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Reliability Analysis of Slope based on Fellenius's and Bishop's Method

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ABSTRACT

This paper uses five reliability methods to analyze the stability of a slope. The stability of the slope is calculated by Fellenius's method and Simplified Bishop's method respectively. The main factors considered in slope stability are the cohesion and friction angle of soil. We assume that the cohesion and friction angle of soil are normally distributed, and their distribution is updated using the Bayesian method based on investigated field test data. We mainly studied the influence of the correlation coefficient between the cohesion and friction angle of soil and the slope angle on the stability of the slope. The results showed that methods of IRSM (Intelligent Response Surface Method) or AFORM (Advanced First-Order Reliability Method) are more accurate in reliability analysis of the slope, while MCS's (Monte Carlo Simulation) results fluctuate greatly for sample points and are not sufficient due to the limitation of calculation time. The probability of failure increases as the value of the correlation coefficient between cohesion and friction angle increases. Results calculated by Fellenius's method are more conservative, and Simplified Bishop's method is recommended for steep slopes.

KEYWORDS: Slope stability analysis, Reliability analysis, Bishop's method, Fellenius's method, Taylor series expansion method, Rosenblueth method, Advanced first order reliability method.

1 INTRODUCTION

A slope is determined as a surface whose one end or side is elevated than the other: an ascending or descending surface. An earth slope is a cantilevered, slanted surface of a soil mound. The failure of a soil mound situated under a slope is called a landslide. This contains a downward and outward gesticulation of the complete soil mass involved in the fiasco. The failure of slopes is primarily due to the effect of gravity and seepage flows in the soil. The durability of slopes is a prominent issue in civil engineering, as Zolkepli MF et al. using the modified method of Fellenius and Bishop [1], Ullah S [2], and Harabinová S et al. analyze the slope stability method [3] He Y and Li Z et al. considered the strength anisotropy of c-φ soil [4], slope failures can induce substantial destruction to downstream infrastructure and estate. Slope stability analysis is a critical aspect of geotechnical engineering, particularly in projects involving excavations, embankments, and natural slopes. The safety of such projects hinges on a thorough understanding of the factors influencing slope stability. Traditionally, the calculation of the factor of safety has been the cornerstone of assessing slope stability, wherein engineers compare the resisting forces against the driving forces to determine if a slope is prone to failure. However, with the advancement of engineering practices and the growing recognition of the inherent uncertainties in natural systems, the need for more comprehensive and reliable methods of analysis has become evident. In this paper, Fellenius's method [5] and simplified Bishop's method [6] are used to analyze the stability of the slope. The main parameters that affect the stability of the slope are the cohesion (c) and internal friction angle (φ) of the soil. While the traditional slope stability safety factor method has been widely employed in practice, it has certain limitations. One of the primary challenges lies in its treatment of uncertainties. Real-world geotechnical systems are inherently subjected to a multitude of uncertainties stemming from factors such as material properties, environmental conditions, and construction processes.

Similarly, Jampani H and Harabinová S et al. point out that tools for slope stability analysis are crucial to appraise the risk of ground motion design proper preservative actions and manage more intricated circumstances [7-8], such as the impact of runoff, scouring and disintegration on slope stability [9-10]. The conventional safety factor approach often overlooks or oversimplifies these uncertainties, potentially leading to overly optimistic or conservative stability assessments. This disparity between theoretical analyses and dynamic real-world conditions underscores the necessity of incorporating reliability analysis methods into slope stability assessments. Reliability analysis offers a way to bridge this gap by considering uncertainties in a more robust manner. It acknowledges the inherent variability in geotechnical parameters and other influential factors.

2 METHODOLOGY

2.1 The determination of distribution parameters

Based on geotechnical experience, the unit weight (γ) of the soil is regarded as a deterministic variable due to its small coefficient of variation [8], which is equal to 19.2 kN/m3.
In the absence of measured data, for the teaching purpose of learning reliability analysis, both c and φ in this article adopt normal distribution [11-12].

To obtain accurate distribution parameters of soil properties, a large amount of measured data is often required. However, due to the influence of the natural environment and other factors, there may be insufficient on-site measured data, so the probability distribution of c and φ can only be determined based on existing experience and it is not accurate enough.

Many scholars have conducted shear tests on different types of soils in different regions and found that the coefficient of variation of general clay's cohesion (c) is generally between 0.20 and 0.50, and the coefficient of variation of friction angle (φ) is generally from 0.12 to 0.56 [13]. Thus, we assume the cohesion and internal friction angle follow the normal distribution of $c \sim N$ (20, 3), $\varphi \sim N$ (25, 5).

