ISSN:1813-1786 (Print) 2313-7770 (Online)

Model Reduction of Unstable Discretized Time Second-Orderly Structured Systems

S. Ali[1], S. Haider[2], M. Ahmad[3], R. M. Mokhtar[4], K. Saleem[5]

[1,2,3,4]*School of Electrical and Electronic Engineering, Engg. Campus, Universiti Sains Malaysia, Pulau Pinang, Malaysia*
[5]*Graduate School of Sciences and Engineering Koc University, Istanbul, Turkey*

[1]sadaqat.ali@student.usm.my

Abstract- Physical world often face complex systems, expressed in terms of mathematical models which are then symbolized as equivalent trimmed structures. In this context disparate inexistent approaches to lessen the order of *unstable* Discretized time second-orderly structured systems (SOSSs) are construed. Gramianns for equating are procured from Discretized time algebric Lyapnov equations (DALEs) as long as system stays *stable*. Nevertheless, if system under consideration is *unstable*, DALEs get unresolvable and curtailment carcass get stuck. To avert this, two structure retaining Discretized time secondOrder equated trimming approaches for *unstable* SOSSs are advised in the manuscript. Given unstable system goes through Bernouli-feedback stabilization framwork and then gramiann get contrived for resulting sustained system. Xformation of gramianns into pos and velo fragments takes place and as a result structural retainment in ROM is gotten. As proposed approach retains SecondOrderly structure along with sustained system dynamics, the said approach provides true approximation of incipient system. Above mentioned procedures are validated on various systems to prove the dominance of the developments.

Keywords- Unstable, SecondOrderly Systems, Finite Grami Anns, Curtailed Model, HenkelSingular Values.

I. INTRODUCTION

The transformation of complex higher order systems (CHOS) to mathematical models interprets H-dimensional O-differential equations or P-differential equations which pose multiple challenges when exposed to investigation and synthesis. Direct processing of CHOS poses a serious threat of high computational costs. So it is desirable to translate CHOS to a system of similar nature but with lower dimensions/orders while carrying key characteristics of CHOS. This type of lower dimensional systems (LDS) are in great demand due to their simplicity of nature and found quite useful in sensitivity analysis, optimization and parameter tuning. Second order systems possess first two derivatives in their system dynamics as is the case of velocity and acceleration to displacement. In almost all fields, systems related to engineering and technology like skyscraper, International space station, mechatronics technology, image processing, community building, underground structures etc. are faced quite often. [1]–[8] and Discretized SOSS is portrayed in (1).

$$\mu_s x[ki+2] + \eta_s x[ki+1] + \mathrm{K}_s x[ki] = B_{s2} u[ki]$$
$$C_{s2} x[ki+1] + C_{s1} x[ki] = y_s[ki] \tag{1}$$

where $\eta_s \in R\sim^{n\times n}$, $\mu_s \in R\sim^{n\times n}$, $\beta s_2 \in R\sim^{n\times m}$, $\mathrm{K}_s \in R\sim^{n\times n}$ Cs_2 and $Cs_1 \in R\sim^{p\times n}$, $x(t) \in R\sim^{n}$, $u_s(t) \in R\sim^{m}$ and $y(t) \in R\sim^{p}$, n is system order, m is No. of input(s), p is nomb O/P(s) of system.
(1) in GenForm as:

$$\gamma q[ki+1] = \alpha q[ki] + B_s u[ki] \tag{2}$$
$$y[ki] = C_s q[ki]$$

with $q[k_i] = [\;x[k_i]^T \quad x[ki+1]^T]^T$

$$\gamma = \begin{bmatrix} Is & 0 \\ 0 & \mu s \end{bmatrix} \qquad \alpha = \begin{bmatrix} 0 & Is \\ -Ks & -\eta s \end{bmatrix}$$

$$B_s = \begin{bmatrix} 0 \\ Bs2 \end{bmatrix} \qquad C = [Cs1 \quad Cs2]$$

The desired LDM given as:

$$\mu_{sr} x_r[ki+2] + \eta_{sr} x_r[ki+1] + \mathrm{K}_{sr} x_r[ki] = B_{s2r} u[ki]$$
$$C_{s2r} x_r[ki+1] + C_{s1r} x_r[ki] = y_{sr}[ki] \tag{3}$$

$\mu_{sr} \in \mathbb{R}_{\sim}{}^{r\times r}, \eta_{sr} \in \mathbb{R}_{\sim}{}^{r\times r}, \mathrm{K}_{sr} \in \mathbb{R}_{\sim}{}^{r\times r}, \beta s_{2r} \in \mathbb{R}_{\sim}{}^{r\times m}, Cs_{1r}$ and $Cs_{2r} \in \mathbb{R}_{\sim}{}^{p\times r}$,

$$\gamma_r q_r[ki+1] = \alpha_r q_r[ki] + B_{sr} u[ki]$$
$$y_{sr}[ki] = C_r q_r[ki] \tag{4}$$

