Topological Attributes of Caffeine [$C_8H_{10}N_4O_2$] and Subdivided Aztec Diamond Networks Via Revan Indices

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Abstract- In this article, we have discussed some topological attributes of two important types of the chemical compounds [$C_8H_{10}N_4O_2$] and subdivided version of aztec diamond ($S.Aztec.D$) networks. We have described the Revan indices and polynomials of these networks by assuming that graphs are planar, finite, and simple and connected with multiple edges. We have calculated different Revan polynomials of these networks such as first Revan, first hyper Revan, modified first Revan, sum connectivity, second Revan, second hyper Revan, modified second Revan and the product connectivity, forgotten, harmonic, symmetric division and inverse sum Revan polynomials. These indices have remarkable applications in fields of science, engineering and technology. We have also discussed the graphical behaviors of these networks which is very effective to know how the Revan polynomials change its behavior and position with the increase in network size and structure.

Keywords- Revan Polynomials, Revan Indices, Caffeine, Aztec Diamond; Molecular Structure

I. INTRODUCTION

Basically, Caffeine [$C_8H_{10}N_4O_2$] is a natural chemical compound which is possess in different plants. In field of chemistry, it is a bitter tasting, white un-smiling powder and soluble in water at room temperature. [$C_8H_{10}N_4O_2$] is also classified as an achiral (reverse of chiral) molecule. [$C_8H_{10}N_4O_2$] is found in he the following items: coffee, cola, tea, year bar mate and many other products. [$C_8H_{10}N_4O_2$] is used in so many drugs to enhance the capacity of our brain and other nervous system. Use of ne [$C_8H_{10}N_4O_2$] refreshes our central nervous parts, heart muscles and also controls the blood pressure. Sometimes [$C_8H_{10}N_4O_2$] raises the blood pressure but it has no effects to those which ch use it regularly. [$C_8H_{10}N_4O_2$] is usually use for mental alertness, headache, memory and athletic achievements. Moreover, it is used for asthma, gallbladder disease, ADHD, depression, low blood pressure and a lot of other situations. Study of graph theory has been enhanced to chemical compounds and networks which give us the idea of chemical graph theory. Thus, with the combination of chemistry and graph theory a new branch called chemical graph theory ($C.G.T$) was invented to characterize molecular structures [1]. Basically, a molecular structure is a graph in which vertices replace with atoms and edges with bonds. Molecular structures have a constant attention on C.G.T. which indeed helps us to prepare few drugs for different diseases [2].

Our technique is to take molecular structures and then modeling of the chemical compound by, conversion it into planar graph and then analyzing it by using topological invariants [3-5]. Topological invariants have many structural properties and wide range of uses in structural chemistry. 'A topological index helps us to determine the chemical, physical, biological properties of a chemical compound'. 'It has also, lot of uses in field of chemistry, biology, information, and quantitative structure-property relationships' etc. [6-8]. In this paper, many invariants of molecular graph of the chemical compound [$C_8H_{10}N_4O_2$] and a subdivided version of $S.Aztec.D.$ networks are determined by using Revan indices. The molecular graphs of the chemical compound [$C_8H_{10}N_4O_2$] and $S.Aztec.D.$ networks are shown in Fig. 1, Fig. 2, Fig. 3, Fig. 4 and Fig. 5 respectively.

The first and second Revan indices [9] are defined as

$$R_1(V) = \sum_{pq\in E(V)} [r_V(p) + r_V(q)] \ \& \ \ R_2(V) = \sum_{pq\in E(V)} [r_V(p) r_V(q)].$$

The first and second Revan polynomials [10] are defined as

$$R_1^{\alpha}(V,x) = \sum_{pq\in E(V)} x^{[r_V(p)+r_V(q)]^{\alpha}} \ \& \ R_2^{\alpha}(V,x) = \sum_{pq\in E(V)} x^{[r_V(p)\, r_V(q)]^{\alpha}}.$$

First hyper Revan index and polynomial [11] are defined as

$$HR_1(V) = \sum_{pq\in E(V)} [r_V(p)+r_V(q)]^2 \ \& \ HR_1(V,x) = \sum_{pq\in E(V)} x^{[r_V(p)+r_V(q)]^2}.$$