However, this is based on other scholars' experience, it is questionable whether it can be used in our project or not. Based on the Bayesian method, we can use limited field measurement data to obtain a more accurate parameter distribution model, thereby improving the effectiveness and accuracy of slope reliability analysis.
Let $\boldsymbol{X} = [c, \varphi]^T$， Assume the prior distribution of $\boldsymbol{X}$ is [14].

$$\boldsymbol{X} \sim N(\boldsymbol{\mu_{c,\varphi}}, \boldsymbol{\phi_{c,\varphi}}) = N\left(\begin{bmatrix}20\\25\end{bmatrix}, \begin{bmatrix}9 & 4.5\\4.5 & 25\end{bmatrix}\right) \tag{1}$$

Because of the influence of measurement technology test environment and other factors, the actually measured value will have errors. Assuming that it is normally distributed near $\boldsymbol{\mu}_{c,\varphi}$, the prior distribution probability of $\boldsymbol{\mu}_{c,\varphi}$ can be obtained as:

$$\boldsymbol{\mu'}_{\boldsymbol{c,\varphi}} \sim N(\boldsymbol{\mu_0}, \boldsymbol{\phi_0}) = N\left(\begin{bmatrix}20\\25\end{bmatrix}, \begin{bmatrix}1 & 0\\0 & 4\end{bmatrix}\right) \tag{2}$$

Where $\boldsymbol{\phi_0}$ is the covariance matrix of $\boldsymbol{\mu_{c,\varphi}}$, which is used to characterize the deviation between the measured value and the true value in the field test. The value of $\boldsymbol{\phi_0}$ is generally calibrated by a large amount of measured data and we assumed it as $\boldsymbol{\phi_0} = \begin{bmatrix}1 & 0\\0 & 4\end{bmatrix}$.

Based on site investigation, 8 points of cohesion and friction angle are collected shown in Table 1. The like hood function is [14-15]

$$L(\boldsymbol{\mu_{c,\varphi}}) \propto N(\overline{\boldsymbol{X}}, \tfrac{1}{n}\boldsymbol{\phi_{c,\varphi}}) \tag{3}$$

The posterior distribution of $\boldsymbol{\mu_{c,\varphi}}$ is

$$\boldsymbol{\mu_{c,\varphi}}'' \sim N(\boldsymbol{\mu_N}, \boldsymbol{\phi_N}) \tag{4}$$

$$\boldsymbol{\phi_N} = (\begin{bmatrix}1 & 0\\0 & 4\end{bmatrix}^{-1} + 8\begin{bmatrix}9 & 4.5\\4.5 & 9\end{bmatrix}^{-1})^{-1} = \begin{bmatrix}0.519 & 0.152\\0.152 & 1.706\end{bmatrix}$$

$$\boldsymbol{\mu_N} = \boldsymbol{\phi_N}\left(n\boldsymbol{\phi_{c,\varphi}}^{-1}\overline{\boldsymbol{X}} + \boldsymbol{\phi_0}^{-1}\boldsymbol{\mu_0}\right) \tag{5}$$

$$\boldsymbol{\mu_N} = \begin{bmatrix} 0.519 & 0.152 \\ 0.152 & 1.706 \end{bmatrix} (8\begin{bmatrix} 9 & 4.5 \\ 4.5 & 25 \end{bmatrix}^{-1} \begin{bmatrix} 19.788 \\ 25.213 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 20 \\ 25 \end{bmatrix}) = \begin{bmatrix} 19.890 \\ 25.154 \end{bmatrix}$$

Where $\overline{\boldsymbol{X}}$ denotes the mean vector of the measured value of c and φ. So, the posterior distribution of $\boldsymbol{X}$ is

$$\boldsymbol{X}'' \sim N\left(\boldsymbol{\mu_N}, \boldsymbol{\phi_{c,\varphi}} + \boldsymbol{\phi_N}\right) = N(\begin{bmatrix} 19.890 \\ 25.154 \end{bmatrix}, \begin{bmatrix} 9.519 & 4.652 \\ 4.652 & 25.706 \end{bmatrix} \tag{6}$$

Table 1. Measured data of cohesion and friction angle

Test No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
c (kPa)	19	19.3	20.7	19	21	20.3	19	20
φ (°)	26	26	24	27	23.7	23.3	24.7	27

Thus, the updated cohesion and friction angle follow the distributions of $c \sim N$ (19.890, 3.085), $\varphi \sim N$ (25.154, 5.070), and the correlation coefficient between cohesion and friction angle is ρ = 0.297.