II. RELATED WORK

Moore came up with his notion of BalancedTruncation in his publication entitled "Principle CompAnalysis in Linear Systems" [9] which is special one in his endeavors. However, model curtailment was introduced by [10] for generalized systems in 1966. Then [11-12] modified it further respectively. On Structure-preservation [13], used integration algorithms depending mostly on numerical techniques while [14] preferred interpolation-based model reduction over the ordinary StateSpace 1st order adjustments using R-Krylof procedures. Similarly [15-17] used the error of Hamiltonian because of reduction as an error empirical interpolation approach, [18] continued to ensure preservation of port-Hamiltonian structure and [19] utilized dominant pole algorithm in his structure preservation effort. Balanced truncation for second order systems was first applied by [20], while [21] used the H2-optimal I-rational Krilof proposition. In [22], author brought in the idea of balancing which eventually paved the way for balanced truncation process development. For first order systems, solicitations in [23], are very much targeting the final developments of the process. However, early reduction mechanism couldn't preserve the original structure in reduced form (RF) [24-25]. But developments in this direction afterwards like modified Anoldi, second Order balanced truncation technique (SOBTT) and matching of moments (Krilof-subspeces) [4], [22], [25-30] etc. retained second order structure in RF of LDS. Approaches like Krilof, Anoldi, or MomentMatching did acquire the reduction of the model but RF got unstable in process. Similarly, a-prior error bounds not found in the due course. But RF of the system is preserved and do reside the a-prior reduction E-bounds for SOBTT [24-25], [31-33]. [20] offered a variety of SOBTTs for stable SOSs. Various practical requirements of existing systems ask for development of RF of CHOS realized to a particular frequency interval exclusively. This brings us to the mechanism of limited frequency LDS (LFLDS) techniques [7], [34-36]. In LFLDS, solicitation of reduced model is truely converging on a defined frequency interval. In LFLDS techniques for maintained systems offered for balancing of controlabity-gramiann with obsevability-gramiann on an underscored frequency hiatus is formulated. The notion for LFLDS in case of S-systems was conferred by [23] and same intellect was expanded by [37] for Gen-systems. Contributions by [34], [38] on the SOBTT intellect have been found quite significant. LFLDS of continual and discretized time maintained SOSSs utilizing SOBTT have been really contributory. For stable systems all goes perfect but when system is unstable by default, this instability cause serious consequences to reduction process as the algebric Lypnov equations (ALEs) get incomprehensible and involved mathematical approach gets halted. Numerous LDS techniques [39-48] etc. for similar purposes have been introduced. For reduction mechanism ahead, these techniques first stabilize the unstable system as a prerequisite. But it's a bit ironic that, these techniques do not take care of the multiple issues involved when LDS of unstable SOSs takes place. None of the techniques mentioned above play a role to reduction process of second order form of the unstable systems. Structure preservation of matrices involved in ROM is not seen a matter of concern. As a result, we are left with a form of LDS that not only prohibit analysis rather it does not allow design procedures to take place and ROM carries no value. Alongside, LFLDS mechanism, underscoring LF deployments of unstable SOSSs, not seen in any of the approaches shown by the literary world. Having seen the facts above, current effort addresses, structure preserving LDS techniques for LFLDS fulfillments of unstable SOSs. To stabilize unstable SOSS, Bernoulli feedback stabilisation (BSF) and limited frequency gramianns (LFGs) are conferred using LF continuous ALEs. The SOS once stabilized, is importuned in continuous ALEs to find out gramianns for balanced truncation. On the way, a highly innovative and convincing approach is supplicated to find out Cholisky factorisation of limited FGs and configuration of continuous ALEs. Procedural gramianns are bifurcated in pos and velo fragments to retain structure preservation in ROM. Additionally, the obtained gramianns are poised with different collocations to find assorted SOBTTs. ROM acquired in this process shows SOS disposition as well as optimization in desired frequency domain. proposed limited interval techniques are collated with infinite range techniques for myriad unstable SOSs to endorse the proposed development and superiority. Results for a benchmark example [49]) and few other examples are portrayed and accomplished Collocation of proposed techniques is analyzed. The ensued publication is structured as below. The section ahead elaborates basics on stable/unstable SOSSs trailed by proposed techniques in section 4. data analysis outcomes discussed in section 5 and publication is concluded.

III. ORDER-2 SYSTEM

The TF matrics of SOS (1) is as:

$$H(z) = (zC_{s2} + C_{s1})(z^2\mu_s + z\eta_s + \mathrm{K}_s)^{-1}B_{s2} \quad (5)$$

or $H = [\mu_s, \eta_s, \mathrm{K}_s, B_{s2}, C_{s1}, C_{s2}]$. System (1) is stable with no zeros of $P(\lambda) = \lambda^2\mu_s + \lambda\eta_s + \mathrm{K}_s$ exists exterior to unitary circled while zeros are in the interior part of the unitary circular zone. Furthermore, sys (1) is controlled only when ranked $[\lambda^2\mu_s + \lambda\eta_s + \mathrm{K}_s, B_{s2}] = n$ for all $\lambda \in C$ and observed when

rank $[\lambda^2\mu_s^T + \lambda\eta_s^T + \mathrm{K}_s^T, \lambda C_{s2}^T + C_{s1}^T] = n$ for all $\lambda \in C$

commensuratingly, system (1) is controlable and observable if and only if the first order system (2) is controlable and observable respectively i.e. ranked $[\lambda \curlyvee -\alpha, B] = 2n$ and ranked $[\lambda\gamma^T - \alpha^T, C^T] = 2n$ for all $\lambda \in C$.

For stable system (1) (having all the eignvalues (EVs) of the pencil $\lambda \curlyvee -\alpha$ inside the unit-circle), the discrete gramianns would be:

$$W_c = \sum_{k=0}^{\infty} \phi_d^k BB^T (\phi_d^T)^k \tag{6}$$

$$W_o = \sum_{k=0}^{\infty} (\phi_d^T)^k C^T C \phi_d \tag{7}$$

are the positive semidefinite, unique and symmetric solutions of the DALEs:

$$\alpha W_c \alpha - \gamma^T W_c \gamma^T = -BB^T \tag{8}$$

$$\alpha^T W_o \alpha - \gamma^T W_o \gamma = -C^T C \tag{9}$$

where $\phi_d = e^{\gamma^{-1}\alpha k} \curlyvee^{-1}$ is the fundamental solution matrix of (2).

Though, when order-2 sys of (1) is unstabled, Lypnov Eqns. (8) and (9) come up with indeterminate configuration which eventually breaks the flow of maintained trimming. To obviate this imposition, in the next component, approaches for MOR of unstabled discretized time systems are solicited.

A. Proclaimed Methods

DALEs will be workable for unstable Discretized time SOS (2) when Burnoulli review stabilisation is extended to make the SOSS stable and consequently utilized to formulate gramianns. Later on gramianns are apportioned in to pos-velo excerpts to retain structure preservation. ROM is gotton using discretized time order-2 balanced trimming (DSOBT).