Modified first Revan index and polynomial [12] are defined as

$$R_1^{m}(V) = \sum_{pq\in E(V)} \frac{1}{[r_V(p)+r_V(q)]} \ \& \ R_1^{m}(V,x) = \sum_{pq\in E(V)} \frac{1}{x^{[r_V(p)+r_V(q)]}}.$$

Modified second Revan index and polynomial [12] are defined as

$$R_2^{m}(V) = \sum_{pq\in E(V)} \frac{1}{[r_V(p)r_V(q)]} \ \& \ R_2^{m}(V,x) = \sum_{pq\in E(V)} \frac{1}{x^{[r_V(p)\, r_V(q)]}}.$$

Second hyper Revan index and polynomial [11] are defined as

$$HR_2(V) = \sum_{pq\in E(V)} [r_V(p)r_V(q)]^2 \ \& \ HR_2(V,x) = \sum_{pq\in E(V)} x^{[r_V(p)r_V(q)]^2}.$$

Sum Revan index and polynomial [13] are defined as

$$SR(V) = \sum_{pq\in E(V)} \frac{1}{\sqrt{r_V(p)+r_V(q)}} \ \& \ SR(V,x) = \sum_{pq\in E(V)} \frac{1}{x^{\sqrt{[r_V(p)+r_V(q)]}}}.$$

Product Revan index and polynomial [14] are defined as

$$PR(V) = \sum_{pq\in E(V)} \frac{1}{\sqrt{r_V(p)r_V(q)}} \ \& \ PR(V,x) = \sum_{pq\in E(V)} \frac{1}{x^{\sqrt{[r_V(p)\, r_V(q)]}}}.$$

Forgotton Revan index and polynomial [15] are defined as

$$FR(V) = \sum_{pq\in E(V)} [r_V(p)^2\, r_V(q)^2] \ \& \ FR(V,x) = \sum_{pq\in E(V)} x^{[r_V(p)^2\, r_V(q)^2]}.$$

Harmonic Revan index and polynomial [10] are defined as

$$HR(V) = \sum_{pq\in E(V)} \frac{2}{[r_V(p)+r_V(q)]} \ \& \ HR(V,x) = \sum_{pq\in E(V)} \frac{2}{x^{[r_V(p)+r_V(q)]}}.$$

Symmetric division Revan index and polynomial [10] are defined as

$$SDR(V) = \sum_{pq\in E(V)} \left[\frac{r_V(p)}{r_V(q)} + \frac{r_V(q)}{r_V(p)}\right] \ \& \ SDR(V,x) = \sum_{pq\in E(V)} x^{\left[\frac{r_V(p)}{r_V(q)} + \frac{r_V(q)}{r_V(p)}\right]}.$$

Inverse sum Revan index and polynomial [10] are defined as

$$IR(V) = \sum_{pq\in E(V)} \left[\frac{r_V(p)r_V(q)}{r_V(p)+r_V(q)}\right] \ \& \ IR(V,x) = \sum_{pq\in E(V)} x^{\left[\frac{r_V(p)r_V(q)}{r_V(p)+r_V(q)}\right]}.$$

II. MATERIAL AND METHODS

Initially, we will find the transformed pattern of molecular structures of [C8H10N4O2] and subdivided aztecd diamond ($S.Aztec.D$) named as V and U.Moreover, we define Revan polynomials and Revan indices of these netwoks by assuming that the graphs are planar, finite, simple and connected with multiple edges. Suppose, $\Delta(V)$ and $\delta(V)$ denote maximum and minimum degree of the graph V among the vertices then Revan vertex is determined as $r_V(p) = \Delta(V) + \delta(V) - d_V(p)$. Further, we define edg partition depending upon degree-based vertices and then calculate the polynomials associated with networks.

III. RESULTS AND DISCUSSION

3.1 Revan Polynomials for [$C_8H_{10}N_4O_2$]

In this section, we determine some Revan polynomials for the molecular structure of **[**$C_8H_{10}N_4O_2$**]**.