2.2 The procedure of Fellenius's and simplified Bishop's method

In the Fellenius slice method [16-17], and simplified Bishop's method, we need to specify the center point of the circular slip surface, for convenience, an area is given where we are going to search for the minimal factor of safety. After a center point is given, the slip surface can be determined under Bishop's assumptions as Fig. 1 shows. In Bishop's approach, it is presumed that the interaction forces between neighbouring slices follow a collinear pattern, leading to a resultant shear force of zero between the slices. Bishop's method's moment equilibrium safety factor may be stated as

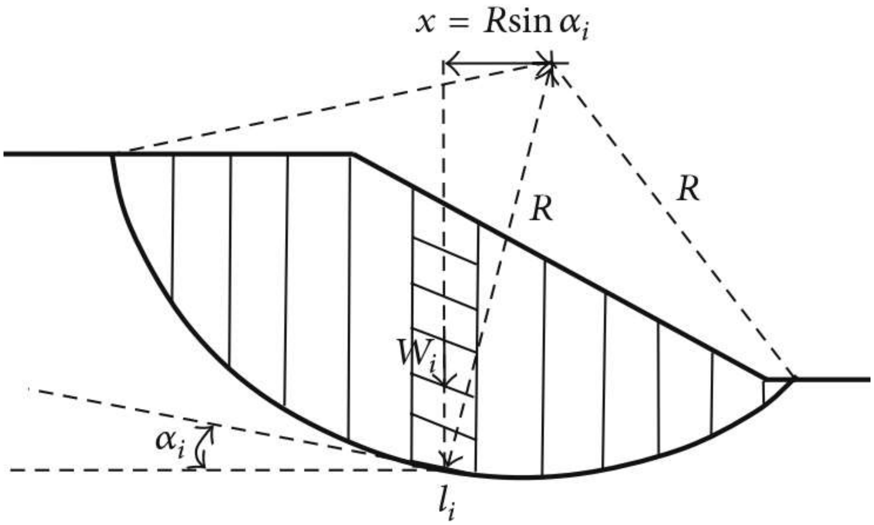

Figure 1. Slope stability analysis: method of slices (Niu, 2014)

$$F = \frac{\sum_j \frac{cl_j + (W_j - u_j l_j)\tan\varphi}{\psi_j}}{\sum_j W_j \sin\alpha_j} \tag{7}$$

Where

$$\psi_j = \cos\alpha_j + \frac{\sin\alpha_j \tan\varphi}{F} \tag{8}$$

Where j signifies the index assigned to each slice, c represents the effective cohesion, φ stands for the effective internal angle of friction, l corresponds to the width of individual slices, W indicates the weight carried by each slice, and u denotes the water pressure at the base of these slices. The solution for F requires an iterative approach due to the presence of the factor of safety on both the left and right sides of the equation.

The Swedish slip circle method, also known as Fellenius's method, operates under the assumption that the friction angle of the soil or rock is negligible ($\tau = c'$). This implies that when the friction angle is treated as zero, the effective stress component becomes zero as well. Consequently, this leads to the shear strength being equated with the cohesion parameter specific to the soil in question. The Swedish slip circle approach involves considering a circular failure boundary and examining stress and strength factors through circular geometry and static analysis. It involves comparing the moment generated by a slope's internal driving forces with the moment created by the forces that counteract slope failure. When the counteracting forces exceed the driving forces, the slope is considered to be in a stable condition according to this method.

$$F = \frac{\sum_j c\, l_j + \left(W_j - u_j l_j\right)\cos\alpha_j \tan\varphi}{\sum_j W_j \sin\alpha_j} \tag{9}$$

2.3 Methods to search for the critical slip surface

Fellenius' approach for identifying the crucial failure boundary [5] is outlined as follows: In scenarios where the internal friction angle of the soil, denoted as φ, equals zero, a two-dimensional critical failure boundary that traverses the base of the slope point A can be established using the guidelines provided in Fig.2 and Table 2. Within Fig. 2, the center E of the critical failure boundary circle can be ascertained through the employment of angles $\beta 1$ and $\beta 2$, which are themselves determined by the slope angle α as outlined in Table 2. The center of the critical failure surface circle may potentially lie along the extension line of segment DE. Numerous points on segment DE can be examined as potential centers for the critical failure surface circle, such as O_1 and O_4. Draw a line FG perpendicular to the line DE via the point Ox if it turns out that point Ox on the line DE provides the minimal slope safety factor. Then, you may test a variety of sites along the line FG that are candidates for the critical failure surface circle center, including O'_1, O'_2, O'_3, and O'_4. The ultimate most minimal safety factor of the examined slope is shown by a point on the line FG that provides the least slope safety factor. The 36-degree approach is another quick way to identify the crucial slip surface. In this method, the center point is assumed to be located at the line of BE, and β_2 is assumed to be equal to 36 degrees.