1) Stablized outcome for Unstable Discretized Time System

In an initial effort, Burnouli review workout gramianns takes place (10)-(13), to stabilize system (2), [50]

$$\alpha W_c \alpha - \gamma^T W_c \gamma^T = -BB^T \tag{10}$$

$$\alpha^T W_o \alpha - \gamma^T W_o \curlyvee = -C^T C \tag{11}$$

Hence $K_c = B^T X_c \curlyvee$ and $K_o = \curlyvee X_o C^T$ are Bernouli stabilized feedback matrics, that's why the matrics X_c and X_o are the respective maintaining workouts of the algebric Bernouli eqns. (12), (13)

$$\alpha^T X_c \alpha - \curlyvee^T X_c \curlyvee = \curlyvee^T X_c BB^T X_c \curlyvee \tag{12}$$

$$\alpha X_o \alpha^T - \curlyvee X_o \curlyvee^T = \curlyvee X_o C^T C X_o \curlyvee^T \tag{13}$$

apportioning of gramianns found from (10) and (11) relents

$$W_{\mathrm{cstb}} = \begin{bmatrix} W_{\mathrm{pcstb}} & W_{12\ \mathrm{cstb}} \\ W_{12\ \mathrm{cstb}}^T & W_{\mathrm{vcstb}} \end{bmatrix},$$

$$W_{\mathrm{ostb}} = \begin{bmatrix} W_{\mathrm{postb}} & W_{12\ \mathrm{ostb}} \\ W_{12\ \mathrm{ostb}}^T & W_{\mathrm{vostb}} \end{bmatrix}$$

where W_{vostb} , W_{postb} , W_{vcstb} and W_{pcstb} are velo and pos obsevability and controlability gramianns in the go.

When pos and velo gramianns multiplied, relents the following denotation.

Denotation 1: Observe a stablised system as in (1) [20]. The SQURT of the eugenvalues of products

1) $W_{\mathrm{pcstb}}W_{\mathrm{postb}}$ are the HenkelValues for pos.
2) $W_{\mathrm{vcstb}}M^T W_{\mathrm{vostb}}M$ are the HenkelValues for velo.
3) $W_{\mathrm{pcstb}}M^T W_{\mathrm{vostb}}M$ are the HenkelValues for pos-velo.
4) $W_{\mathrm{vcstb}}W_{\mathrm{postb}}$ are the HenkelValues for velo-pos.

B. Maintained Xformation For Stabilised Systems

For order-2 maintained Xformation, disparate maintaining approaches can be considered to stablize sys. for velo-pos HankelValues acquisition.

Denotation 2: The stablized SOSS is pronounced:

1) Pos-Maintained if $W_{\mathrm{pcstb}}=W_{\mathrm{postb}}=\mathrm{diag}(\zeta^{\mathrm{pstb}}{}_1,\ldots, \zeta^{\mathrm{pstb}}{}_n)$.
2) velo equated if $W_{\mathrm{vcstb}} = W_{\mathrm{vostb}} = \mathrm{diag}(\zeta^{\mathrm{vstb}}{}_1,\ldots, \zeta^{\mathrm{vstb}}{}_n)$.
3) pos-velo equated if $W_{\mathrm{pcstb}} = W_{\mathrm{vostb}} = \mathrm{diag}(\zeta^{\mathrm{pvstb}}{}_1, \ldots, \zeta^{\mathrm{pvstb}}{}_n)$.
4) velo-pos equated if $W_{\mathrm{vcstb}} = W_{\mathrm{postb}} = \mathrm{diag}(\zeta^{\mathrm{vpstb}}{}_1, \ldots, \zeta^{\mathrm{vpstb}}{}_n)$.

here HSVs for finite hiatus descendingly represented by ζ. It can be either velo or pos or both velo-pos arrangements. Maintained Xformation may be derived through Cholsky factrization of gramianns given as:

$$W_{pcstb} = R_{pstb}R_{pstb}^T \quad W_{vcstb} = R_{vstb}R_{vstb}^T$$
$$W_{postb} = L_{pstb}L_{pstb}^T \quad W_{vostb} = L_{vstb}L_{vstb}^T \tag{14}$$

where $R_{pstb}, R_{vstb}, L_{pstb}, L_{vstb} \in R^{n\times n}$ are LowerTriangular nonsingular Cholsky factors to calculate HSVs through ClassicalSingular values as below:

$$\left(\zeta_i^{pstb}\right)^2 = \lambda_i\left(W_{pcstb}W_{postb}\right)$$
$$= \lambda_i\left(R_{pstb}R_{pstb}^T L_{pstb}L_{pstb}^T\right)$$
$$= \lambda_i\left(L_{pstb}^T R_{pstb}R_{pstb}^T L_{pstb}\right) = \sigma_i^2\left(R_{pstb}L_{pstb}\right)$$

where σ_{istb} 's are clasical SVs. So $\zeta_i^{vstb} = \sigma_i(R_{vstb}^T M^T L_{vstb})$, $\zeta_i^{pvstb} = \sigma_i\left(R_{pstb}^T M^T L_{vstb}\right)$, and $\zeta_i^{vpstb} = \sigma_i\left(R_{vstb}^T L_{pstb}\right)$. Above products are decomposed singularly to provide:

$$R_{pstb}^T L_{pstb} = U_{pstb}\Sigma_{pstb}V_{pstb}^T$$
$$R_{vstb}^T M^T L_{vstb} = U_{vstb}\Sigma_{vstb}V_{vstb}^T \tag{15}$$
$$R_{pstb}^T M^T L_{vstb} = U_{pvstb}\Sigma_{pvstb}V_{pvstb}^T$$
$$R_{vstb}^T L_{pstb}^T = U_{vpstb}\Sigma_{vpstb}V_{vpstb}^T$$

where $U_{pstb}, V_{pstb}, U_{vstb}, V_{vstb}, U_{pvstb}, V_{pvstb}, U_{vpstb}, V_{vpstb}$ are the orthgonal matrics, But:

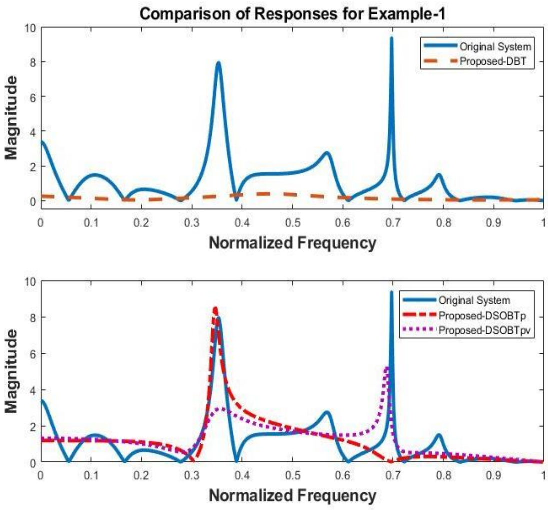

FIGURE 1: Depiction of retorts for Example-1

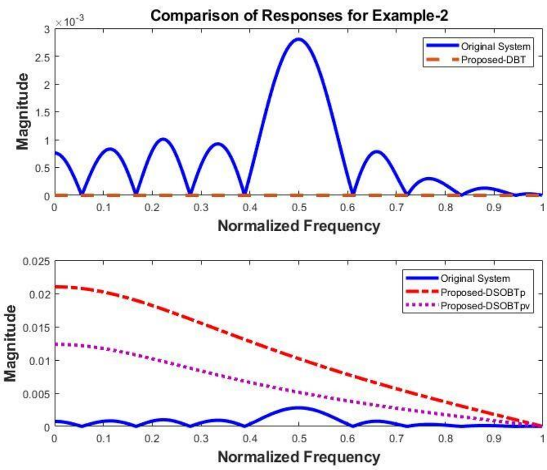

FIGURE 2: Depiction of retorts for Example-2

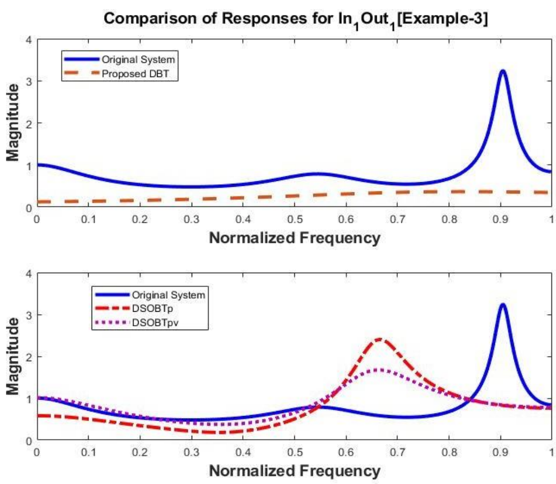

FIGURE 3: Depiction of retorts for $I_1 - O_1$ Example-3

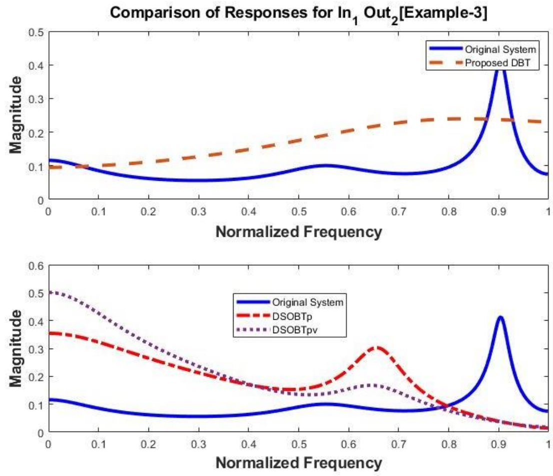

FIGURE 4: Depiction of retorts for $I_1 - O_2$ Example-3

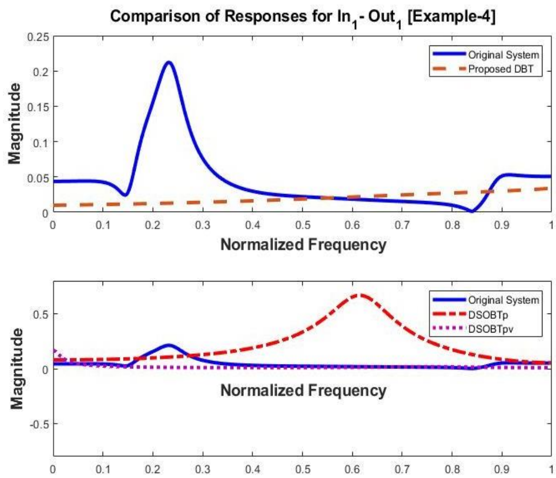

FIGURE 5: Depiction of retorts for $I_1 - O_1$ Example-4

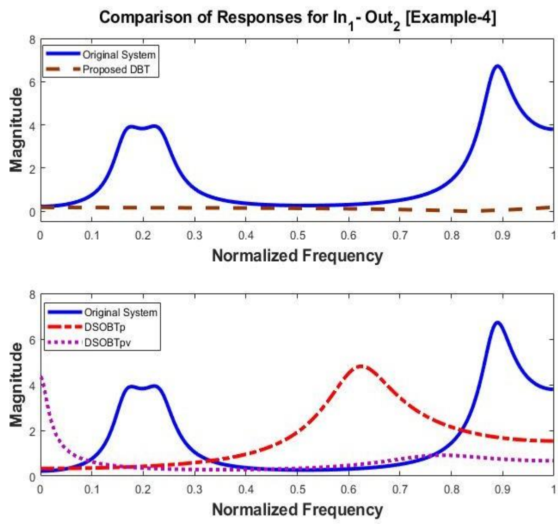

FIGURE 6: Depiction of retorts for $I_1 - O_2$ Example-4

$$\Sigma_{pstb} = \text{diag}\left(\zeta_1^{pstb}, \dots, \zeta_n^{pstb}\right)$$
$$\Sigma_{vstb} = \text{diag}\left(\zeta_1^{vstb}, \dots, \zeta_n^{vstb}\right)$$
$$\Sigma_{pvstb} = \text{diag}\left(\zeta_1^{pvstb}, \dots, \zeta_n^{pvtb}\right)$$
$$\Sigma_{vpstb} = \text{diag}\left(\zeta_1^{vpstb}, \dots, \zeta_n^{vpstb}\right)$$

are NonsingularMatrics. Equation (15) is there for maintained Xformation (Plstb, Prstb) for manipulations of denotation 2.

1) Maintained Trimming for Stablized Discretized Time SOS

As given above, the maitained Xformation relents HSVs.

