.8277	32.3847	44.8037
	KHI	49.1838	31.0670	43.8019	31.7104	31.3619	84.0996
	MZD	14.1181	24.8534	12.4223	26.8850	25.7778	20.3524
Jun.	ISL	35.9269	29.8209	35.9369	30.7229	30.2677	52.2430
	LHR	31.0143	32.6053	29.5252	33.7709	33.1571	44.6602
	KHI	62.8724	31.8987	46.9074	32.4885	32.1346	94.8481
	MZD	18.0095	28.1461	24.5417	29.5580	29.0195	34.4707
Jul.	ISL	15.7012	28.6585	30.3060	29.8531	29.4277	52.5201
	LHR	42.7179	30.9222	33.4795	31.7543	31.2604	63.6654
	KHI	40.2154	30.4743	38.1224	31.1831	30.8497	81.6191
	MZD	32.7610	28.3513	33.6025	29.2368	28.8116	53.1282
Aug.	ISL	70.1441	28.3463	63.6551	28.7618	28.5456	118.8418
	LHR	34.6738	30.3749	32.7264	31.2551	30.8168	60.3060
	KHI	40.0319	29.2317	41.4665	29.9224	29.6004	75.2718
	MZD	44.8385	28.0061	39.2614	28.6864	28.3068	70.2633
Sep.	ISL	34.8713	26.3915	38.3880	27.1270	26.7921	60.9677
	LHR	42.5769	29.5133	40.3923	30.2201	29.8493	71.3782
	KHI	32.6827	29.2411	35.4465	30.1142	29.6995	57.0263
	MZD	29.8819	25.9049	31.7608	26.7852	26.3605	48.7069
Oct.	ISL	20.6278	21.6324	24.1421	22.6795	22.2123	33.7500
	LHR	19.1159	25.6734	35.2603	26.6655	26.3500	48.5218
	KHI	28.2317	28.8236	13.3284	30.2051	29.2784	44.9679
	MZD	16.5424	21.0626	19.9884	22.3172	21.7885	27.7420
Nov.	ISL	23.1212	16.1626	23.1827	16.8775	16.5149	38.0473
	LHR	22.7237	19.9676	28.6204	20.7866	20.4585	41.3930
	KHI	28.1232	24.7052	31.2919	25.5883	25.1821	46.1670
	MZD	10.4284	14.9577	14.7338	16.2466	15.7674	20.8267
Dec.	ISL	15.2405	11.6538	16.1647	12.3924	12.0282	28.0598
	LHR	12.1168	14.5056	17.5141	15.5782	15.1880	23.7023
	KHI	28.3928	20.4592	20.1059	21.3320	20.7929	41.7699
	MZD	10.4116	10.6410	10.2720	11.7547	11.1759	16.4429

Table 5: Goodness of fit tests for FD, WD and LLD using ML Estimates.

Month	Sites	FD			WD			LLD		
		KS	AIC	BIC	KS	AIC	BIC	KS	AIC	BIC
Jan.	ISL	0.6078	90.5276	93.3300	0.9701	83.9897	86.7921	0.8559	83.1670	85.9694
	LHR	0.4011	94.1104	96.9128	0.9936	84.9164	87.7188	0.9235	86.2219	89.0243
	KHI	0.0151	154.4436	157.2460	0.9234	106.5575	109.3599	0.7667	113.1238	115.9262
	MZD	0.1874	121.9142	124.7166	0.9930	106.2673	109.0697	0.9232	111.8645	114.6669
Feb.	ISL	0.6502	118.0981	120.9005	0.0228	146.8657	149.6681	0.8817	120.5813	123.3837
	LHR	0.6129	109.6650	112.4677	0.8212	115.9739	118.7763	0.9300	108.1620	110.9644
	KHI	0.8251	107.6181	110.4205	0.8555	104.6634	107.4658	0.9172	102.3786	105.1810
	MZD	0.5550	117.7	120.5	0.765	107.9	110.7	0.997	107.9746	110.7

			117	141	8	212	236	0		77
Mar.	ISL	0.9657	115.5594	118.3618	0.5589	120.3507	123.1531	0.9474	115.3981	118.2005
	LHR	0.9552	115.1280	117.9304	0.5291	124.5160	127.3184	0.9979	117.2090	120.0114
	KHI	0.7289	101.7236	104.5259	0.9223	94.5962	97.3986	0.9870	94.7173	97.5197
	MZD	0.4625	132.432	135.2344	0.5147	137.2733	140.0757	0.9336	131.8907	134.6931
Apr.	ISL	0.7391	121.6362	124.4386	0.8431	116.9925	119.7949	0.9317	118.5188	121.3212
	LHR	0.8788	125.1174	127.9198	0.8712	123.7684	126.5708	0.9900	123.2487	126.0511
	KHI	0.0561	116.6144	119.4168	0.9867	83.9344	86.7368	0.9212	90.6691	93.47155
	MZD	0.8820	125.6033	128.4057	0.5743	128.2506	131.053	0.9916	124.5182	127.3206
May	ISL	0.8357	123.5124	126.3148	0.6114	128.1515	130.9539	0.9413	123.9987	126.8011
	LHR	0.1579	111.7131	114.5155	0.8788	97.4544	100.2569	0.6083	102.4985	105.3009
	KHI	0.8199	66.5229	69.3253	0.5469	71.5368	74.3392	0.8662	63.1846	65.9870
	MZD	0.7055	132.8167	135.6191	0.7903	137.8163	140.6187	0.7301	135.0406	137.843
Jun.	ISL	0.5795	85.8049	88.6073	0.4931	85.7939	88.5963	0.6585	87.3439	90.1463
	LHR	0.6280	100.4081	103.2105	0.6227	102.3928	105.1952	0.6228	102.4649	105.2673
	KHI	0.5530	56.2123	59.0147	0.2526	68.7430	71.5454	0.9261	57.9696	60.7720
	MZD	0.4320	120.5675	123.3698	0.9920	107.3043	110.1067	0.8514	111.6183	114.4207
Jul.	ISL	0.0077	123.8231	126.6255	0.3039	93.1119	95.9143	0.8001	90.8163	93.6187
	LHR	0.7050	77.6786	80.4810	0.3331	88.1062	90.9086	0.9843	79.6799	82.4823
	KHI	0.1864	75.8712	78.6736	0.3841	77.5210	80.3234	0.9138	65.8431	68.6455
	MZD	0.7299	86.76129	89.56368	0.9896	85.45058	88.25298	0.9965	84.56519	87.36759
Aug.	ISL	0.4774	40.1824	42.9848	0.3305	44.1427	46.9451	0.9770	37.1213	39.9237
	LHR	0.7483	86.4470	89.2494	0.4809	88.4969	91.2993	0.9050	82.0946	84.8970
	KHI	0.2332	74.35	77.15	0.658	72.25	75.05	0.896	66.8777	69.68

			46	70	8	66	90	2		01
	MZD	0.4961	67.79876	70.60115	0.1839	73.50094	76.30334	0.8356	67.70463	70.50703
Sep.	ISL	0.5855	77.5443	80.3467	0.9685	73.2063	76.0087	0.9985	72.5229	75.3253
	LHR	0.3989	72.6014	75.4038	0.7088	74.8819	77.6843	0.9387	69.7684	72.5708
	KHI	0.3038	87.6009	90.4032	0.3217	84.1023	86.9047	0.7961	82.8113	85.6137
	MZD	0.5839	86.77421	89.5766	0.8658	84.23274	87.03514	0.9365	84.39215	87.19454
Oct.	ISL	0.4114	98.4378	101.2402	0.7557	92.0273	94.8297	0.8059	95.5038	98.3062
	LHR	0.0782	107.4929	110.2953	0.9594	81.0724	83.8748	0.8746	86.8992	89.7016
	KHI	0.6961	99.0204	101.8228	0.0215	133.5505	136.3529	0.9841	99.8374	102.6399
	MZD	0.2759	110.1721	112.9745	0.8708	102.2175	105.0199	0.8465	106.2231	109.0255
Nov.	ISL	0.9046	73.8324	76.63487	0.6141	73.6625	76.4649	0.9167	71.2920	74.0944
	LHR	0.2895	86.3593	89.1617	0.9985	76.3031	79.1055	0.9158	79.3722	82.1746
	KHI	0.4268	87.5744	90.37686	0.5565	83.0703	85.8727	0.9592	84.7409	87.5433
	MZD	0.3848	116.1045	118.9069	0.9925	101.2007	104.0031	0.8941	105.5255	108.3279
Dec.	ISL	0.5953	78.1753	80.9777	0.6703	75.6934	78.4958	0.8731	71.5953	74.3977
	LHR	0.2948	105.0329	107.8353	0.9921	89.4742	92.2766	0.9458	95.1798	97.9822
	KHI	0.8610	78.5336	81.3360	0.2783	93.2743	96.0767	0.9967	81.3032	84.1056
	MZD	0.7871	99.27853	102.0809	0.7457	99.81469	102.6171	0.9211	98.24484	101.0472

Table 6: Goodness of fit tests for FD, WD and LLD using Bayesian estimates.

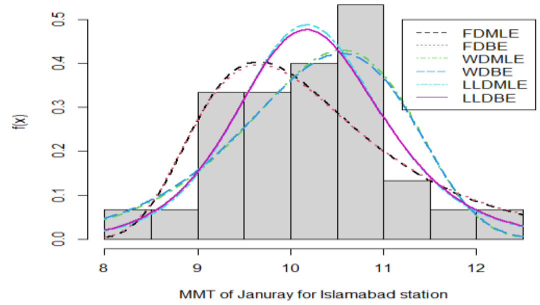
Month	Sites	FD			WD			LLD		
		KS	AIC	BIC	KS	AIC	BIC	KS	AIC	BIC
Jan.	ISL	0.6017	99.7912	102.5936	0.9760	93.7326	96.5350	0.8768	93.7759	96.5783
	LHR	0.4254	104.257	107.0596	0.9975	95.6313	98.4337	0.9273	97.7173	100.5198
	KHI	0.0136	163.7694	166.5718	0.9246	118.1644	120.9668	0.7320	125.3574	128.1598
	MZD	0.1956	129.7274	132.5298	0.9839	115.1216	117.924	0.9221	121.2434	124.0458

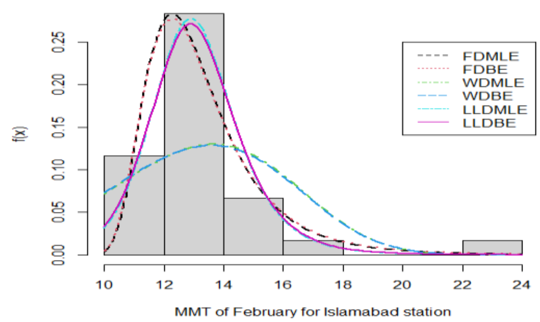
Feb.	ISL	0.5956	127.6138	130.4162	0.0272	155.3598	158.1622	0.8490	131.0201	133.8225
	LHR	0.6618	120.3090	123.1115	0.8266	126.4519	129.2543	0.9572	119.8403	122.6427
	KHI	0.7797	119.3944	122.1968	0.8690	116.6767	119.4791	0.9312	115.3516	118.1540
	MZD	0.5661	126.5925	129.3948	0.8238	117.4976	120.3	0.9976	118.3549	121.1573
Mar.	ISL	0.9825	126.2221	129.0245	0.6111	130.9688	133.7712	0.9645	127.0377	129.8401
	LHR	0.9744	126.7342	129.5366	0.5648	135.8198	138.6222	0.9995	129.6625	132.4649
	KHI	0.6920	114.3985	117.2009	0.9308	107.7029	110.5053	0.9915	108.6688	111.4715
	MZD	0.4751	142.2073	145.0097	0.5819	147.0653	149.8677	0.9027	142.6986	145.501
Apr.	ISL	0.7906	133.1036	135.9060	0.8830	128.8277	131.6301	0.9382	131.0727	133.8751
	LHR	0.8828	137.3336	140.1360	0.9200	136.1707	138.9731	0.9951	136.4825	139.2849
	KHI	0.0489	129.1361	131.9385	0.9722	98.0341	100.8365	1.066e-08	231.2897	234.0921
	MZD	0.8846	136.7037	139.5061	0.6353	139.3967	142.1991	0.9924	136.6372	139.4396
May	ISL	0.8772	135.7677	138.5701	0.6670	140.3102	143.1125	0.9638	137.1640	139.9664
	LHR	0.1789	124.9286	127.7310	0.8311	111.4238	114.2262	0.6015	117.0823	119.8847
	KHI	0.8513	81.2041	84.0065	0.5950	86.0260	88.8284	0.8744	78.9630	81.7654
	MZD	0.7587	144.5574	147.3598	0.8220	149.4594	152.2618	0.7782	147.589	150.3914
Jun.	ISL	0.6399	99.7788	102.5812	0.5655	99.8298	102.6323	0.7009	102.0973	104.8997
	LHR	0.6920	114.2663	117.0687	0.6929	116.2240	119.0264	0.6653	117.0877	119.8900
	KHI	0.5924	71.4398	74.2422	0.2763	83.4160	86.2183	0.9154	74.0379	76.8403
	MZD	0.4504	133.0403	135.8427	0.9842	120.5011	123.3035	0.8580	125.4588	128.2612
Jul.	ISL	0.0082	136.0536	138.8560	0.2611	106.7451	109.5475	0.7506	105.527	108.3303
	LHR	0.7476	92.0705	94.8729	0.3816	102.0629	104.8653	0.9807	94.8952	97.6976
	KHI	0.2034	90.1093	92.9117	0.4012	91.6978	94.5002	0.8847	81.5305	84.3329
	MZD	0.7862	100.4474	103.2498	0.9959	99.2514	102.0539	0.9976	99.25554	102.0579
Aug.	ISL	0.5102	55.3908	58.1932	0.3496	59.1877	61.9901	0.9624	53.4036	56.2060
	LHR	0.7796	100.3835	103.1859	0.5042	102.3760	105.1784	0.9187	97.1736	99.9760
	KHI	0.2548	88.5012	91.3036	0.6199	86.5200	89.3224	0.8738	82.3195	85.1219
	MZD	0.1466	82.08916	84.89155	0.1944	87.57326	90.37565	0.1161	82.91949	85.72188
Sep.	ISL	0.6112	91.2111	94.0135	0.9824	87.1217	89.9241	0.9981	87.3441	90.1465
	LHR	0.4197	86.8910	89.6934	0.7645	89.1149	91.9173	0.9169	85.1200	87.9224
	KHI	0.3338	101.3435	104.1459	0.3618	98.0675	100.8699	0.7921	97.7046	100.5071
	MZD	0.6409	100.0961	102.8985	0.8950	97.74614	100.5485	0.9393	98.73039	101.5328
Oct.	ISL	0.4545	110.6575	113.4599	0.7579	104.6601	107.4625	0.8398	108.765	111.5677
	LHR	0.0851	119.8980	122.7004	0.9198	94.7885	97.5909	0.8588	101.2316	104.0340
	KHI	0.6412	112.4427	115.2451	0.0242	145.5605	148.3629	0.9712	114.227	117.0303
	MZD	0.3087	121.8966	124.699	0.9214	114.4404	117.2428	0.8914	119.0552	121.8576
Nov.	ISL	0.9165	85.6969	88.49932	0.6782	85.6216	88.4240	0.9284	84.2017	87.0041
	LHR	0.3226	98.6114	101.4138	0.9953	89.1010	91.9034	0.9037	92.8789	95.6813
	KHI	0.4653	100.6805	103.4829	0.6162	96.4625	99.2649	0.9621	98.8802	101.6827
	MZD	0.4060	126.2197	129.0221	0.9855	112.1786	114.981	0.9019	117.1384	119.9408
Dec.	ISL	0.6316	88.5505	91.3529	0.7213	86.3120	89.1144	0.8734	83.2625	86.0649
	LHR	0.3230	115.3868	118.1892	0.9875	100.7161	103.5185	0.9306	106.9766	109.7790
	KHI	0.8906	91.2830	94.0854	0.2996	105.4134	108.2158	0.9930	94.8608	97.6632
	MZD	0.8319	108.7118	111.5142	0.7805	109.4219	112.2243	0.9431	108.6957	111.4981