Niu et al. [18], introduced the Genetic-Traversal Random Search Method, aimed at identifying the critical slip surface. Points A, B, and C in Fig. 3 illustrate the probable failure circle, which was influenced by the genetic algorithm. Thus, the possible failure circle may be represented by the three parameters a, b, and c. This enables the utilization of optimization algorithms such as the Genetic Algorithm, Particle Swarm Optimization, Simulated Annealing Algorithm, Trust Region Reflective, Active Set, Interior Point, and SQP, among others, to minimize the factor of safety.

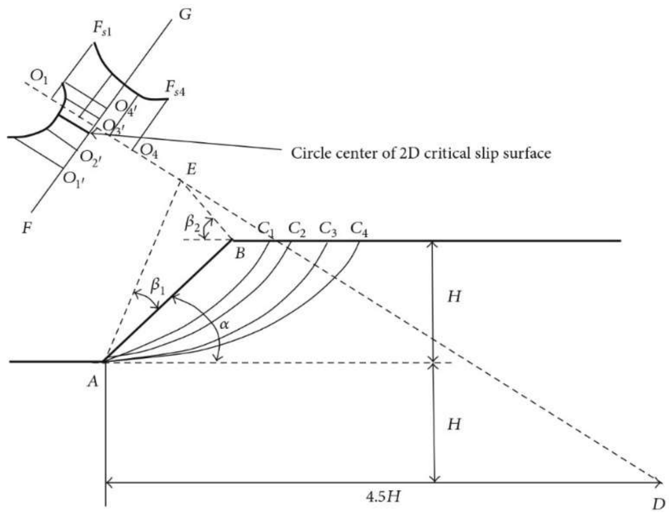

Figure 2. Determination of potential failure surfaces (Niu, 2014)

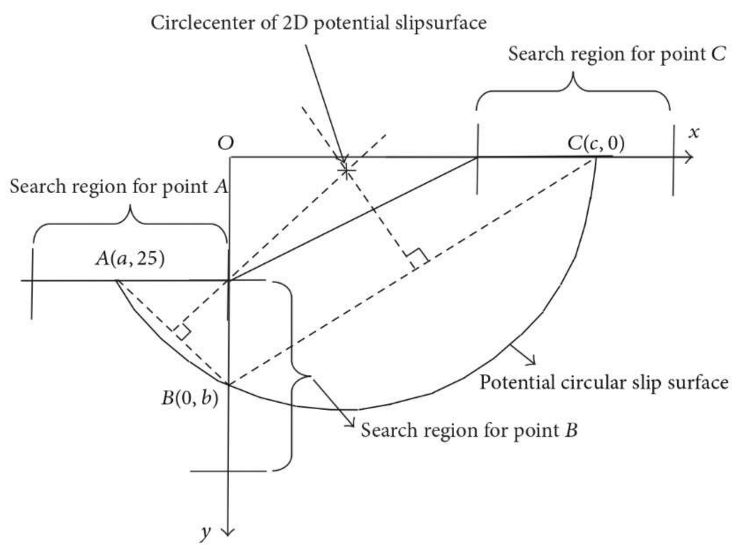

Figure 3. Potential slip surface determined and represented with points A, B, and C [18]

The results obtained by different methods are shown in Fig.4. We can conclude that the difference between the results obtained by optimization method [9], 4.5 H method and 36 degree method [10] is very small. To see the difference clearly, we can use the regression analysis to obtain the expression of a factor of safety obtained by these 3 methods, the results are.

$$\mathrm{FOS}_{4.5\mathrm{H}} = -0.0069 + 0.0542c + 1.4103\tan\varphi \quad (10)$$

$$\mathrm{FOS}_{36\mathrm{dg}} = -0.0123 + 0.0542c + 1.4366\tan\varphi \quad (11)$$

$$\mathrm{FOS}_{\mathrm{optm}} = -0.0134 + 0.0478c + 1.6446\tan\varphi \quad (12)$$

Table 2. Determination of β1 and β2 with slope angle α

Slope angle α	Slope ratio 1: m	β1	β2
60°	01:0.58	29°	40°
45°	01: 1.0	28°	37°
33°41′	01: 1.5	26°	35°
26°34′	01: 2.0	25°	35°
18°26′	01: 3.0	26°	35°
14°02′	01: 4.0	25°	36°
11°19′	01: 5.0	25°	39°