Algorithm 1: FFDSOBTp

Input: Given a stable SOSS $H = [\mu_s, \eta_s, \mathrm{K}_s, B_{s2}, C_{s1}, C_{s2}]$ and frequency hiatus $\hat{\Delta}$:

Output: The FLROM $H_r = [\mu_{sr}, \eta_{sr}, \mathrm{K}_{sr}, B_{s2r}, C_{s1r}, C_{s2r}]$

1. Jot down the DLFGs $W_{\hat{p}c\hat{\Delta}}, W_{\tilde{p}o\hat{\Delta}}, W_{\hat{v}c\hat{\Delta}}$ and $W_{\tilde{v}o\hat{\Delta}}$ using (10) and (11).
2. Jot down the Cholsky facets $R_{p\hat{\Lambda}}, R_{u\hat{\Lambda}}, L_{p\hat{\Lambda}}$ and $L_{n\hat{\Lambda}}$ of DLFGs using (14).
3. Compute SVD for the products:

$$R_{p\dot{\Delta}}^T L_{p\dot{\Delta}} = [U_{p1\hat{\Delta}} \quad U_{p2\hat{\Delta}}] \begin{bmatrix} \tilde{\Sigma}_{p1\hat{\Delta}} & 0 \\ 0 & \tilde{\Sigma}_{p2\hat{\Delta}} \end{bmatrix} [V_{p1\Delta} \quad V_{p2\hat{\Delta}}]^T$$

$$R_{v\hat{\Delta}}^T M^T L_{v\dot{\Delta}} = [U_{v1\hat{\Delta}} \quad U_{v2\hat{\Delta}}] \begin{bmatrix} \tilde{\Sigma}_{v1\hat{\Delta}} & 0 \\ 0 & \tilde{\Sigma}_{v2\dot{\Delta}} \end{bmatrix} [V_{v1\grave{\Delta}} \quad V_{v2\grave{\Delta}}]^T$$

where the matrices $[U_{p1\Delta} \quad U_{p2\Delta}]$, $[V_{p1\Delta} \quad V_{p2\Delta}]$, $[U_{v1\Delta} \quad U_{v2\Delta}]$ and $[V_{v1\Delta} \quad V_{v2\Delta}]$ are orthogonal and

$$\tilde{\Sigma}_{p1\hat{\Delta}} = \text{diagnn}\left(\hat{\zeta}_1^{p\hat{\Delta}}, \dots, \hat{\zeta}_r^{p\dot{\Delta}}\right) \tilde{\Sigma}_{p2\hat{\Delta}} = \text{diagnn}\left(\hat{\zeta}_{r_+}^{p\hat{\Delta}}, \dots, \hat{\zeta}_n^{p\Delta}\right.$$

$$\tilde{\Sigma}_{v1\Delta} = \text{diagnn}\left(\hat{\zeta}_1^{v\Delta}, \dots, \hat{\zeta}_r^{v\hat{\Delta}}\right) \tilde{\Sigma}_{v2\hat{\Delta}} = \text{diagnn}\left(\hat{\zeta}_{r_+}^{v\hat{\Delta}}, \dots, \hat{\zeta}_n^{v\dot{\Delta}}\right.$$

4. Calculate the ROM

$$\mu_{sr} = P_{l\grave{\Delta}}^T M P_{r\hat{\Delta}} \quad \eta_{sr} = P_{l\grave{\Delta}}^T D P_{r\hat{\Delta}} \quad \mathrm{K}_{sr} = P_{l\grave{\Delta}}^T K P_{r\hat{\Delta}}$$
$$B_{s2r} = P_{l\hat{\Delta}}^T B_{s2} \quad C_{s1r} = C_1 P_{r\hat{\Delta}} \quad C_{s2r} = C_2 P_{r\hat{\Delta}}$$

where $P_{l\tilde{\Delta}} = L_{v\tilde{\Delta}} V_{v1\hat{\Delta}} \tilde{\Sigma}_{p1\tilde{\Delta}}^{-1/2}$ and $P_{r\hat{\Delta}} = R_{p\dot{\Delta}} U_{p1\hat{\Delta}} \tilde{\Sigma}_{p1\tilde{\Delta}}^{-1/2}$

Algorithm 2: (LFDSOBTpv)

Input: Find here a stable SOSS $H = [\mu_s, \eta_s, \mathrm{K}_s, B_{s2}, C_{s1}, C_{s2}]$ and frequency hiatus $\hat{\Delta}$:

Output: The FLROM $H_r = [\mu_{sr}, \eta_{sr}, \mathrm{K}_{sr}, B_{s2r}, C_{s1r}, C_{s2r}]$

1. Jot down the DLFGs $W_{\hat{p}c\hat{\Delta}}$ and $W_{\tilde{v}o\hat{\Delta}}$ using (10) and (11).
2. Jot down the Cholsky factors $R_{p\dot{\Delta}}$ and $L_{v\hat{\Delta}}$ of DLFGs $W_{\hat{p}c}\hat{\Delta}$ and $W_{\tilde{v}o}\hat{\Delta}$ as given in (14).
3. Compute SVD for the products:

$$= [U_{pv1\hat{\Delta}}^T \quad U_{pv2\hat{\Delta}}] \begin{bmatrix} \tilde{\Sigma}_{\hat{p}\hat{\Delta}1} M^T L_{v\hat{\Delta}} & 0 \\ 0 & \tilde{\Sigma}_{\hat{p}\hat{v}2\hat{\Delta}} \end{bmatrix} [V_{\hat{p}\hat{v}1\hat{\Delta}} \quad V_{\hat{p}\hat{v}2\hat{\Delta}}]^T$$

where $[U_{\hat{p}v1\Delta} \quad U_{\hat{p}v2\hat{\Delta}}]$ and $[V_{\hat{p}v1\hat{\Delta}} \quad V_{\hat{p}\hat{v}2\hat{\Delta}}]$ are orthogonal and $\tilde{\Sigma}_{\hat{p}v1\hat{\Delta}} = \text{diagnn}\left(\hat{\zeta}_1^{\hat{p}\hat{v}}\hat{\Delta}, \dots, \hat{\zeta}_r^{\hat{p}\hat{v}}\right)$,

$$\tilde{\Sigma}_{\hat{p}\hat{v}2\Delta} = \text{diagnn}\left(\hat{\zeta}_{r+1}^{pv\hat{\Delta}}, \dots, \hat{\zeta}_n^{\hat{p}v\Delta}\right)$$