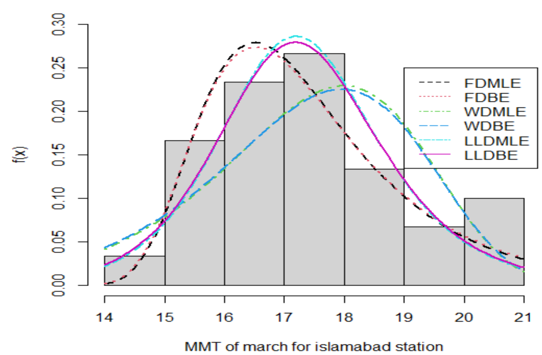
The results are listed in Tables 5-6. Conclusions regarding the selection of appropriate probability distribution(s) for MMT series of four selected sites are summarized as follows:

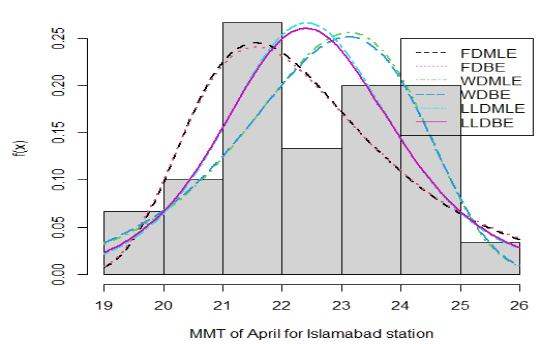
- Based on the p-values of KS test it is concluded that both on ML and Bayesian estimates of LLD is appropriated for MMT of Islamabad site however FD is not suitable for MMT for the month of July (with red color) and similarly WD is also inappropriate for the month of February at 5% level of significance. The values of AIC and BIC discover that WD and LLD are finest fixed distributions as it has minimum AIC and BIC values with slight variation. Thus it is concluded that WD and LLD are more precise for modeling the MMT for Islamabad site based on ML and Bayesian estimation method. The values of AIC and BIC indicated that LLD is the best fitted distribution because it has lesser AIC and BIC values with slight variations. It is also concluded FD also suitable for modeling the MMT for Islamabad station based on ML and Bayesian estimation method. Moreover, PDF graphs are also constructed to compare estimation methods and are presented in Figure 2. Graphs also indicate that LLD and FD are more close to the observed data series based on ML and Bayesian estimation method. So, we recommend that LLD and FD can be used to model the MMT of Islamabad site.

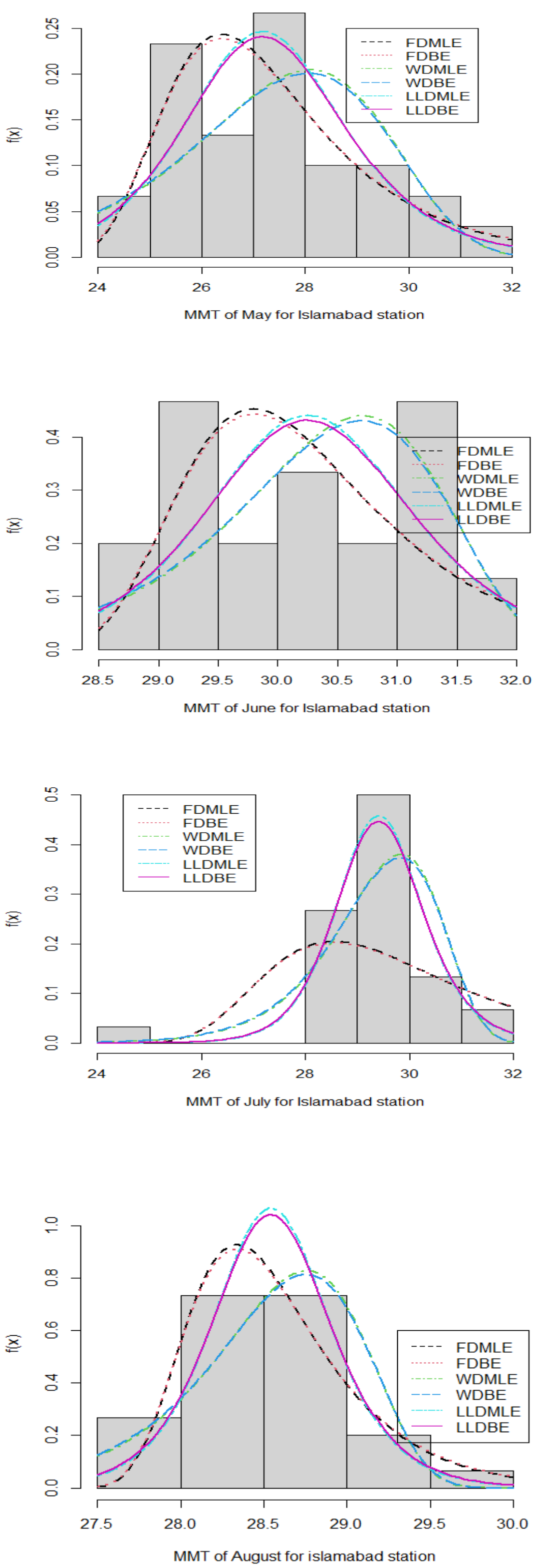
- It is observed that the p-values for every month of KS test for FD, WD and LLD are greater than 5% of level of significance. Hence it is concluded that FD,WD and LLD is the most appropriate for modeling the MMT for Lahore site based on ML and Bayesian estimation method. AIC and BIC values identified that WD and LLD are most appropriate fitted distributions as it has lesser AIC and BIC values. Consequently it is established that WD and LLD are more suitable for modeling the MMT for Lahore site based on ML and Bayesian estimation method. In addition, PDF graphs evaluate the estimation methods and are presented in Figure 3 and graphs also depict that WD and LLD are most suitable and these distributions are more close to the observed data series based on ML and Bayesian estimation method.
- To find out overall p-values for each and every month of KS test for FD, WD and LLD are more than 5% level of significance. Hence it is concluded that FD,WD and LLD is most accurate for modeling the MMT for Muzaffarabad site based on ML and Bayesian estimation method. Graphs also indicate that WD and LLD and are more close to the observed data series based on ML and Bayesian estimation method. So it is said that WD and LLD can be used to modeling the MMT of Muzaffarabad site.
- The resulted p-values for all the month of KS test for WD, FD and LLD are greater than pre-selected 5% level of significance except April. Hence it is accomplished that LLD is best fitted distribution on the basis of ML and Bayesian estimation method for modeling the MMT of Karachi station except the month of April. WD and LLD is best fitted distribution on the basis of AIC and BIC as it shows lesser AIC and BIC values with slender variations. For Karachi station, it is concluded that WD and LLD are more suitable for modeling the MMT which is based on ML and Bayesian estimation method. Additionally, month wise PDF graphs are also constructed to justify estimation methods and are shown in Figure 5 respectively. Graphs also indicate that WD and LLD are more close to the under estimated data series based on ML and Bayesian estimation Method. For Karachi station, we recommend that LLD and FD can be used to modeling the MMT.











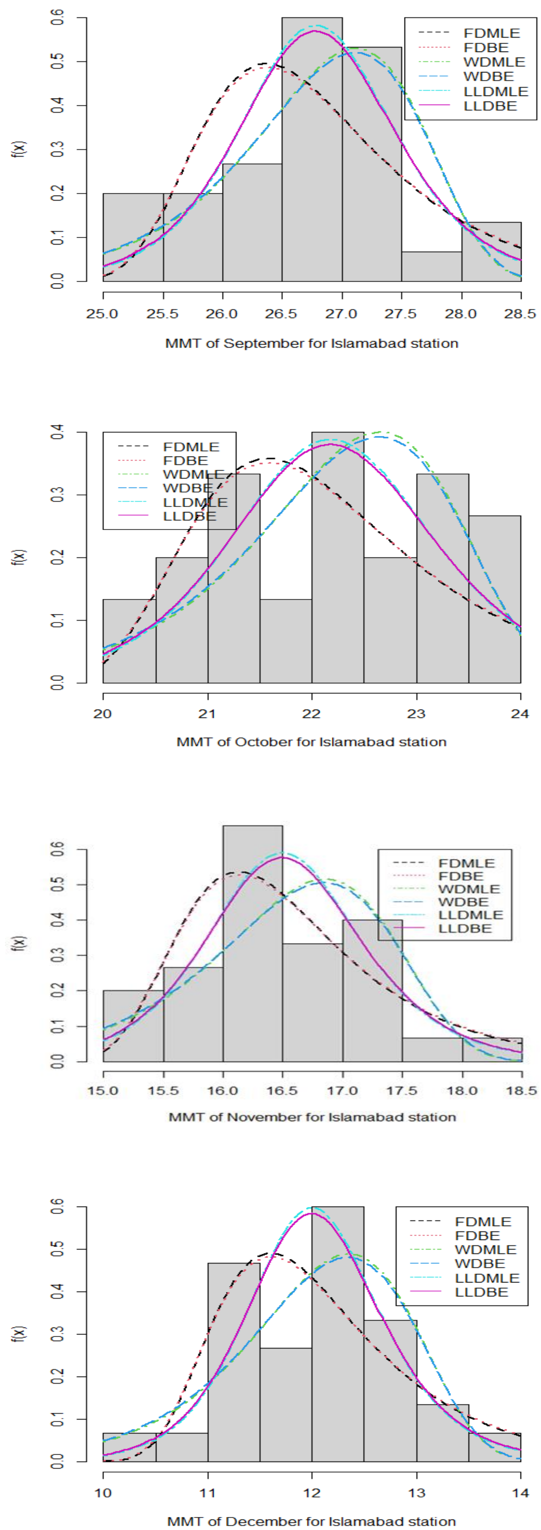
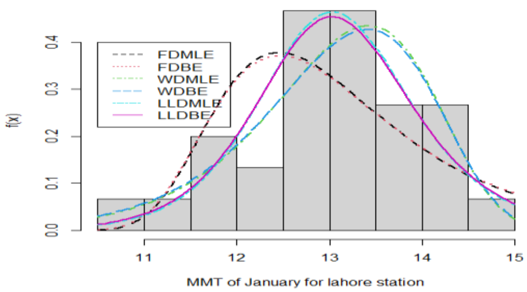
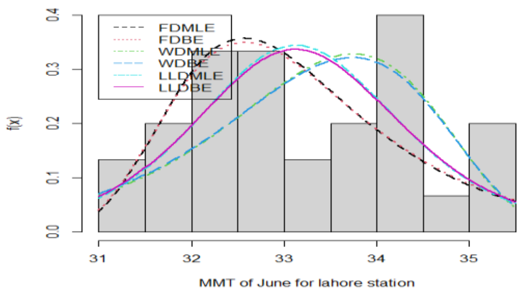
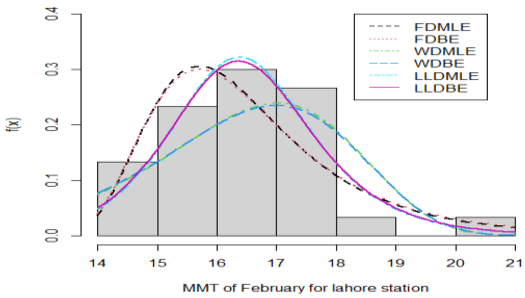
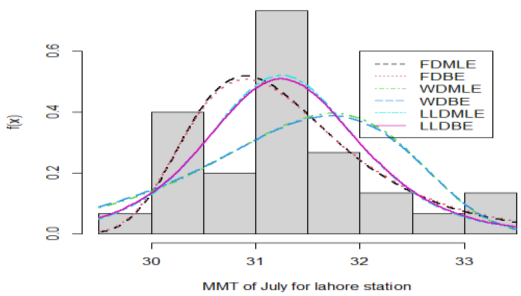


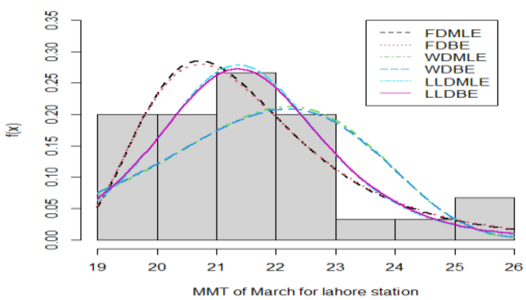
Figure 2. Month wise PDF plots of FD, WD and LLD for Islamabad site.