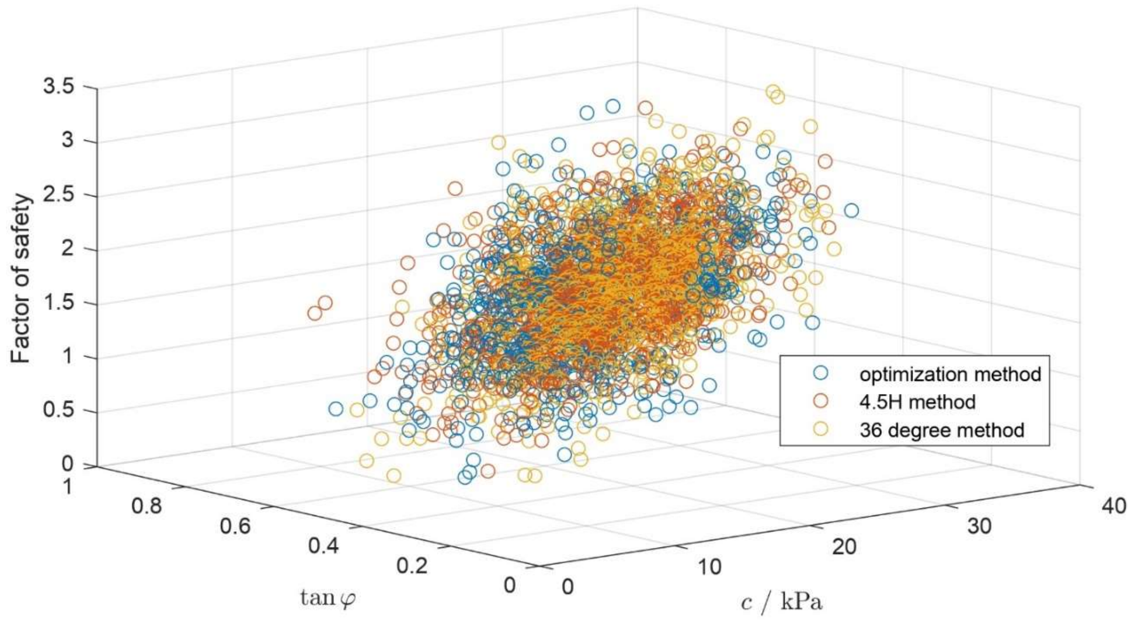

Figure 4. Scatter plot of the factor of safety obtained by different methods.

The values of R^2 are 0.9995, 0.9995, and 0.9822, respectively. By generating random samples of cohesion and friction angle, the results showed that there are about 78% of points of a factor of safety obtained by the optimization method are less than results obtained by the other two methods, showing that the optimization method is more powerful than the other two methods. However, the results obtained by these three methods are so close, that the average difference of results is less than 0.04, indicating that all the three methods are reliable. For convenience, we will use the 4.5H method to conduct the reliability analysis.

2.4 Methods of reliability analysis

The following methods of reliability analysis are used for this research. Kubica, J., Ahmed, B., et al. used some of these methods for the dynamic reliability calculation of random structures [19].

- Taylor Series Expansion Method.
- Rosenblueth Method.
- Advanced First Order Reliability Method.
- Iterative Response Surface Method.
- Monte Carlo Simulation.

Monte Carlo validated simulations and all the other methods have been used by plenty of research across the globe in almost every field of life like flow-thermal-solid coupling analysis on turbine [20], modular sequence-enforcing fault tree model [21], vehicle driving cycles [22], and on RC beams [23].

3 RESULTS

3.1 Reliability index and probability of failure

The results are shown in Table 3, in which the number of MCS samples is 10000. When calculated by Bishop's Method, the probability of slope instability is about 4.6%, for comparison, when calculated by Fellenius's method, the probability of slope instability is about 16.5%. The reason is that modified Fellenius's method ignores the forces between the slices, causing some constraints to be omitted, thus it tends to underestimate the factor of safety, while simplified Bishop's method tends to overestimate the factor of safety [24]. Also, according to data from practical construction sites, Bishop's method is a better fit for reality [25-26].