4. Calculate the ROM

$$\mu_{sr} = I_n \quad \eta_{sr} = P_{l\Delta}^T D P_{r\hat{\Delta}} \quad K_r = P_{l\Delta}^T K P_{r\hat{\Delta}}$$
$$B_{s2r} = P_{l\Delta}^T B_{s2} \quad C_{s1r} = C_1 P_{r\hat{\Delta}} \quad C_{s2r} = C_2 P_{r\hat{\Delta}}$$

where $P_{l\hat{\Delta}} = L_{v\hat{\Delta}} V_{\hat{p}\hat{v}1\hat{\Delta}_\Delta} \tilde{\Sigma}_{\hat{p}v1\hat{\Delta}}^{-1/2}$ and $P_{r\hat{\Delta}} = R_{p\hat{\Delta}} U_{\hat{p}v1\hat{\Delta}} \tilde{\Sigma}_{\hat{p}v1\hat{\Delta}}^{-1/2}$

Remark: The gramianns for equations (10) and (11) are exclusive, symetric and determinate workouts of DALEs, resulting ROMs are stable

IV. Data Analysis and Observations

The proclaimed MOR approaches are deployed to disparate nonstable systems like piezoelectric and a few autonomous cases (appendices) and outcomes presented. A modular structure based in Los Angeles refurbished with piezoelectric struts [51] portrayed in Fig.5 and commensurating coeffisient matrics:

$$\mu_s = \begin{bmatrix} 9.3 & 0 & 0 \\ 0 & 6.3 & 0 \\ 0 & 0 & 4.7 \end{bmatrix}, \eta_s = \begin{bmatrix} 7.4 & 2 & 7 \\ 5 & 6.1 & 0 \\ 1 & 3 & 2.1 \end{bmatrix}$$
$$K_s = \begin{bmatrix} 8.8 & 3.7 & 0 \\ -2.3 & 8 & 1 \\ -3 & 0 & 7.6 \end{bmatrix} \quad (16)$$

Here a diagonal extension of modular structure from three story to twelve with three scores of similar piezoelectric elements is provided and reduced model of this apparently twelve story structure is accomplished. 12th order system is reduced to r = 3 using proclaimed infinite and finite interval contrivances. The proclaimed contrivances have been implemented on Discretized time piezoelectric system model. These contrivances depicts an overwhelming riposte for ROM in pos as well as pos-velo. Fig. 12 and Fig. 14 show riposte for pos and pos-velo on overall band of frequency (infinite range). It is again observed that these contrivances retain second-orderly structure and engage system dynamics in stabilized form, therefore, these contrivances seem more promising. The results ratify the true framwork and superiority of the suggested contrivances.

SISO Systems: The 9th order SISO system portrayed in Example-1 is reduced to r = 3 using proclaimed DSOBTTp and DSOBTTpv approaches as provided in algo 1 and 2. The ripostes of incipient and reduced systems have been depicted in Fig.1. It is notable that the reduced model of DSOBTTp and DSOBTTpv contrivances is very truely surmising the incipient SOS. Furthermore, 9th level SISO system in Example2 is trimmed to r = 3 and ripostes for incipient and trimmed systems are presented in Fig.2.

MIMO Systems: The tenth level MIMO-sys of Example3 with 1-2 I/O is trimmed to r = 5 and riposte for incipient and reduced forms using proclaimed approaches DSOBTTp and DSOBTTpv are shown in Fig.3 and Fig.4. The ROMs are truely surmising incipient large order MIMO sys of Example3 for the construed approaches of DSOBTTp and DSOBTTpv. At last the 8th order MIMO sys portrayed in Example4 with 1-2 I/O is trimmed to r = 4 and riposte for incipient and reduced forms obtained through applied techniques are shown in Fig.5 and Fig.6. This explanation shows once more that the response of the proclaimed approaches is up to the mark.

V. Conclusion

Disparate inexistent system reduction approaches for Discretized time nonstable Level-2 systems are construed. These approaches take care of resolvability of DALEs for precarious Discretized time SOSSs. System is maintained using Burnouli feedback stablization mechanism and next gramianns are computed for maintained system. Furthermore, gramians are apportioned in to pos and velo portions to acquire structural attainment in ROM and maintained trimming goes solicited. For structural attainment, gramianns are apportioned into pos and velo portions. The pos and velo controlability and obsrvability gramians are maintained in disparate arrangements to get pos and velo or both HSVs. States where magnitudnal value stay less for HSVs are trimmed to get stable ROMs for precarious, incipient SOSSs. Proclaimed approaches are tested on benchmark as well as autonomous cases and outcomes for SISO and MIMO achievements are portrayed. The sequels accredit the true formation of the proclaimed approaches.

References

[1] Z.-H. Wang, Z. Miao, and Y.-L. Jiang, “Structure-preserving model reduction based on multiorder arnoldi for discrete-time second-order systems,” International Journal of Systems Science, pp. 1–15, 2024.

[2] B. B. Duddeti, A. K. Naskar, and K. Subhashini, “Order reduction of lti systems using balanced truncation and particle swarm optimization algorithm,” Circuits, Systems, and Signal Processing, vol. 42, no. 8, pp. 4506–4552, 2023.

[3] A. Laub and W. Arnold, “Controllability and observability criteria for multivariable linear second-order models,” IEEE Transactions on Automatic Control, vol. 29, no. 2, pp. 163–165, 1984.

[4] R. W. Freund, “Padé-type model reduction of second-order systems and higher-order linear dynamical systems,” Dimension Reduction of Large-Scale Systems, vol. 45, pp. 193–226, 2005.

[5] R. Craig, “An introduction to computer methods,” in John Wiley and Sons. New York, 1981.

[6] J. Clark, N. Zhou, and K. Pister, “Modified nodal analysis for mems with multi-energy domains,” in International Conference on Modeling and Simulation of Microsystems, Semiconductors, Sensors and Actuators, San Diego, CA, 2000, pp. 31–34.

[7] S. K. Suman and A. Kumar, “Linear system of order reduction using a modified balanced truncation method,” Circuits, Systems, and Signal Processing, vol. 40, no. 6, pp. 2741–2762, 2021.

[8] U. Zulfiqar, V. Sreeram, and X. Du, “Time-limited pseudo-optimal-model order reduction,” IET Control Theory & Applications, vol. 14, no. 14, pp. 1995–2007, 2020.

[9] B. Moore, “Principal component analysis in linear systems: Controllability, observability, and model reduction,” IEEE transactions on automatic control, vol. 26, no. 1, pp. 17–32, 1981.

[10] E. Davison, “A method for simplifying linear dynamic systems,” IEEE Transactions on automatic control, vol. 11, no. 1, pp. 93–101, 1966.