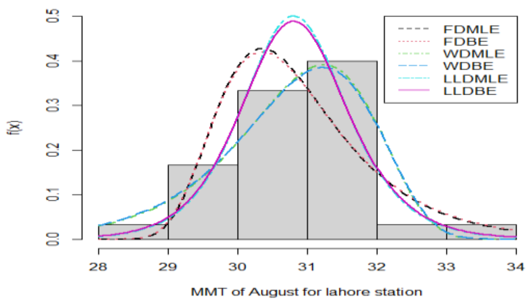

FDMLE
FDBE
WDMLE
WDBE
LLDMLE
LLDBE
f(x)
MMT of January for lahore station

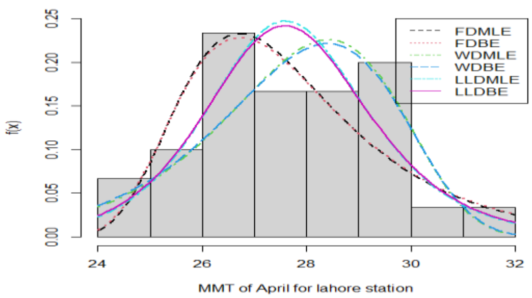

FDMLE
FDBE
WDMLE
WDBE
LLDMLE
LLDBE
f(x)
MMT of June for lahore station

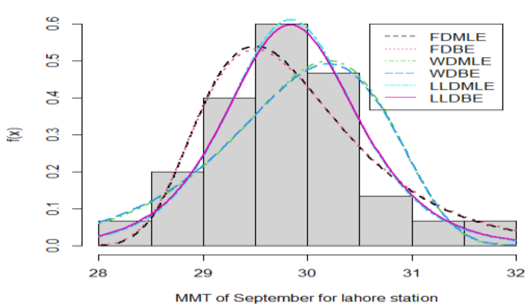

FDMLE
FDBE
WDMLE
WDBE
LLDMLE
LLDBE
f(x)
MMT of February for lahore station

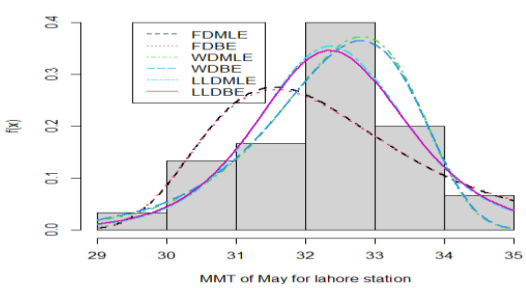

FDMLE
FDBE
WDMLE
WDBE
LLDMLE
LLDBE
f(x)
MMT of July for lahore station

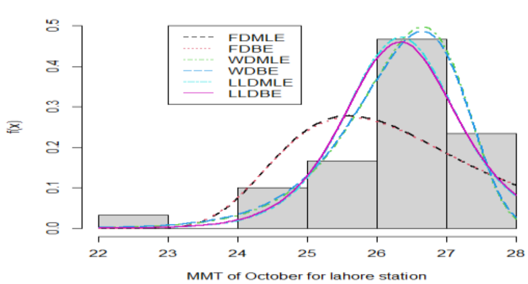

FDMLE
FDBE
WDMLE
WDBE
LLDMLE
LLDBE
f(x)
MMT of March for lahore station

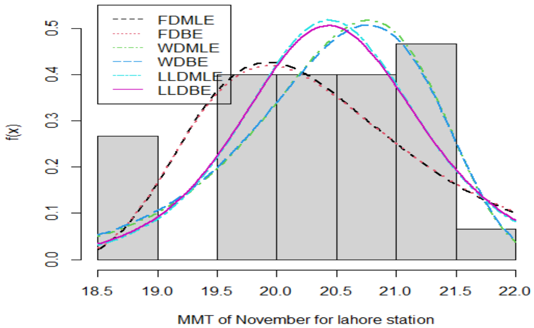

FDMLE
FDBE
WDMLE
WDBE
LLDMLE
LLDBE
f(x)
MMT of August for lahore station


FDMLE
FDBE
WDMLE
WDBE
LLDMLE
LLDBE
f(x)
MMT of April for lahore station


FDMLE
FDBE
WDMLE
WDBE
LLDMLE
LLDBE
f(x)
MMT of September for lahore station


FDMLE
FDBE
WDMLE
WDBE
LLDMLE
LLDBE
f(x)
MMT of May for lahore station


FDMLE
FDBE
WDMLE
WDBE
LLDMLE
LLDBE
f(x)
MMT of October for lahore station



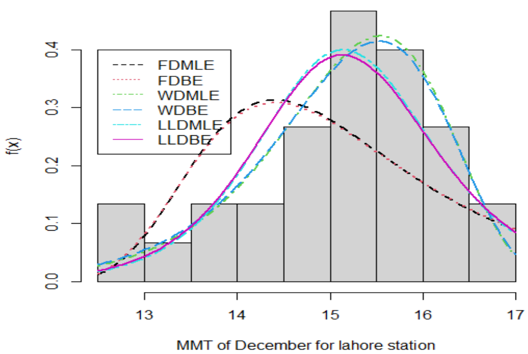
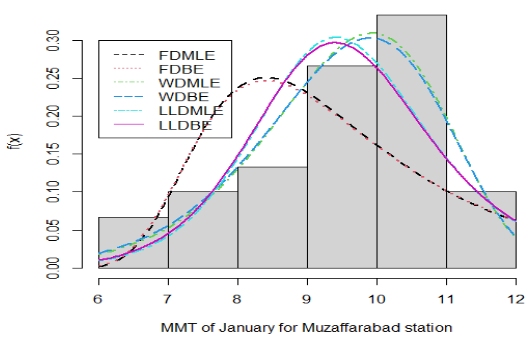
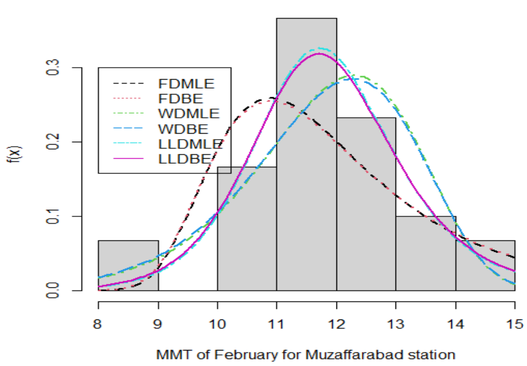
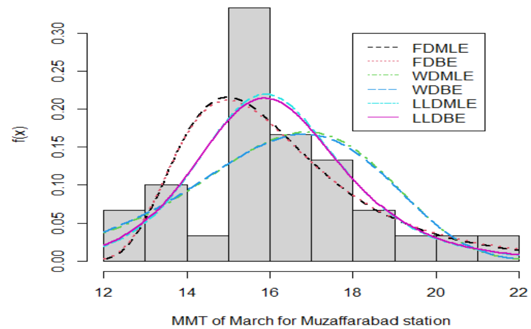
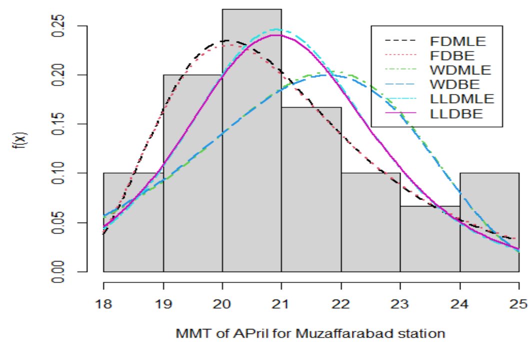


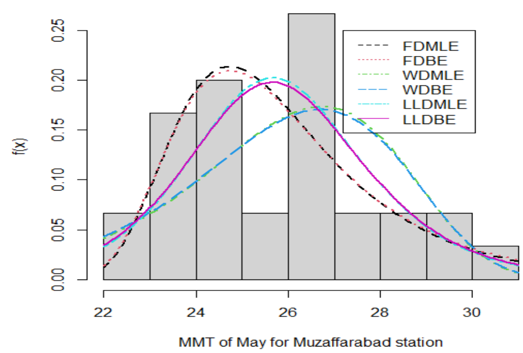
Figure 3. Month wise PDF plots of FD, WD and LLD for Lahore site.