Table 3. Probability of failure of five reliability methods based on bishops and Fellenius's method

Method	Simplified Bishop's Method		Fellenius's Method	
	Reliability Index	Probability of failure	Reliability Index	Probability of failure
Taylor	1.6453	0.0500	0.9431	0.1728
Rosenblueth	1.6582	0.0486	0.9613	0.1682
AFORM	1.6816	0.0463	0.9758	0.1646
IRSM	1.6816	0.0463	0.9760	0.1645
MCS	1.6912	0.0454 （0.0418~0.0490）	0.9613	0.1682 (0.1620~0.1744)

3.2 The effect of slope angle on the factor of safety and probability of failure

As mentioned in several other articles [25-26], the factor of safety and probability of failure of the slope is highly correlated with the shape of the slope itself, mostly with the angle of the slope. Also, the factor of safety of some methods might be more sensitive to the slope angle than others. To analyze the effect of slope angle on the factor of safety, several different sets of slope angles are chosen for calculation, and the results are shown in Fig. 5. As we can see from Fig. 5, the probability of failure is strongly corrected with the quadratic function of the tangent of the slope angle, which indicates that as the slope angle increases, the probability of failure increases faster and faster. This may be due to the fact that, as the slope angle increases, the forces in between slices play an increasing part in maintaining the slope's stability, and thus cannot be omitted. The results also show that for steeper slopes, the results of Fellenius's method stray farther from reality, and thus are far less recommended than Bishop.

One thing to be noted is that, as the probability of failure (pf) of Fellenius's method is significantly greater than Bishop's method, and the calculation of reliability index (β) which directly connects to the probability of failure comes in the form of a square root, it's important to determine the positive or negative sign of β, especially in Fellenius's method. The solution is to substitute the mean point of (c, φ) into the factor of the safety function to see if FOS > 1. If FOS > 1, the slope is relatively stable and will not collapse under the average condition, thus β should be positive. However, if FOS < 1, it means that the slope itself is unstable, and β should be negative. In this case, the design parameters of the slope should be changed to avoid collapsing.

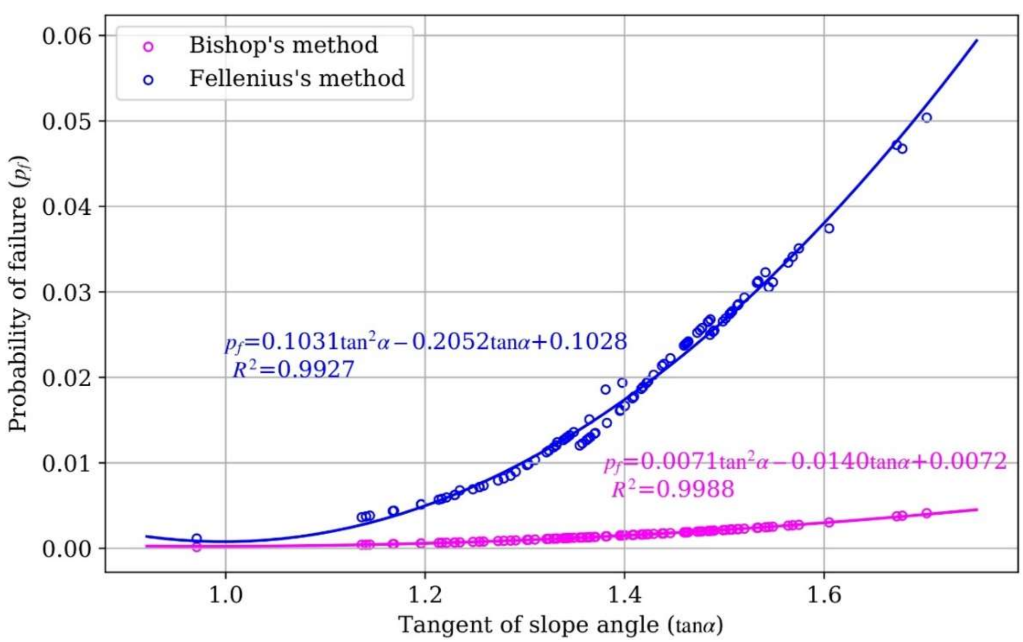

Figure 5. Scatter plot of the probability of failure versus the tangent of slope angle (tan α)

3.3 The effect of correlation between cohesion and friction angle

The correlation between c and φ is generally represented by ρ (Pearson linear correlation coefficient). Most scholars [21-23] believe that the correlation between c and φ is negative, and φ is between −0.8 and −0.3. However, some scholars believe that there is a positive correlation between c and φ, and ρ is between 0.2 and 0.9 [26-27]. Change the value of the ρ, and use Bishop's method, the probability of failure is calculated by five reliability methods as shown in Fig. 6. It can be found that as the ρ increases, the probability of failure gradually increases.