[11] M. Chidambara, “Two simple techniques for the simplification of large dynamic systems,” in Joint Automatic Control Conference, no. 7, 1969, pp. 669–674.

[12] E. Davison and F. Man, “Interaction index for multivariable control systems,” in Proceedings of the Institution of Electrical Engineers, vol. 117, no. 2. IET, 1970, pp. 459–462.

[13] S. Lall, P. Krysl, and J. E. Marsden, “Structure-preserving model reduction for mechanical systems,” Physica D: Nonlinear Phenomena, vol. 184, no. 1-4, pp. 304–318, 2003.

[14] C. Beattie and S. Gugercin, “Interpolatory projection methods for structure-preserving model reduction,” Systems & Control Letters, vol. 58, no. 3, pp. 225–232, 2009.

[15] B. M. Afkham and J. S. Hesthaven, “Structure preserving model reduction of parametric hamiltonian systems,” SIAM Journal on Scientific Computing, vol. 39, no. 6, pp. A2616–A2644, 2017.

[16] S. K. Suman and A. Kumar, “Investigation and implementation of model order reduction technique for large scale dynamical systems,” Archives of Computational Methods in Engineering, vol. 29, no. 5, pp. 3087–3108, 2022.

[17] A. M. Burohman, B. Besselink, J. M. Scherpen, and M. K. Camlibel, “From data to reduced-order models via generalized balanced truncation,” IEEE Transactions on Automatic Control, vol. 68, no. 10, pp. 6160–6175, 2023.

[18] S. Chaturantabut, C. Beattie, and S. Gugercin, “Structure-preserving model reduction for nonlinear port-hamiltonian systems,” SIAM

Journal on Scientific Computing, vol. 38, no. 5, pp. B837–B865, 2016.

[19] S. W. Werner, "Structure-preserving model reduction for mechanical systems," Ph.D. dissertation, 2021.

[20] T. Reis and T. Stykel, "Balanced truncation model reduction of secondorder systems," Mathematical and Computer Modelling of Dynamical Systems, vol. 14, no. 5, pp. 391–406, 2008.

[21] S. A. Wyatt, "Issues in interpolatory model reduction: Inexact solves, second-order systems and daes," Ph.D. dissertation, Virginia Tech, 2012.

[22] Y. Chahlaoui, D. Lemonnier, A. Vandendorpe, and P. Van Dooren, "Second-order balanced truncation," Linear Algebra and Its Applications, vol. 415, no. 2-3, pp. 373–384, 2006.

[23] W. Gawronski and J.-N. Juang, "Model reduction in limited time and frequency intervals," International Journal of Systems Science, vol. 21, no. 2, pp. 349–376, 1990.

[24] D. G. Meyer and S. Srinivasan, "Balancing and model reduction for second-order form linear systems," IEEE Transactions on Automatic Control, vol. 41, no. 11, pp. 1632–1644, 1996.

[25] B. Salimbahrami and B. Lohmann, "Structure preserving order reduction of large scale second order systems," IFAC Proceedings, vol. 37, no. 11, pp. 233–238, 2004.

[26] ——, "Order reduction of large scale second-order systems using krylov subspace methods," Linear Algebra and its Applications, vol. 415, no. 2-3, pp. 385–405, 2006.

[27] Z. Bai and Y. Su, "Dimension reduction of large-scale second-order dynamical systems via a second-order arnoldi method," SIAM Journal on Scientific Computing, vol. 26, no. 5, pp. 1692–1709, 2005.

[28] Z. Bai, K. Meerbergen, and Y. Su, "Arnoldi methods for structurepreserving dimension reduction of second-order dynamical systems," in Dimension Reduction of Large-Scale Systems, 2005, pp. 173–189.

[29] A. Padoan, F. Forni, and R. Sepulchre, "Balanced truncation for model reduction of biological oscillators," Biological Cybernetics, vol. 115, no. 4, pp. 383–395, 2021.

[30] Z. Salehi, P. Karimaghaee, and M.-H. Khooban, "Mixed positive-bounded balanced truncation," IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 68, no. 7, pp. 2488–2492, 2021.

[31] A. M. Burohman, B. Besselink, J. Scherpen, and M. K. Camlibel, "From data to reduced-order models via generalized balanced truncation," arXiv preprint arXiv:2109.11685, 2021.

[32] C. Grussler, T. Damm, and R. Sepulchre, "Balanced truncation of k-positive systems," IEEE Transactions on Automatic Control, vol. 67, no. 1, pp. 526–531, 2021.

[33] Z. Salehi, P. Karimaghaee, and M.-H. Khooban, "Model order reduction of positive real systems based on mixed gramian balanced truncation with error bounds," Circuits, Systems, and Signal Processing, vol. 40, no. 11, pp. 5309–5327, 2021.

[34] S. Haider, I. M. Ghafoor, Abdul, and F. M. Malik, "Frequency limited gramians-based structure preserving model order reduction for discrete time second-order systems," International Journal of Control, vol. 92, no. 11, pp. 2608–2619, 2019.

[35] Z. Salehi, P. Karimaghaee, and M.-H. Khooban, "A new passivity preserving model order reduction method: conic positive real balanced truncation method," IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 52, no. 5, pp. 2945–2953, 2021.

[36] P. Benner and S. W. Werner, "Frequency-and time-limited balanced truncation for large-scale second-order systems," Linear Algebra and its Applications, vol. 623, pp. 68–103, 2021.

[37] H. I. Toor, M. Imran, A. Ghafoor, D. Kumar, V. Sreeram, and A. Rauf, "Frequency limited model reduction techniques for discrete-time systems," IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 67, no. 2, pp. 345–349, 2019.

[38] K. S. Haider, A. Ghafoor, M. Imran, and F. M. Malik, "Frequency interval gramians based structure preserving model order reduction for second order systems," Asian Journal of Control, vol. 20, no. 2, pp. 790–801, 2018.

[39] J. Yang, C. S. Chen, J. D. Abreu-Garcia, and Y. Xu, "Model reduction of unstable systems," International Journal of Systems Science, vol. 24, no. 12, pp. 2407–2414, 1993.

[40] N. Mirnateghi and E. Mirnateghi, "Model reduction of unstable systems using balanced truncation," in IEEE 3rd International Conference on System Engineering and Technology (ICSET), 2013, pp. 193–196.