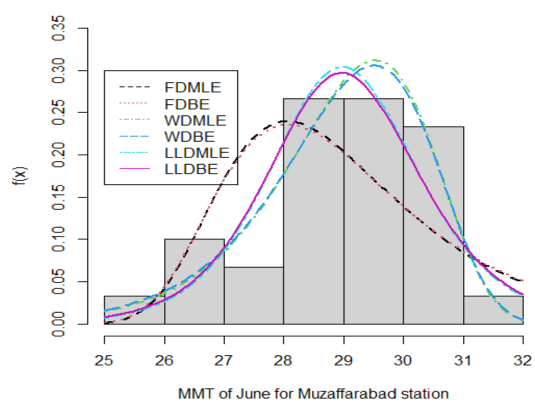


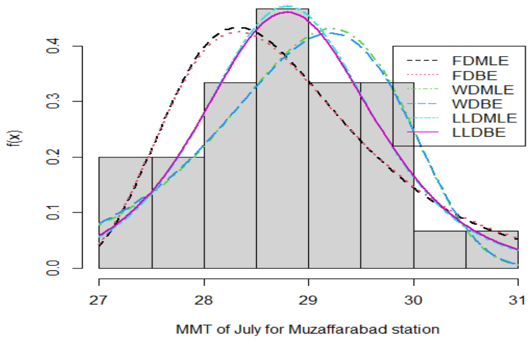


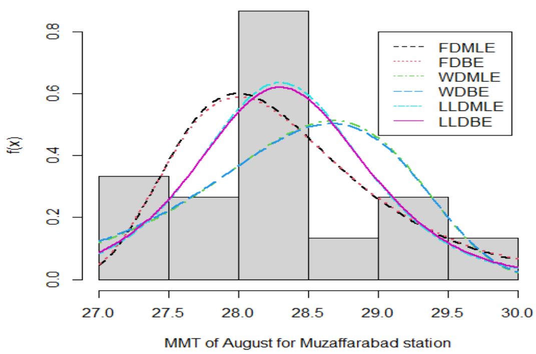


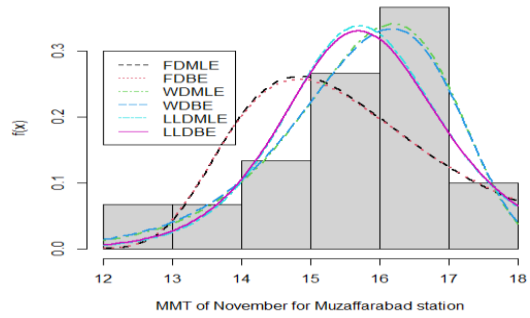


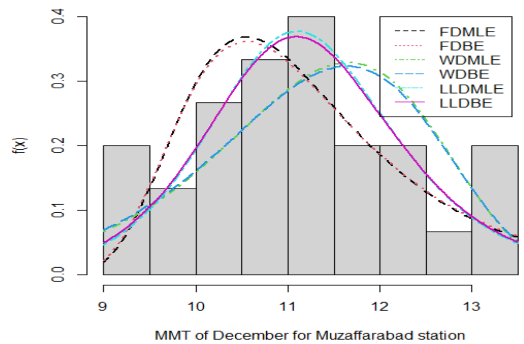


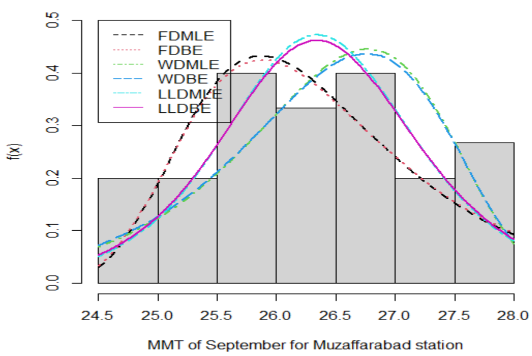












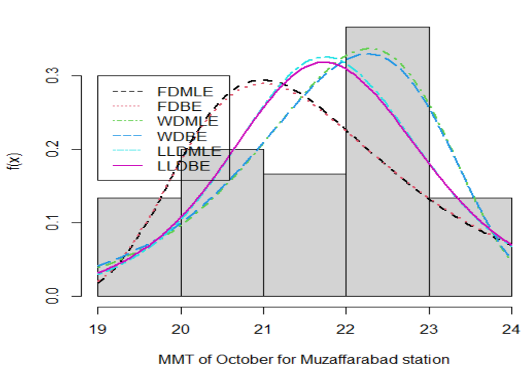
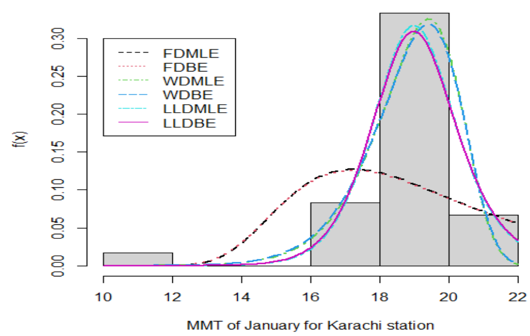
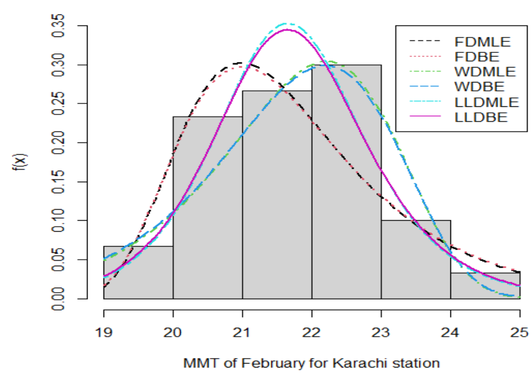
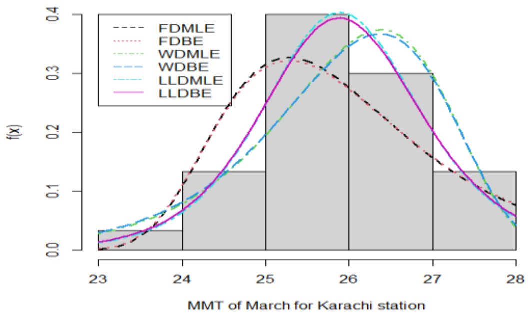
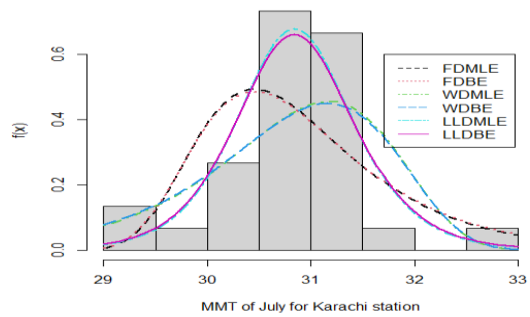


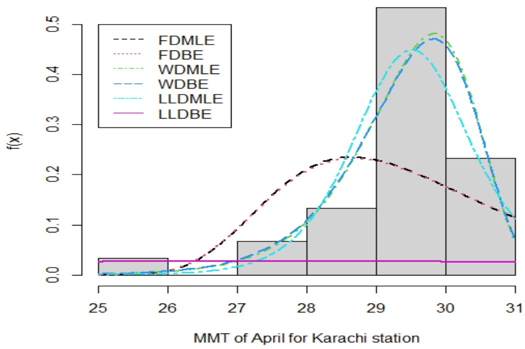
Figure 4. Month wise PDF plots of FD, WD and LLD for Muzaffarabad site.

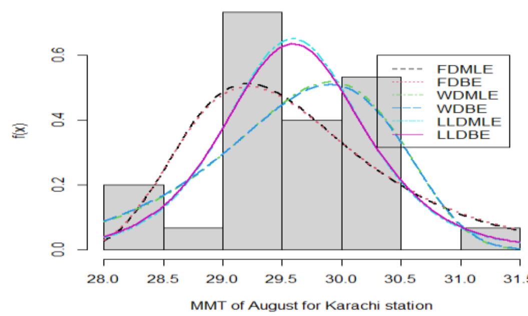


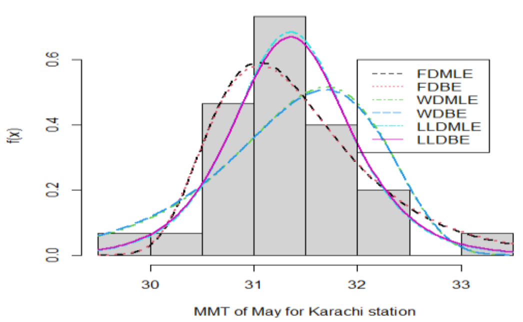


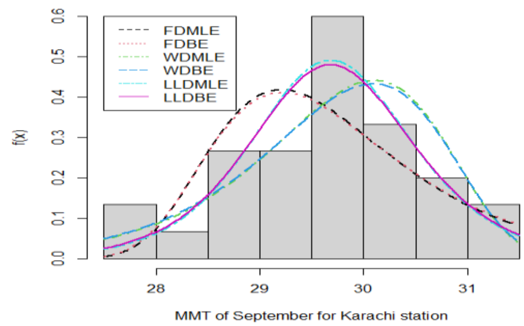

FDMLE
FDBE
WDMLE
WDBE
LLDMLE
LLDBE
f(x)
MMT of March for Karachi station

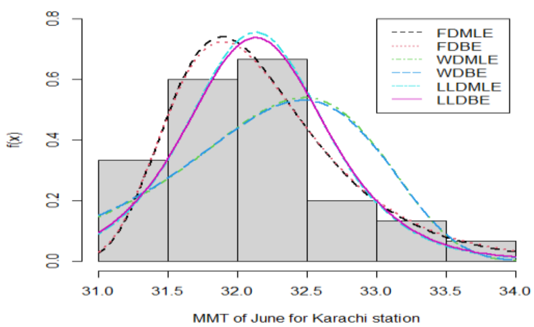

FDMLE
FDBE
WDMLE
WDBE
LLDMLE
LLDBE
f(x)
MMT of July for Karachi station

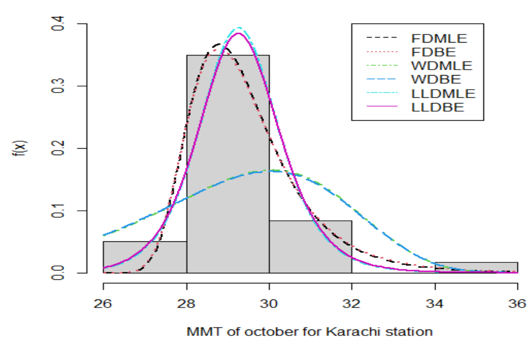

FDMLE
FDBE
WDMLE
WDBE
LLDMLE
LLDBE
f(x)
MMT of April for Karachi station


FDMLE
FDBE
WDMLE
WDBE
LLDMLE
LLDBE
f(x)
MMT of August for Karachi station


FDMLE
FDBE
WDMLE
WDBE
LLDMLE
LLDBE
f(x)
MMT of May for Karachi station


FDMLE
FDBE
WDMLE
WDBE
LLDMLE
LLDBE
f(x)
MMT of September for Karachi station


FDMLE
FDBE
WDMLE
WDBE
LLDMLE
LLDBE
f(x)
MMT of June for Karachi station


FDMLE
FDBE
WDMLE
WDBE
LLDMLE
LLDBE
f(x)
MMT of october for Karachi station

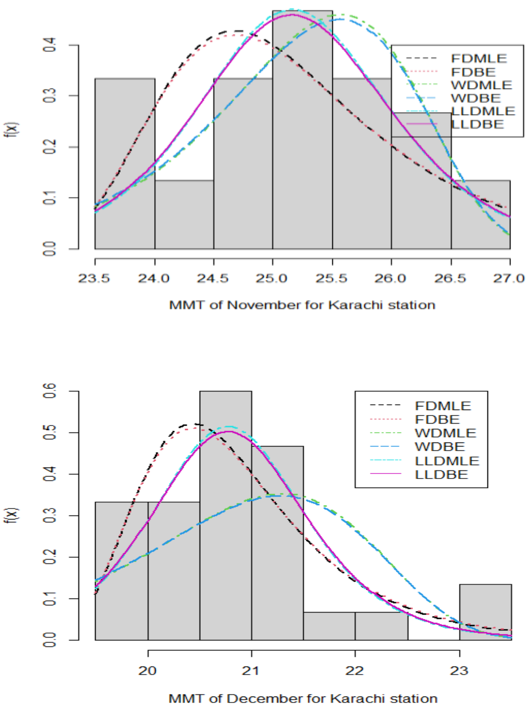


Figure 5. Month wise PDF plots of FD, WD and LLD for Karachi site.

Parameters Estimates and Goodness of Fit Tests for LND and GEVD

In this study LND and GEVD are also considered for the analysis of MMT series for four sites and their parameters are estimated by only ML method. The estimates of parameters and goodness of fit tests are provided in Tables 7-8 respectively.

Table 7: ML estimates for LND and GEVD.

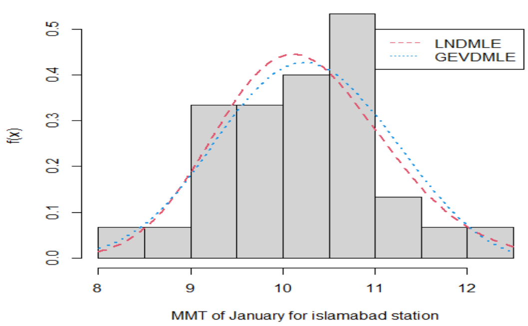
Month	Stations	LND		GEVD		
		$\hat{\mu}$	$\hat{\sigma}$	$\hat{\eta}$	$\hat{\alpha}$	$\hat{\theta}$
Jan.	ISL	2.3211	0.0882	-0.3039	0.9072	9.9211
	LHR	2.5664	0.0726	-0.3677	0.9755	12.7614
	KHI	2.9328	0.0997	-9.6645	3.3714	17.9886
	MZD	2.2393	0.1555	-0.5365	1.5159	9.1846
Feb.	ISL	2.5773	0.1417	0.1191	1.3002	12.3783
	LHR	2.7996	0.0831	-0.1225	1.2517	15.9151
	KHI	3.0756	0.0569	-0.2665	1.2326	21.2558
	MZD	2.4647	0.1171	-0.3650	1.4024	11.4184
Mar.	ISL	2.8527	0.0879	-0.1325	1.3783	16.7606
	LHR	3.0690	0.0734	-0.0450	1.3340	20.8585
	KHI	3.2537	0.0420	-0.4321	1.1478	25.6088
	MZD	2.7795	0.1251	-0.1033	1.7996	15.3600
Apr.	ISL	3.1108	0.0704	-0.4085	1.6418	22.0382
	LHR	3.3197	0.0620	-0.3078	1.7105	27.1222
	KHI	3.3801	0.0384	338.1959	27.2623	25.4806
	MZD	3.0472	0.0841	-0.1852	1.6571	20.4271
May	ISL	3.3070	0.0640	-0.1573	1.6013	26.6415
	LHR	3.4739	0.0384	-0.5810	1.3431	32.0391
	KHI	3.4461	0.0207	-0.1851	0.6225	31.1297
	MZD	3.2516	0.0802	-0.1213	1.8505	25.0256
Jun.	ISL	3.4101	0.0304	-0.4719	0.9842	30.0440
	LHR	3.5019	0.0359	-0.3117	1.1745	32.7953
	KHI	3.4711	0.0186	-0.0666	0.5085	31.9169
	MZD	3.3636	0.0500	-0.5021	1.5359	28.5823
Jul.	ISL	3.3792	0.0410	337.7830	27.2465	24.7806
	LHR	3.4440	0.0271	-0.0785	0.7286	30.9553
	KHI	3.4284	0.0231	367.7242	28.2922	29.1769
	MZD	3.3602	0.0313	-0.3205	0.9103	28.5078
Aug.	ISL	3.3516	0.0147	-0.2107	0.4065	28.3914
	LHR	3.4278	0.0289	-0.2164	0.8725	30.4821
	KHI	3.3871	0.0236	341.9650	27.3992	28.0801
	MZD	3.3440	0.0244	-0.1735	0.6395	28.0662
Sep.	ISL	3.2871	0.0279	-0.3497	0.7704	26.5372
	LHR	3.3965	0.0240	-0.2277	0.6965	29.5991
	KHI	3.3905	0.0299	-0.3418	0.9100	29.4090
	MZD	3.2713	0.0340	-0.3960	0.9309	26.0969
Oct.	ISL	3.0986	0.0482	-0.6371	1.2121	22.0076
	LHR	3.2669	0.0388	276.253	24.9239	22.9902
	KHI	3.3804	0.0476	0.0422	1.0001	28.8121
	MZD	3.0777	0.0591	-0.5082	1.3759	21.4401
Nov.	ISL	2.8044	0.0440	-0.2597	0.7143	16.2689
	LHR	3.0153	0.0410	173.6090	20.2583	18.8166
	KHI	3.2250	0.0358	-0.4658	0.9592	24.9341
	MZD	2.7499	0.0843	-0.5052	1.3877	15.3866
Dec.	ISL	2.4868	0.0622	-0.2583	0.7446	11.7753
	LHR	2.7131	0.0730	-0.6471	1.1958	14.9363
	KHI	3.0378	0.0426	-0.0033	0.7143	20.4684
	MZD	2.4149	0.1012	-0.2330	1.0850	10.8159

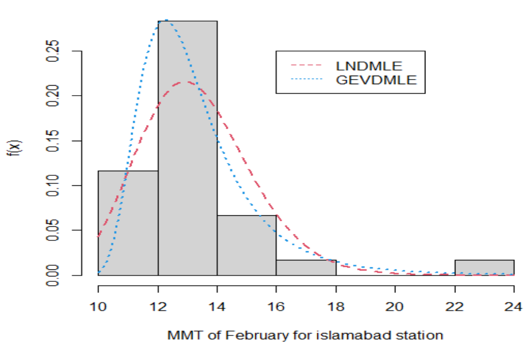
Table 8: Goodness of fit tests for LND and GEVD.