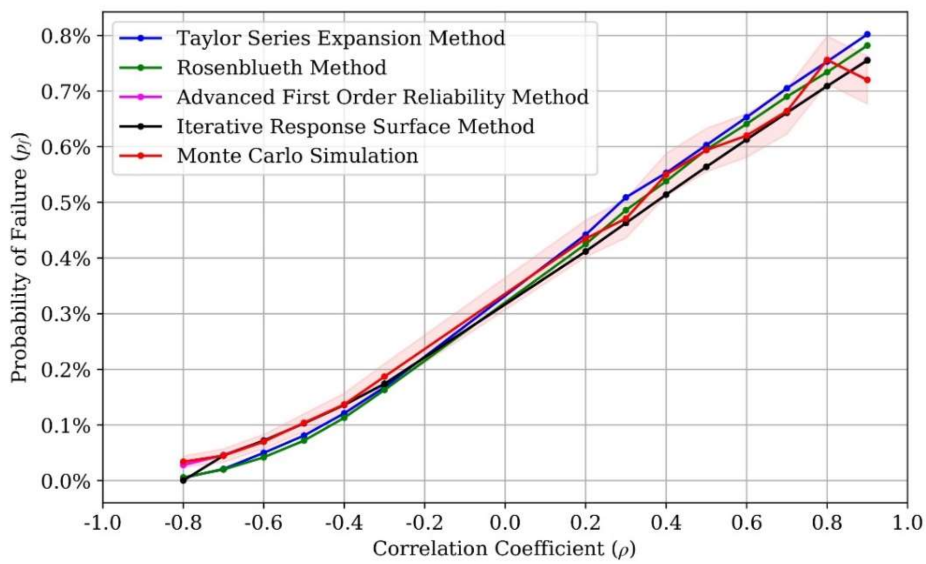

Figure 6. Probability of failure of different ρ based on Simplified Bishop's Method

The regression expression of a factor of safety shows in Eq. (10a), that a factor of safety is a linear function of c and tan φ, and the mean and standard deviation of a factor of safety can be calculated using Eq. (11a) and (11b). The probability of failure is equal to $1 - f\ (\beta)$, and the β can be expressed as Eq.(11c), so when the value of $\rho_{c,\ \tan\ \varphi}$ increases, the probability of failure will increase, too. Because φ follows the normal distribution, tan φ is also a similar normal distribution, which means $\rho \propto \rho_{c,\ \tan\ \varphi}$, and the probability of failure will increase while ρ grows.

$$\mu_{FOS} = \beta_0 + \beta_1 \mu_c + \beta_2 \mu_{tan\varphi} \tag{13}$$

$$\sigma_{FOS} = \sqrt{(\beta_1 \sigma_c)^2 + (\beta_2 \sigma_{tan\varphi})^2 + 2\,\beta_1\ \beta_2\, \rho_{c,tan\varphi} \sigma_c \sigma_{tan\varphi}} \tag{14}$$

$$\beta = (\mu_{FOS} - 1) / \sigma_{FOS} \tag{15}$$

4 DISCUSSION

The regression expression of a factor of safety is a non-linear relationship, so the results obtained by the Taylor method and Rosenblueth method are rough estimates, and the results obtained by the IRSM method or AFORM are more accurate. The number of MCS's sample is only 10,000, causing the results fluctuate obviously. However, an increase in the number of samples needs more time for calculation. Using the importance sampling of the MCS method to obtain more accurate results can be considered.

5 CONCLUSION

1. Compared with Bishop's method, the factor of safety calculated by Fellenius's method is more conservative.
2. When c and φ are positively correlated, as the correlation increases, the reliability value decreases. When c and φ are negatively correlated, as the correlation increases, the value of reliability increases, and the calculation of the probability of failure is conservative.
3. The results obtained by iterative RSM or AFORM are more accurate, and the result of MCS fluctuates greatly due to the number of samples is small.
4. For steep slopes, we recommend using the simplified Bishop's method instead of Fellenius's method to analyze the stability of the slope.

6 ACKNOWLEDGMENTS

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7 REFERENCES

1. Zolkepli MF, Ishak MF, Zaini MSI. Slope stability analysis using modified Fellenius's and Bishop's method. IOP Conf Ser Mater Sci Eng. 2019;527(1).
2. Ullah S, Khan MU, Rehman G. a Brief Review of the Slope Stability Analysis Methods. Geol Behav. 2020;4(2):73– 7.
3. Harabinová S, Kotrasová K, Kormaníková E, Hegedüsová I. Analysis of Slope Stability. Civ Environ Eng. 2021;17(1):192–9.
4. He Y, Li Z, Wang W, Yuan R, Zhao X, Nikitas N. Slope stability analysis considering the strength anisotropy of c-φ soil. Sci Rep. 2022;12(1):1–12.