[41] K. Mustaqim, D. K. Arif, E. Apriliani, and D. Adzkiya, "Model reduction of unstable systems using balanced truncation method and its application to shallow water equations," in Journal of Physics: Conference Series, vol. 855, no. 1, 2017, p. 012029.

[42] C. Boess, A. S. Lawless, N. K. Nichols, and A. Bunse-Gerstner, "State estimation using

model order reduction for unstable systems," Computers & Fluids, vol. 46, no. 1, pp. 155–160, 2011.

[43] C. Magruder, C. Beattie, and S. Gugercin, "L2-optimal model reduction for unstable systems using iterative rational krylov algorithm," Technical Report, Virginia Technical, Mathematics Department, Tech. Rep., 2009.

[44] P. Benner, M. Castillo, E. S. Quintana-Ortí, and G. Quintana-Ortí, "Parallel model reduction of large-scale unstable systems," in Advances in Parallel Computing, 2004, vol. 13, pp. 251–258.

[45] K. Zhou, G. Salomon, and E. Wu, "Balanced realization and model reduction for unstable systems," International Journal of Robust and Nonlinear Control: IFAC-Affiliated Journal, vol. 9, no. 3, pp. 183–198, 1999.

[46] D. K. Arif, D. Adzkiya, E. Apriliani, and I. N. Khasanah, "Model reduction of non-minimal discrete-time linear-time-invariant systems," Malaysian Journal of Mathematical Sciences, vol. 11, pp. 377–391, 2017.

[47] A. G. Pillai and E. Rita Samuel, "Pso based lqr-pid output feedback for load frequency control of reduced power system model using balanced truncation," International Transactions on Electrical Energy Systems, vol. 31, no. 9, p. e13012, 2021.

[48] T. Breiten and P. Schulze, "Structure-preserving linear quadratic gaussian balanced truncation for port-hamiltonian descriptor systems," arXiv preprint arXiv:2111.05065, 2021.

[49] M. Ahmadloo and A. Dounavis, "Parameterized model order reduction of electromagnetic systems using multiorder arnoldi," IEEE transactions on advanced packaging, vol. 33, no. 4, pp. 1012–1020, 2010.

[50] P. Benner, J. Saak, and M. M. Uddin, "Balancing based model reduction for structured index-2 unstable descriptor systems with application to flow control," Numerical Algebra, Control and Optimization, vol. 6, no. 1, pp. 1–20, 2016.

[51] Y. Chahlaoui and P. Van Dooren, "A collection of benchmark examples for model reduction of linear time invariant dynamical systems." 2002.

ISSN:1813-1786 (Print) 2313-7770 (Online)

APPENDIX

Different unstable discrete time SISO and MIMO SOSs are

Example 1: A 9th order system

$$K=\begin{bmatrix} -1 & -.4 & .2 & -1 & -1 & .4 & .2 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\eta=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1.71 & 0.6264 & 0.19 & 0.38 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\beta_2=[1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]^T$, $C_1=[1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1]$,

$C_2=[1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1], \mu=I_9$

Example 2: A 9th order system

$$K=\begin{bmatrix} 0.8 & -1.2 & -0.7 & 41 & -0.7 & -5.4 & -3.4 & 1.6 & 4 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$K=\begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 & -8 & 0.1 & 0 & 0 & -115 & 0.1 \\ 0.1 & 0 & 0 & 0 & 0.1 & -8 & 0.1 & 0 & 0.2 & 0.1 \\ 0.1 & 0.1 & 1.1 & 0 & 0.9 & 0.1 & -8 & 0.1 & 0 & 0 \\ 0.1 & 0.1 & -8 & 0.1 & 0.1 & -8 & 0.1 & 0 & 0 & 0.2 \\ 0.1 & 0.1 & 0.2 & 0.1 & -8 & 0.1 & 0 & 0.1 & 0 & 0.1 \\ 0.1 & 0.2 & 0 & 0.1 & -8 & 0.1 & 0 & 0.2 & 0.1 & 0.2 \\ 0.1 & 0 & 1 & 0 & 0.1 & -8 & 0.1 & 0 & 0.2 & 0.1 \\ 0.1 & 0 & 0 & 0 & 0.1 & -8 & 0.1 & 0 & 0.2 & 0.1 \\ 0.1 & 0 & 0 & 0 & 0.1 & -8 & 0.1 & 0 & 0.2 & 0.1 \\ 0.1 & 0 & 0 & 0 & 0.1 & -8 & 0.1 & 0 & 0.2 & 0.1 \end{bmatrix};$$

$$\eta=\begin{bmatrix} 8.3 & 9.2 & 0.1 & 0.8 & 0.1 & 2.4 & 0.1 & 2.6 & 0.9 & 0.1 \\ 0 & 0.1 & 0.8 & 0.8 & 0.2 & 0.1 & 0.2 & 0.2 & 0.1 & 0 \\ 0.8 & 0.2 & 0.1 & 0.2 & 0.8 & 0.2 & 0.1 & 0.2 & 0.5 & 0.2 \\ 0.1 & 0.5 & 0.8 & 0.2 & 0.1 & 0.8 & 0.1 & 0.3 & 0.2 & 0.8 \\ 0.8 & 0.2 & 0.1 & 0.2 & 0.8 & 0.2 & 0.1 & 0.2 & 0.9 & 0.1 \\ 0.3 & 0.1 & 0.2 & 0.9 & 0 & 0.8 & 0.2 & 0.1 & 0.2 & 0.8 \\ 0.8 & 0.2 & 0.1 & 0.82 & 0.08 & 0.22 & 0.11 & 0.82 & 0.95 & 0.12 \\ 0.31 & 0.13 & 0.11 & 0.12 & 0.08 & 0.22 & 0.1 & 0.2 & 0.3 & 0.9 \\ 0.8 & 0.2 & 0.1 & 0.2 & 0.8 & 0.2 & 0.1 & 0.2 & 0.5 & 0.2 \\ 0.8 & 0.8 & 0.2 & 0.1 & 0.8 & 0.1 & 0.2 & 0.9 & 0.2 & 0.4 \end{bmatrix}$$

$$\beta_2=\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T, C_1=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$C_2=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \mu=I_{10}$$