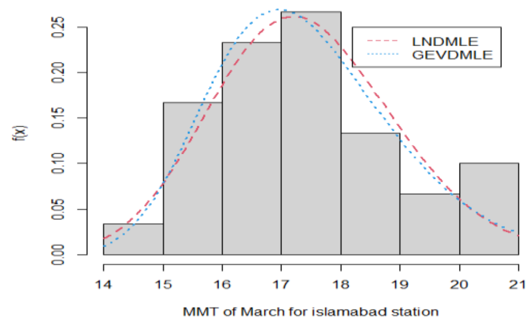
Month	Stations	LND			GEVD		
		KS	AIC	BIC	KS	AIC	BIC
Jan.	ISL	0.8540	82.777	85.5800	0.9654	84.1534	88.3570
	LHR	0.8190	85.8133	88.6157	0.9859	86.2581	90.4616
	KHI	0.1936	126.8032	129.6056	9.992e-16	2e+10	2e+10
	MZD	0.5746	111.8407	114.6431	0.9646	107.4042	111.6078
Feb.	ISL	0.2824	126.5343	129.3367	0.6457	120.0843	124.2879
	LHR	0.9833	107.8673	110.6697	0.8912	109.6215	113.8251
	KHI	0.9290	101.6922	104.4945	0.9759	103.6758	107.8794
	MZD	0.9656	108.3746	111.1770	0.9257	108.5881	112.7917
Mar.	ISL	0.9359	114.4107	117.2131	0.9884	115.7058	119.9094
	LHR	0.9896	116.5947	119.3971	0.9966	116.7103	120.9139
	KHI	0.9913	94.20634	97.0087	0.9749	94.5569	98.76055
	MZD	0.9136	131.1953	133.9977	0.8199	132.6262	136.8297
Apr.	ISL	0.9023	116.6413	119.4437	0.8632	116.9662	121.1697
	LHR	0.9947	121.5122	124.3146	0.9981	122.7938	126.9974
	KHI	0.3529	96.37635	99.1787	1.094e-10	433.5711	437.7747
	MZD	0.9662	123.476	126.2783	0.9894	125.0180	129.2216
May	ISL	0.9493	122.6735	125.4759	0.9663	123.9451	128.1487
	LHR	0.3029	102.0491	104.8514	0.8803	98.4764	102.6800
	KHI	0.8185	63.4675	66.2699	0.9414	65.3806	69.58425
	MZD	0.7083	132.8856	135.6880	0.7953	134.0287	138.2323
Jun.	ISL	0.6522	84.2878	87.0902	0.5188	84.5751	88.7787
	LHR	0.6537	99.6587	102.4612	0.6661	100.6286	104.8322
	KHI	0.7754	58.4969	61.2993	0,7092	57.8794	62.0829
	MZD	0.6388	111.2316	114.0340	0.9854	108.9701	113.1737
Jul.	ISL	0.1645	100.3498	103.1522	1.117e-10	437.7012	441.9048
	LHR	0.8293	79.3915	82.1939	0.8773	79.2083	83.4119
	KHI	0.5941	68.8274	71.6298	1.042e-10	383.8993	388.1029
	MZD	0.9918	82.9940	85.7964	0.9994	84.3531	88.5566
Aug.	ISL	0.8533	37.3382	40.1406	0.0100	39.2040	43.4076
	LHR	0.9558	82.2270	85.0294	0.9456	84.5139	88.7175
	KHI	0.6872	67.6397	70.4421	1.051e-1	373.2842	377.4878
	MZD	0.6052	67.0585	69.8609	0.7958	68.17922	72.3828
Sep.	ISL	0.9826	71.6988	74.5012	0.9952	73.0026	77.2062
	LHR	0.8862	69.2894	72.0917	0.8100	71.1043	75.3079
	KHI	0.7975	82.0625	84.8649	0.6617	83.4131	87.6167
	MZD	0.9233	82.6748	85.4772	0.9943	83.45139	87.6549
Oct.	ISL	0.7178	93.2181	96.0205	0.8697	91.0583	95.2619
	LHR	0.3374	90.2477	93.0501	1.155e-10	409.7754	413.9790
	KHI	0.3411	109.3379	112.1403	0.1301	101.0138	105.2174

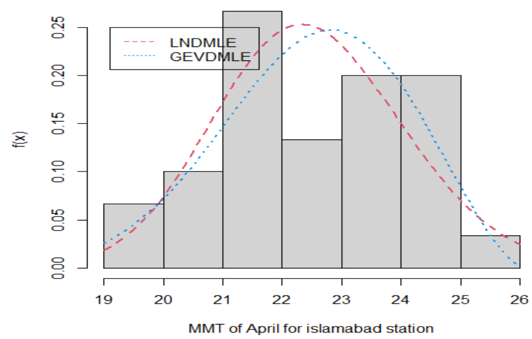
	MZD	0.6860	104.1690	106.9717	0.9257	102.7083	106.9119
Nov.	ISL	0.9371	70.0041	72.8065	0.9419	71.6992	75.9028
	LHR	0.6440	78.4760	81.2784	1.321e-10	329.5273	333.7309
	KHI	0.9324	82.8614	85.6638	0.7399	82.8161	87.0197
	MZD	0.7119	105.7744	108.5768	0.9811	102.8646	107.0682
Dec.	ISL	0.8802	71.7825	74.5849	0.9226	73.8218	78.0254
	LHR	0.5845	94.9218	97.7242	0.9966	89.4148	93.6184
	KHI	0.8737	82.1511	84.9535	0.9141	80.4682	84.6718
	MZD	0.9404	96.6183	99.4207	0.9367	98.14791	102.3515

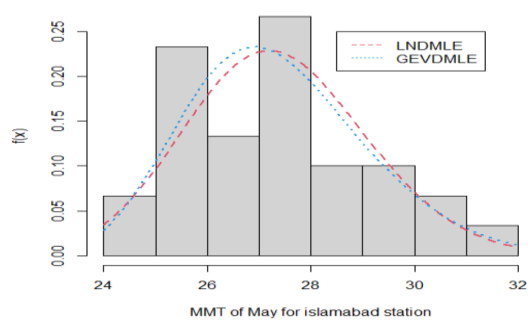
The P-values of KS test for LND and GEVD are listed in Table 8 as a measure of goodness. Since all the P-values of KS test for LND are greater than 5 percent level of significance therefore, it is determined that this distribution is seemed to be good for MMT of all selected sites based on ML estimation method. In addition GEVD also suitable based on the p- values of KS test except for the month of July and August. Moreover, AIC and BIC values are calculated and presented in Table 8 to select the preferable distribution having smallest value of AIC and BIC respectively. Generally, the analysis of AIC and BIC reveals that the LND generally fit best the MMT series as compared to GEVD. Moreover, PDF graphs are also constructed and are presented in Figures 6-9.

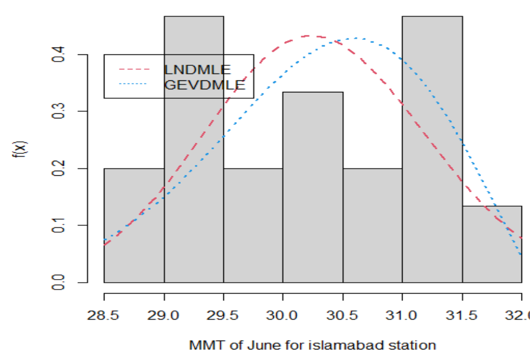


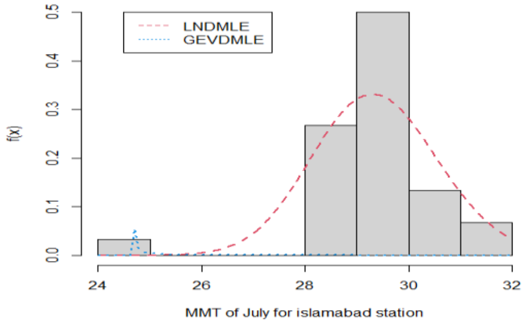


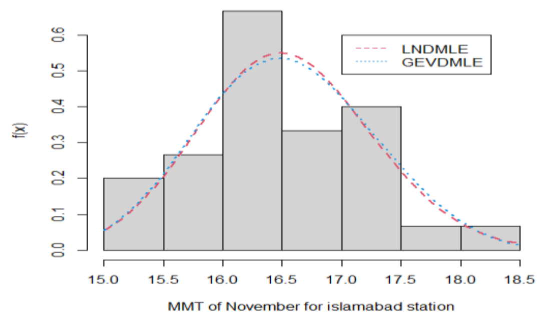


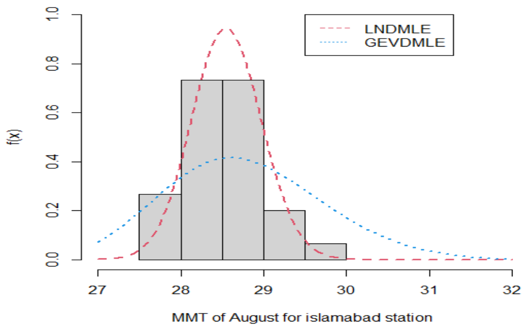


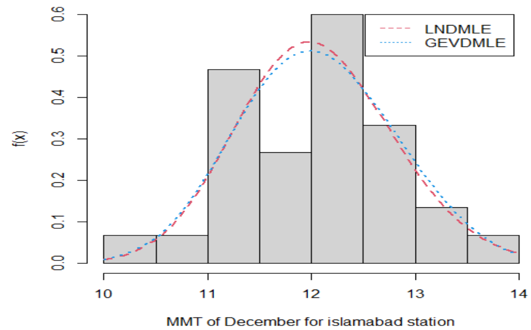


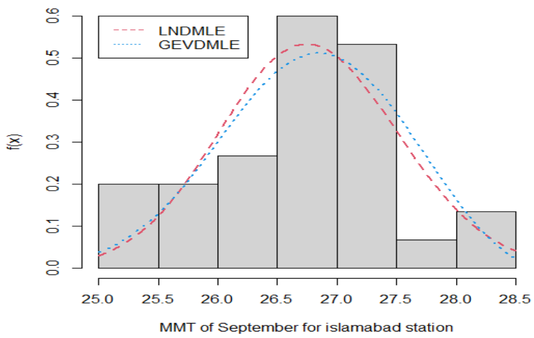












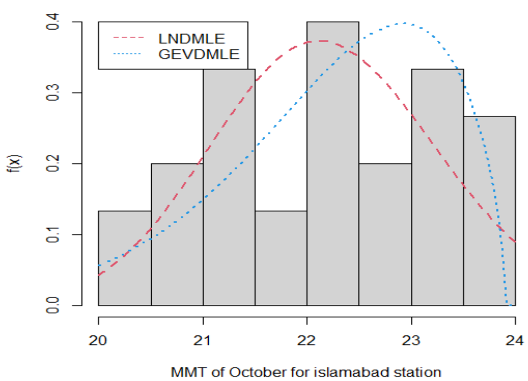
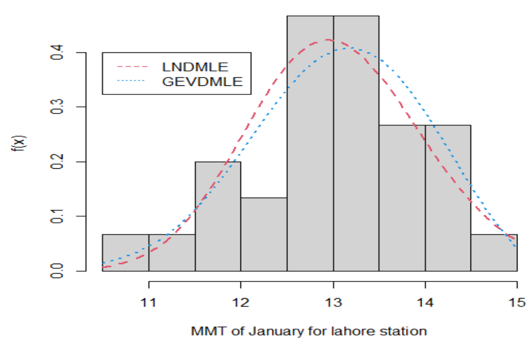
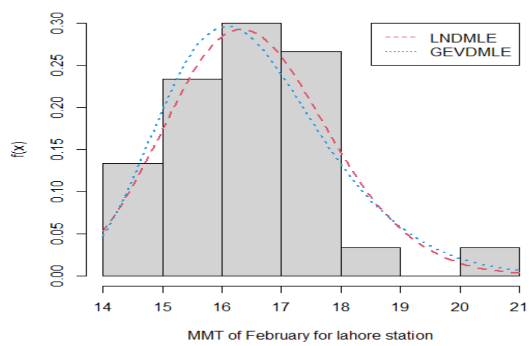
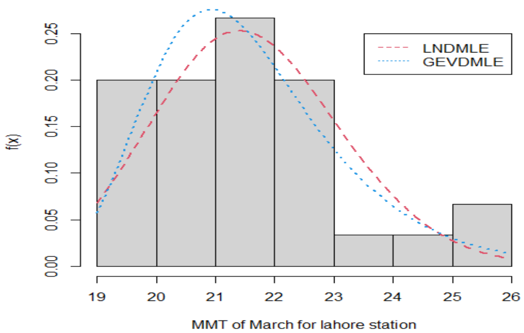
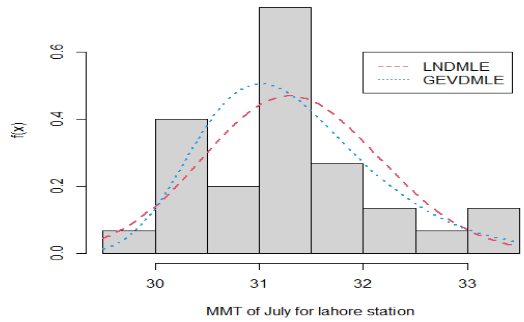


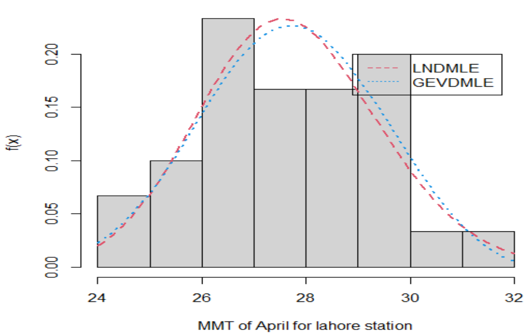
Figure 6. Month wise PDF plots of LND and GEVD for Islamabad site.