5. W. Fellenius. Calculation of the stability of earth dams. In Proceeding of the 2nd congress on large dams. U.S. Government Printing Office, 1936, DOI: 10.3208/sandf.42.2_93.
6. Alan W. Bishop. The use of the slip circle in the stability analysis of slopes. Géotechnique, 5, 3 1955.ISSN 0016-8505, DOI: 10.1680/geot.1955.5.1.7.
7. Jampani H, Bhupathi N. Stability Analysis of Slope with Different Soil Types and Its Stabilization Techniques. Indian Geotech Conf 2017 GeoNest. 2017;(14-16 December, 2017):14–7.
8. Strenk, P., Wartman, J., Grubb, D., Humphrey, D., & Natale, M. (2007). Variability and Scale-Dependency of Tire-Derived Aggregate. Journal of Materials in Civil Engineering, 19, 233-241.
9. M. R. Taha, M. Khajehzadeh, and A. El-Shafie, "Slope stability assessment using optimization techniques: an overview," Electronic Journal of Geotechnical Engineering, vol. 15, pp. 1901–1915, 2010.
10. Kim, M., Kim, H. S., & Chung, C. K. (2020). A three-dimensional geotechnical spatial modeling method for borehole dataset using optimization of geostatistical approaches. KSCE Journal of Civil Engineering, 24, 778-793.
11. Caoyuan Niu, Lehu Wang, and Xiaoliang Xu. Study on impact from statistical characteristics of soil mass shear strength parameters on slope reliability. Water Resources and Hydropower Engineering, 48(12):195–198, 2017.
12. Lihong Chen, Zuyu Chen, and Jinmei Liu. Probability distribution of soil strength. Rock and Soil Mechanics, 26(1):37–40, 2005.
13. Dazhao Gao. Principles of Soil Mechanics Reliability. China Construction Industry Press, 1989.
14. Christopher M Bishop. Pattern recognition and machine learning. springer, 2006.
15. Andrew Gelman, John B Carlin, Hal S Stern, David B Dunson, Aki Vehtari, and Donald B Rubin. Bayesian data analysis. CRC press, 2013, DOI: 10.1201/b16018.
16. Alfat S, Zulmasri LOM, Asfar S, Rianse MS, Eso R. Slope stability analysis through variational slope geometry using Fellenius Method. J Phys Conf Ser. 2019;1242(1).
17. A.R.Arora D. (ARORA) SOIL MECHANICS AND FOUNDATION ENGINEERING.pdf. Particle Size Analysis. 2003. p. 903.
18. Wen-jie Niu. Determination of Slope Safety Factor with Analytical Solution and Searching Critical Slip Surface with Genetic-Traversal Random Method. The Scientific World Journal, 2014, 2014. ISSN 2356-6140, DOI: 10.1155/2014/950531.
19. Kubica, J., Ahmed, B., Muhammad, A., Aslam, M. U. (2024). Dynamic reliability calculation of random structures by conditional probability method. Eksploatacja i Niezawodność – Maintenance and Reliability. https://doi.org/10.17531/ein/181133

20. Qian, W., Zeng, X., Huang, S., Yin, X. (2023). Reliability analysis of multi-site damage with failure dependency of the turbine based on flow-thermal-solid coupling analysis and the Monte Carlo validated simulations. Eksploatacja i Niezawodność – Maintenance and Reliability, 25(3). https://doi.org/10.17531/ein/168771
21. Schober, P., Boer, C., & Schwarte, L. (2018). Correlation Coefficients: Appropriate Use and Interpretation. Anesthesia & Analgesia, 126, 1763–1768.
22. Franke, G. (2010). Product Moment Correlation. https://doi.org/10.1002/9781444316568.WIEM02068.
23. Szmidt, E., & Kacprzyk, J. (2010). Correlation of Intuitionistic Fuzzy Sets., 169-177. https://doi.org/10.1007/978-3-642-14049-5_18.
24. Takashi Tsuchida, A.M.R.G. Athapaththu. Practical slip circle method of slices for calculation of bearing capacity factors [J]. Soils and Foundations. 2014, 54(6): 1127-1144, DOI: 10.1016/j.sandf.2014.11.008.
25. Zhaohui Wang, Shengxie Xiao, and Wenfang Liu. Comparison of analytical methods for slope stability. Journal of Chongqing Jiaotong University, 24(06):101–105, 2005.
26. Schober, P., Boer, C., & Schwarte, L. (2018). Correlation Coefficients: Appropriate Use and Interpretation. Anesthesia & Analgesia, 126, 1763–1768.
27. Adler, J., & Parmryd, I. (2010). Quantifying colocalization by correlation: The Pearson correlation coefficient is superior to the Mander's overlap coefficient. Cytometry Part A, 77A.