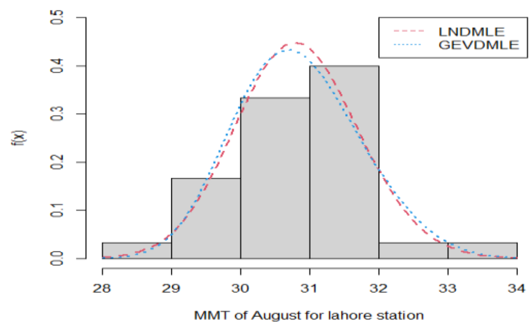


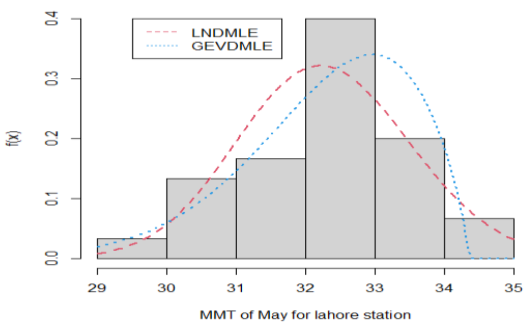


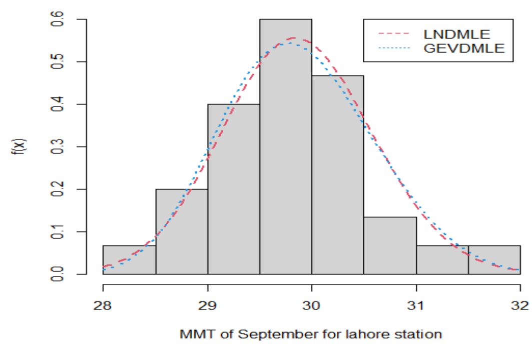

LNDMLE
GEVDMLE
f(x)
MMT of March for lahore station

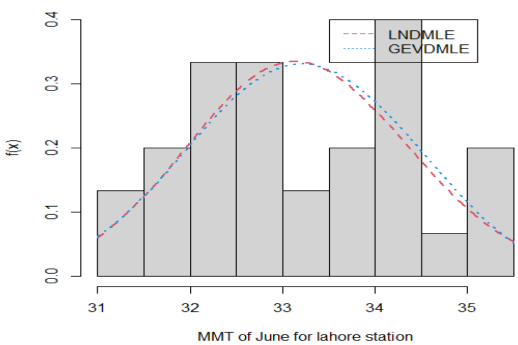

LNDMLE
GEVDMLE
f(x)
MMT of July for lahore station

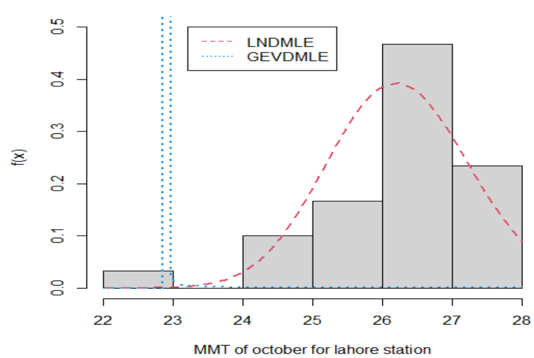

LNDMLE
GEVDMLE
f(x)
MMT of April for lahore station


LNDMLE
GEVDMLE
f(x)
MMT of August for lahore station


LNDMLE
GEVDMLE
f(x)
MMT of May for lahore station


LNDMLE
GEVDMLE
f(x)
MMT of September for lahore station


LNDMLE
GEVDMLE
f(x)
MMT of June for lahore station


LNDMLE
GEVDMLE
f(x)
MMT of october for lahore station

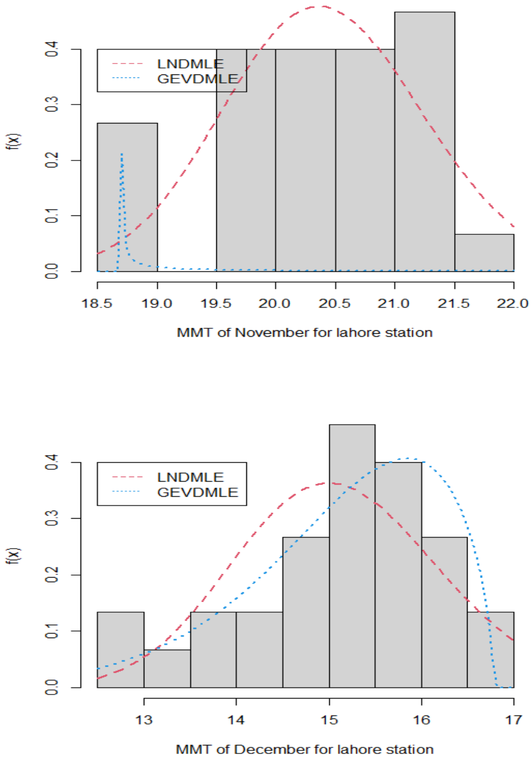
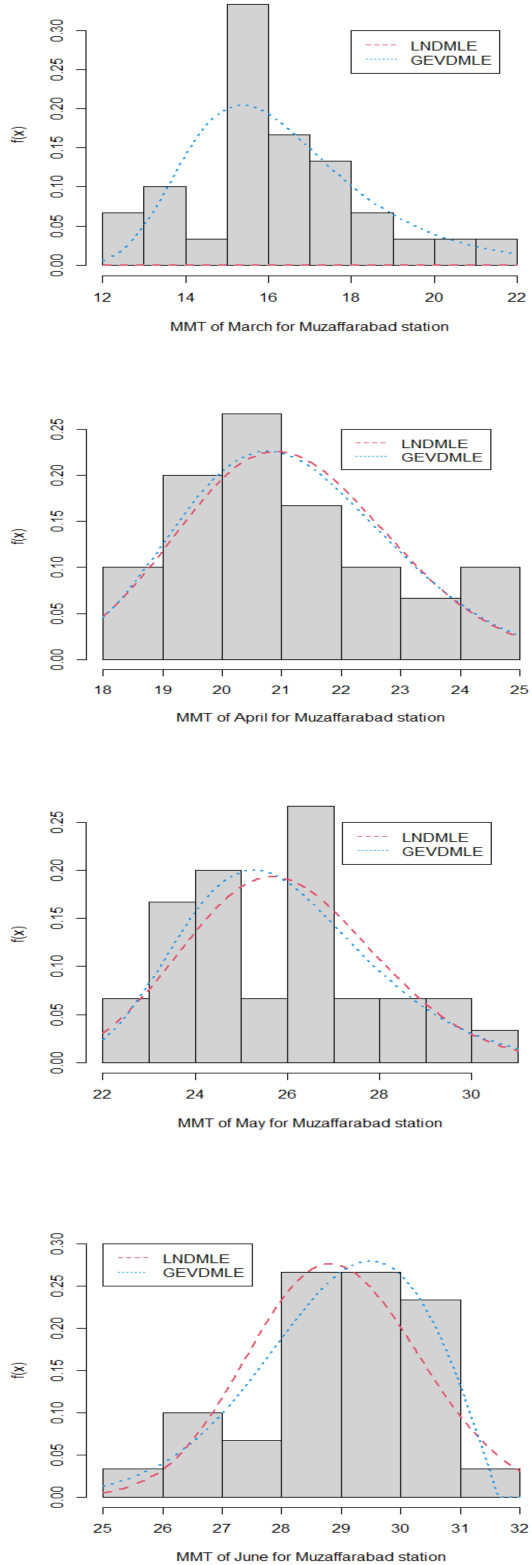
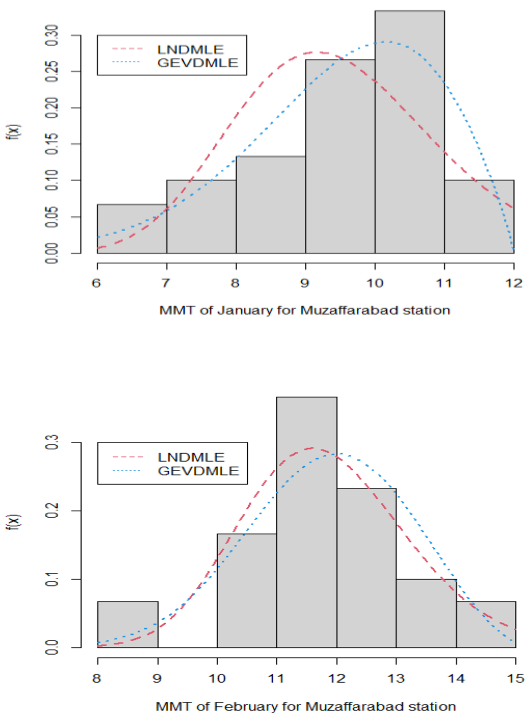


Figure 7. Month wise PDF plots of LND and GEVD for Lahore site.





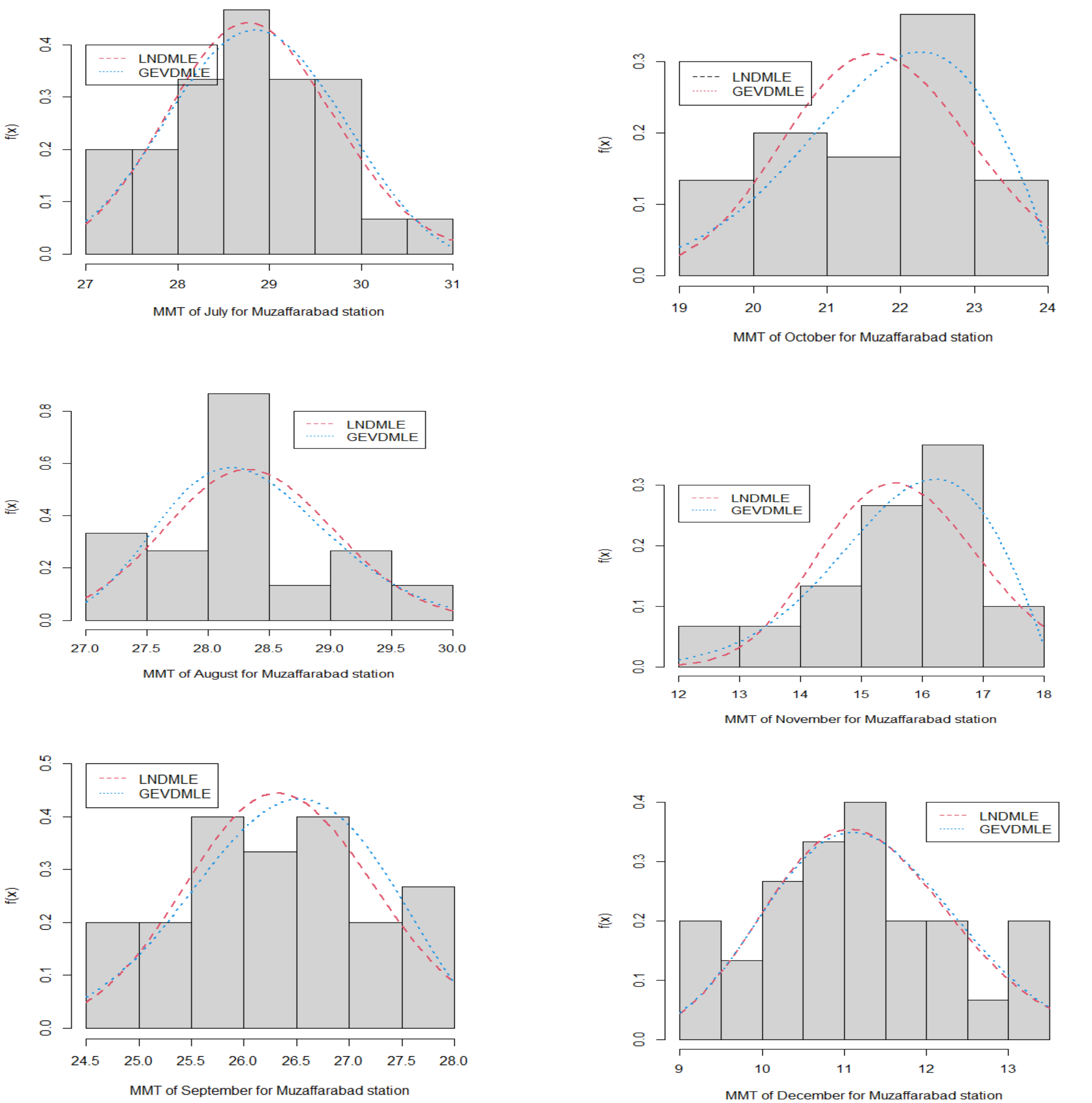
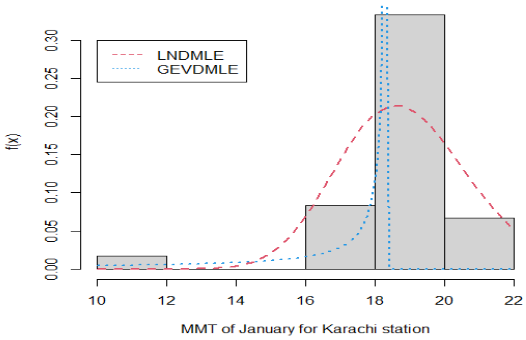
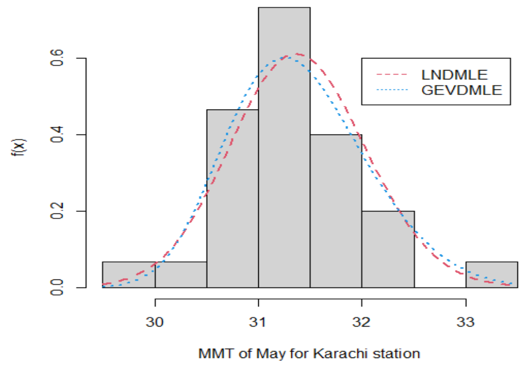
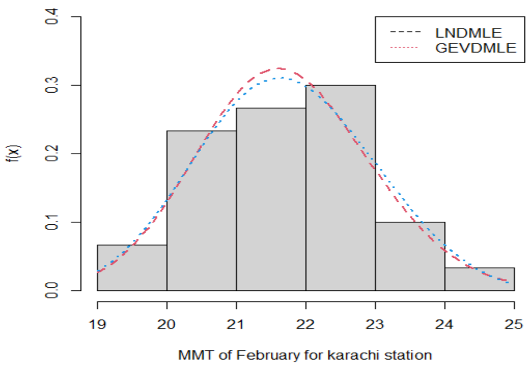
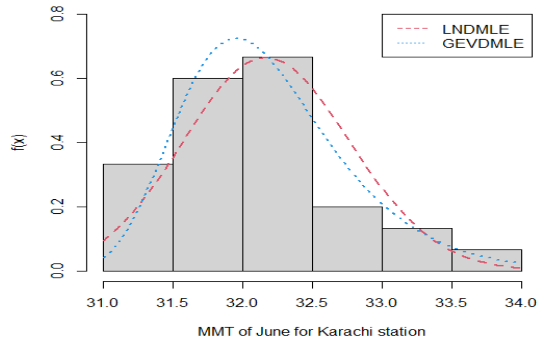


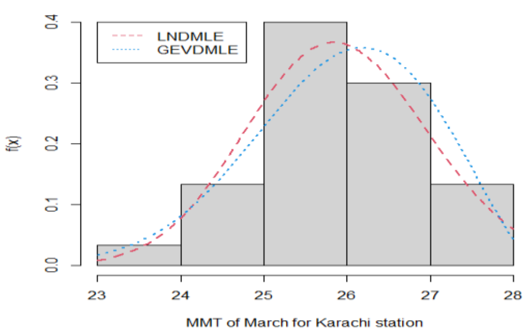
Figure 8. Month wise PDF plots of LND and GEVD for Muzaffarabad site.

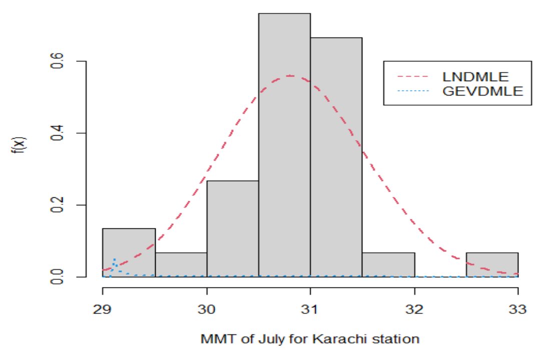

LNDMLE
GEVDMLE
f(x)
MMT of January for Karachi station

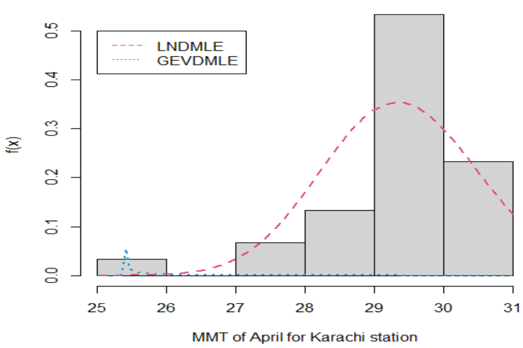

LNDMLE
GEVDMLE
f(x)
MMT of May for Karachi station

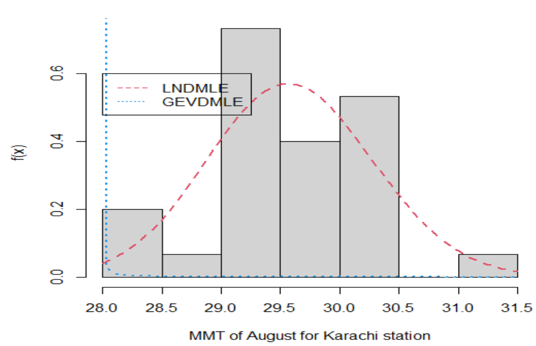

LNDMLE
GEVDMLE
f(x)
MMT of February for karachi station


LNDMLE
GEVDMLE
f(x)
MMT of June for Karachi station


LNDMLE
GEVDMLE
f(x)
MMT of March for Karachi station


LNDMLE
GEVDMLE
f(x)
MMT of July for Karachi station


LNDMLE
GEVDMLE
f(x)
MMT of April for Karachi station


LNDMLE
GEVDMLE
f(x)
MMT of August for Karachi station

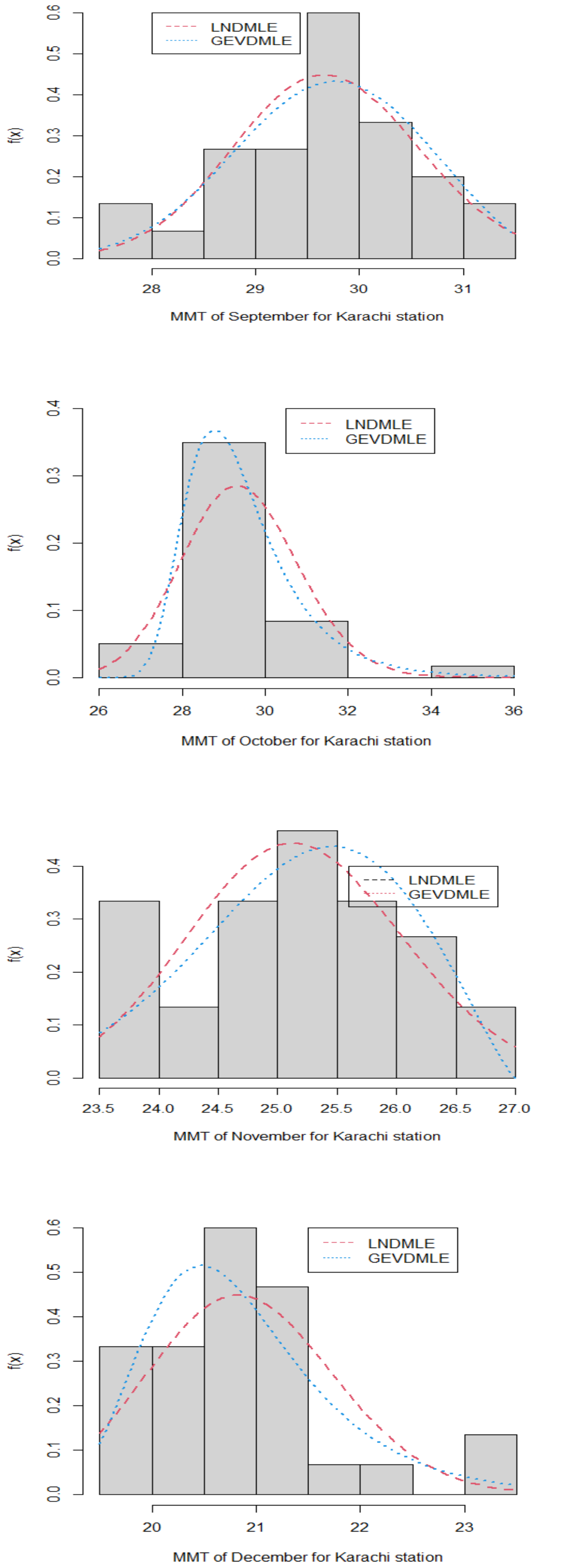


Figure 9. Month wise PDF plots of LND and GEVD for Karachi site.

Quantiles Estimation

The quantile estimates for various for each site can be obtained using the quantile function of the LLD. These quantiles have been calculated using ML and the Bayesian estimation methods. The results are illustrated in Table 9. The results of *Table 9* show that the estimated quantiles for F = 0.99 (100 year return period) are greater than the average values of at given MMT site. Moreover, the magnitude of 100 year return period is greater the maximum value of the observed MMT (see *Table 1*) for the four sites. However, quantile estimates do not exist (represents as steric) for the month of April for Karachi site. These results are useful for the officials and researchers dealing with calamities and cultivation related studies for proper planning and effective management

Table 9. Quantiles estimates (in degree Celsius (°C)) of LLD with non-exceedance probability F/return period (years)

Month	Sites	ML Method				Bayesian Method			
		0.1(1)	0.5(2)	0.9(5)	0.99(100)	0.1(1)	0.5(2)	0.9(5)	0.99(100)
Jan.	ISL	9.1594	10.2278	11.420	12.8823	9.1320	10.2242	11.4470	12.9490
	LHR	11.9463	13.0789	14.3188	15.80656	11.91737	13.0753	14.3457	15.8734
	KHI	17.3778	19.0347	20.8497	23.02847	17.33339	19.0299	20.8925	23.1338
	MZD	7.9006	9.5591	11.5659	14.2396	7.8533	9.54755	11.6072	14.3652
Feb.	ISL	11.1660	13.010	15.1575	17.9077	11.1203	13.004	15.2078	18.0406
	LHR	14.8162	16.4394	18.2403	20.43167	14.77691	16.4348	18.2788	20.5281
	KHI	20.1944	21.6981	23.3137	25.21455	20.15780	21.6950	23.3494	25.29928
	MZD	10.2438	11.8177	13.6334	15.93482	10.20335	11.8119	13.6742	16.0431
Mar.	ISL	15.4578	17.2795	19.3158	21.81295	15.4147	17.2757	19.3615	21.92608
	LHR	19.5507	21.4342	23.4991	25.98025	19.50589	21.4308	23.5458	28.80873
	KHI	24.5933	25.9212	27.3208	28.93459	24.56043	25.9186	27.3520	29.00687
	MZD	13.7211	16.0498	18.7739	22.27697	13.66327	16.0431	18.8375	22.4453
Apr.	ISL	20.5134	22.4858	24.6478	27.24512	20.4660	22.4798	24.6917	27.3547
	LHR	25.5283	27.6655	29.9816	32.73112	25.47800	27.6619	30.0330	32.85331
	KHI	28.3267	29.5240	30.7718	32.19393	****	****	****	****
	MZD	18.8908	21.0154	23.3789	26.26273	18.83920	21.0108	23.4329	26.39585
May	ISL	25.1097	27.2542	29.5818	32.34941	25.05962	27.2516	29.6354	32.47547
	LHR	30.8743	32.3894	33.9790	35.80286	30.83483	32.3847	34.0124	35.8824
	KHI	30.5717	31.3622	32.1731	33.08201	30.55316	31.3619	32.1920	33.1231
	MZD	23.1998	25.7831	28.6539	32.15306	23.13988	25.7778	28.7165	32.3073
Jun.	ISL	29.0478	30.2690	31.5415	32.99135	29.02114	30.2677	31.5678	33.05057
	LHR	31.5997	33.1582	34.7935	36.67035	31.56534	33.1571	34.8292	36.75040
	KHI	31.4155	32.1340	32.8690	33.69035	31.39878	32.1346	32.8877	33.72981
	MZD	27.2738	29.0253	30.8893	33.06050	27.22750	29.0195	30.9295	33.15768
Jul.	ISL	28.2518	29.4285	30.6542	32.05022	28.22202	29.4277	30.6850	32.11846
	LHR	30.2237	31.2599	32.3317	33.54342	30.19996	31.2604	32.3581	33.60009
	KHI	30.0517	30.8511	31.6718	32.59246	30.03035	30.8497	31.6915	32.63641

	MZD	27.6715	28.8130	30.0015	31.35459	27.64435	28.8116	30.0281	31.4145
Aug.	ISL	28.0349	28.5457	28.5456	29.64422	28.02270	28.5456	29.0783	29.6709
	LHR	29.7408	30.8184	31.9349	33.19974	29.71429	30.8168	31.9603	33.2568
	KHI	28.7694	29.6011	30.4568	31.41900	28.74885	29.6004	30.4772	31.4637
	MZD	27.4557	28.3068	29.1843	30.17302	27.43537	28.3068	29.2060	30.2199
Sep.	ISL	25.8676	26.7942	27.7540	28.84072	25.84379	26.7921	27.7753	28.8895
	LHR	28.9649	29.8492	30.7605	31.78690	28.94448	29.8493	30.7824	31.8341
	KHI	28.6042	29.7009	30.8396	32.13220	28.57695	29.6995	30.8661	32.1917
	MZD	25.2255	26.3624	27.5507	28.90868	25.19786	26.3605	27.5769	28.9686
Oct.	ISL	20.8446	22.2153	23.6761	25.38019	20.81229	22.2123	23.7065	25.4521
	LHR	25.2148	26.3534	27.5435	28.90369	25.18343	26.3500	27.5706	28.9674
	KHI	27.9168	29.2800	30.7098	32.35005	27.88223	29.2784	30.7445	32.4285
	MZD	20.1712	21.7944	23.5482	25.62372	20.12942	21.7885	23.5844	25.7136
Nov.	ISL	15.6097	16.5159	17.4749	18.58503	15.58821	16.5149	17.4967	18.63494
	LHR	19.4270	20.4611	21.5503	22.80526	19.40093	20.4585	21.5739	22.86060
	KHI	24.0392	25.1842	26.3836	27.75797	24.01170	25.1821	26.4095	27.81754
	MZD	14.2296	15.7735	17.4850	19.56538	14.18869	15.7674	17.5217	19.65989
Dec.	ISL	11.1443	12.0298	12.9858	14.11600	11.12234	12.0282	13.0080	14.16851
	LHR	13.8790	15.1937	16.6330	18.35980	13.84335	15.1880	16.6632	18.43726
	KHI	19.7522	20.7929	21.8884	23.14997	19.72743	20.7929	21.9159	23.21093
	MZD	9.8109	11.1796	12.7393	16.85446	9.778002	11.1759	12.7737	17.0100

IV. CONCLUSION

The present study illustrates the probability distribution models in MMT analyses that can best describe climatic characteristics of selected sites. Five probability distribution models such as three-parameter distribution (GEV) and four two-parameter distributions (FD, WD, LLD, LND) were assessed using the Bayesian and ML estimation method, goodness of fit tests-based analysis to identify the most suitable distribution model for MMT. Therefore, KS test has been applied as an evaluator to judge the appropriateness of selected distributions. Moreover, AIC and BIC have also been used for preference and endorsement the most appropriate distribution for MMT at selected sites. It has been observed that overall log-logistic and log-normal distributions are the best fitted distributions for all the months for four sites and some of the sites favor the Frechet, Weibull and GEV distribution to describe the MMT.

ACKNOWLEDGEMENT

We would like to thank the Pakistan Meteorological Department Lahore for providing the data.

REFERENCES

[1] Afzaal, M., Haroon, M. A., & Zaman, Q. (2009). Interdecadal oscillations and the warming trend in the area-weighted annual mean temperature of Pakistan. *Pakistan Journal of Meteorology*, *6*(11), 13-19.

[2] Afsar, S., Abbas, N., & Jan, B. (2013). Comparative study of temperature and rainfall fluctuation in Hunza-nagar District. *Journal of Basic & Applied Sciences*, *9*, 151.

[3] Chuanyan, Z., Zhongren, N., & Guodong, C. (2005). Methods for modelling of temporal and spatial distribution of air temperature at landscape scale in the southern Qilian Mountains, China. *Ecological Modelling*, *189*(1-2), 209-220.

[4] Cramer, W., Yohe, G., & Field, C. B. (2014). *Detection and attribution of observed impacts* (pp. 979-1037). Cambridge University Press.

[5] Cao, F., & Gao, T. (2019). Effect of climate change on the centennial drought over China using high-resolution NASA-NEX downscaled climate ensemble data. *Theoretical and Applied Climatology*, *138*(1), 1189-1202.

[6] Chaudhry, Q. Z., & Sheikh, M. M. (2002). Climate change and its impact on the water resources of mountain regions of Pakistan. *Pakistan Journal of Geography*, *11*(172), 45-59.

[7] Chaudhry, Q. Z., & Rasul, G. (2007). Global Warming and Expected snowline shift along Northern Mountains of Pakistan. In *Proceedings of 1st Asiaclic Symposium, Yokohama, Japan*.

[8] de Carvalho Pedrosa, A. M., & de Almeida Gama, S. M. (2004). *Introdução computacional à probabilidade e estatística*.

[9] de Assis, J. P., de Sousa, R. P., Batista, B. D. O., Linhares, P. C. F., Cardoso, E. D. A., Paula, J. D. A., & Neves, A. M. (2018). Probabilistic modeling of monthly temperature historical series in Mossoró, northeastern Brazil. *Journal of Agricultural Science (Toronto)*, *10*(12), 534-542.

[10] Florio, E. N., Lele, S. R., Chi Chang, Y., Sterner, R., & Glass, G. E. (2004). Integrating AVHRR satellite data and NOAA ground observations to predict surface air temperature: a statistical approach. *International Journal of Remote Sensing*, *25*(15), 2979-2994.

[11] Good, P., Giannakopoulos, C., Flocas, H., Tolika, K., Anagnostopoulou, C., & Maheras, P. (2008). Significant changes in the regional climate of the Aegean during 1961–2002. *International Journal of Climatology: A*

Journal of the Royal Meteorological Society, *28*(13), 1735-1749.

[12] Hansen, J., Ruedy, R., Sato, M., & Lo, K. (2010). Global surface temperature change. Rev. Geophys., 48, RG4004.

[13] Hüsler, J., & Li, D. (2007). Statistical analysis of extreme values with applications to insurance, finance, hydrology and other fields.

[14] Hossian, M. M., Abdulla, F., & Rahman, M. H. (2016). Selecting the probability distribution of monthly maximum temperature of Dhaka (capital city) in Bangladesh. *Jahangirnagar University Journal of Statistical Studies*, *33*, 33-45.

[15] Jakob, D., & Walland, D. (2016). Variability and long-term change in Australian temperature and precipitation extremes. *Weather and Climate Extremes*, *14*, 36-55.

[16] Jain, S. K., & Kumar, V. (2012). Trend analysis of rainfall and temperature data for India. *Current Science*, 37-49.

[17] Karmeshu, N. (2012). Trend detection in annual temperature & precipitation using the Mann Kendall test–a case study to assess climate change on select states in the northeastern United States.

[18] Karmakar, S., & Shrestha, M. L. (2000). Recent climatic changes in Bangladesh. SMRC No. 4. *SAARC Meteorological Research Centre, Dhaka*.

[19] Khan, Z. Ahmad (2021, August 20). *Karachi*. *Encyclopedia Britannica*. https://www.britannica.com/place/Karachi.

[20] Khan, A. M., Qureshi, R., Arshad, M., & Mirza, S. N. (2018). Climatic and flowering phenological relationships of western himalayan flora of Muzaffarabad district, Azad Jammu and Kashmir, Pakistan. *Pak. J. Bot*, *50*(3), 1093-1112.

[21] Nisar, M. I., Ansari, N., Khalid, F., Amin, M., Shahbaz, H., Hotwani, A., ... & Jehan, F. (2021). Serial population-based serosurveys for COVID-19 in two neighbourhoods of Karachi, Pakistan. *International Journal of Infectious Diseases*, *106*, 176-182.

[22] Murugappan, A., Subbarayan, S., & Monson, C. C. Global Climate Change and Occurrence of Kidney Stones. *IPHE is preparing a Membership Directory for Members of different centres. Kindly mail to Secretary General (iphe. india@ gmail. com) your contact details viz. Mobile No., Tel. No., Email ID, Address for communication and Permanent Address.*, 9.

[23] Mechler, R., Singh, C., Ebi, K., Djalante, R., Thomas, A., James, R., ... & Revi, A. (2020). Loss and Damage and limits to adaptation: recent IPCC insights and implications for climate science and policy. *Sustainability Science*, *15*(4), 1245-1251.

[24] Ninyerola, M., Pons, X., & Roure, J. M. (2000). A methodological approach of climatological modelling of air temperature and precipitation through GIS techniques. *International Journal of Climatology: A Journal of the Royal Meteorological Society*, *20*(14), 1823-1841.

[25] Nury, A. H., Koch, M., & Alam, M. J. B. (2013, August). Time series analysis and forecasting of temperatures in the Sylhet division of Bangladesh. In *4th International Conference on Environmental Aspects of Bangladesh (ICEAB), August* (pp. 24-26).

[26] Peng, B., Zhou, Y., Gao, P., & Ju, W. (2011). Suitability assessment of different interpolation methods in the gridding process of station collected air temperature: A case study in Jiangsu Province. *China. J. Geo-Inf. Sci*, *4*, 539-548.

[27] Polsgrove, T., & Dwyer-Cianciolo, A. M. (2016). Human mars entry, descent and landing architecture study overview. In *AIAA SPACE 2016* (p. 5494).

[28] Pérez-Muñnuzuri, V., & Gelpi, I. R. (2000, October). Application of nonlinear forecasting techniques for meteorological modeling. In *Annales Geophysicae* (Vol. 18, No. 10, pp. 1349-1359). Springer-Verlag.

[29] Pachauri, R. K., Allen, M. R., Barros, V. R., Broome, J., Cramer, W., Christ, R., ... & van Ypserle, J. P. (2014). *Climate change 2014: synthesis report. Contribution of Working Groups I, II and III to the fifth assessment report of the Intergovernmental Panel on Climate Change* (p. 151). Ipcc.

[30] Qasim, M., Tayyab, H. M., & Ahmad, E. (2010). Use of salvage wood for reconstruction in earthquake affected sites of Muzaffarabad, Azad Jammu & Kashmir. *World Applied Sciences Journal*, *8*(7), 914-916.

[31] Raggad, B. (2018). Stationary and non-stationary extreme value approaches for modelling extreme temperature: the case of Riyadh City, Saudi Arabia. *Environmental Modeling & Assessment*, *23*(1), 99-116.

[32] Rashid, M. B., & Karmakar (2019). S. generalized extreme value (GEV) distribution to measure the changing probabilities of winter temperature in Dhaka. *The Journal of NOAMI, 36(1-2): 73-84. (June & December 2019)*

[33] Rodríguez-Lado, L., Sparovek, G., Vidal-Torrado, P., Dourado-Neto, D., & Macías-Vázquez, F. (2007). Modelling air temperature for the state of São Paulo, Brazil. *Scientia Agricola*, *64*,460-467.

[34] Rockström, J., Steffen, W., Noone, K., Persson, Å., Chapin III, F. S., Lambin, E., ... & Crutzen, P. & Foley, J. (2009). Planetary boundaries: Exploring the safe operating space for humanity. *Ecology and Society*, *14*(2), 32.

[35] Rahimzadeh, F., Asgari, A., & Fattahi, E. (2009). Variability of extreme temperature and precipitation in Iran during recent decades. *International Journal of Climatology: A Journal of the Royal Meteorological Society*, *29*(3), 329-343.

[36] Schaefer, D., & Domroes, M. (2009). Recent climate change in Japan–spatial and temporal characteristics of trends of temperature. *Climate of the Past*, *5*(1), 13-19.

[37] Salinger, M. J. (2005). Increasing climate variability and change: reducing the vulnerability. In *Increasing Climate Variability and Change* (pp. 1-3). Springer, Dordrecht.

[38] Siegert, M., Alley, R. B., Rignot, E., Englander, J., & Corell, R. (2020). Twenty-first century sea-level rise could exceed IPCC projections for strong-warming futures. *One Earth*, *3*(6), 691-703.

[39] Sarraf, A., Vahdat, S. F., & Behbahaninia, A. (2011). Relative humidity and mean monthly temperature forecasts in Ahwaz Station with ARIMA model in time series analysis. In *International Conference on Environment and Industrial Innovation IPCBEE, Singapore* (Vol. 12). IACSIT Press Singapore.

[40] Sivakumar, T., Thennarasu, A., & Rajkumar, J. S. I. (2012). Trend analysis of the climatic parameters (2001–2007) for the Northeastern Zone of Tamil Nadu. *Int J Res Environ Sci Technol*, *2*(3), 83-86.

[41] Sajjad, S. H., Hussain, B., Khan, M. A., Raza, A., Zaman, B., & Ahmed, I. (2009). On rising temperature trends of Karachi in Pakistan. *Climatic change*, *96*(4), 539-547.

[42] Torsen, E., Akinrefon, A. A., Rueben, B. Z., & Mbaga, Y. V. (2015). Probability distribution of maximum temperature in Adamawa State, Nigeria. *Journal of Mathematics (IOSR-JM)*, *11*(4), 1-6.

[43] Vivekanandan, N. (2018). Comparison of probability distributions in extreme value analysis of rainfall and temperature data. *Environmental earth sciences*, *77*(5), 1-10.

[44] Wuebbles, D. J., Fahey, D. W., Hibbard, K. A., Arnold, J. R., DeAngelo, B., Doherty, S., & Walsh, J. (2017). Climate science special report: Fourth national climate assessment (NCA4), Volume I.

[45] Yue, S., & Hashino, M. (2003). Long term trends of annual and monthly precipitation in Japan 1. *JAWRA Journal of the American Water Resources Association*, *39*(3), 587-596.

[46] Zahid, M., & Rasul, G. (2011). Frequency of extreme temperature and precipitation events in Pakistan 1965–2009. *Science International*, *23*(4), 313-319.

[47] Zorn, M. R., & Waylen, P. R. (1997). Seasonal response of mean monthly streamflow to El Nino/Southern Oscillation in north central Florida. *The Professional Geographer*, *49*(1), 51-62.