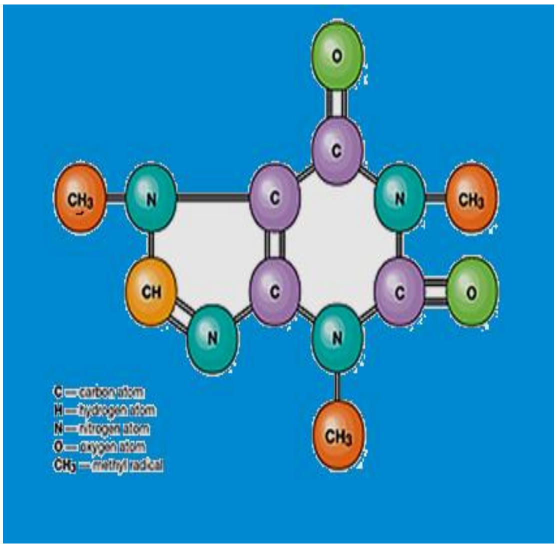

Fig.1. Molecular structure of $[C_8H_{10}N_4O_2]$

From the Fig.1 we determine the following table.

Table 1: Edge Partitions of $[C_8H_{10}N_4O_2]$

Types of edges$((r_V(p), r_V(q))$	No. of Edges
$(1,3)$	5
$(2,3)$	2
$(3,3)$	7
$(2,2)$	1

Since the vertices of the graph $V$are of degree $1,2$ or 3. So $\Delta(V) = 3$ *and* $\delta(V) = 1$. Thus, $r_V(p) = 4 - d_V(p)$. Similarly, $r_V(q) = 4 - d_V(q)$.
On these observations we compute the following table.

Table 2: Revan Edge Partition of $[C_8H_{10}N_4O_2]$

Types of Edges$((r_V(p), r_V(q))$	No. of Revan Edges ($|RE|$)
$(3,1)$	5
$(2,1)$	2
$(1,1)$	7
$(2,2)$	1

3.1.1 Theorem
The general first Revan Polynomial for the molecular structure of $[C_8H_{10}N_4O_2]$ is
$R_1{}^{\alpha}(V,x) = 6x^{4^{\alpha}} + 2x^{5^{\alpha}} + 7x^{6^{\alpha}}$.
Proof: Using definition of Revan Polynomial and Table2, we have

$$R_1{}^{\alpha}(V,x) = \sum_{pq\in E(V)} x^{[r_V(p)+r_V(q)]^{\alpha}}$$

$$= |RE_{1,3}|x^{[r_V(p)+r_V(q)]^{\alpha}} + |RE_{2,3}|x^{[r_V(p)+r_V(q)]^{\alpha}} + |RE_{3,3}|x^{[r_V(p)+r_V(q)]^{\alpha}} + |RE_{2,2}|x^{[r_V(p)+r_V(q)]^{\alpha}}$$

$$= 5x^{(1+3)^{\alpha}} + 2x^{(2+3)^{\alpha}} + 7x^{(3+3)^{\alpha}} + x^{(2+2)^{\alpha}}$$
$$= 5x^{4^{\alpha}} + 2x^{5^{\alpha}} + 7x^{6^{\alpha}} + x^{4^{\alpha}}$$
$$= 6x^{4^{\alpha}} + 2x^{5^{\alpha}} + 7x^{6^{\alpha}}.$$

On the basis of this result, we get the following consequences in

3.1.2 Corollary
For the molecular structure of $[C_8H_{10}N_4O_2]$ we compute the following Revan polynomials
(1) $R_1(V,x) = 6x^4 + 2x^5 + 7x^6$
(2) $HR_1(V,x) = 6x^{16} + 2x^{25} + 7x^{36}$
(3) $R_1{}^m(V,x) = 6x^{\frac{1}{4}} + 2x^{\frac{1}{5}} + 7x^{\frac{1}{6}}$
(4) $SR_1(V,x) = 6x^{\frac{1}{\sqrt{4}}} + 2x^{\frac{1}{\sqrt{5}}} + 7x^{\frac{1}{\sqrt{6}}}$.

Proof:
(1) By putting $\alpha = 1$ in theorem 3.1.1 we conclude that
$R_1(V,x) = 5x^4 + 2x^5 + 7x^6 + x^4$
$= 6x^4 + 2x^5 + 7x^6$.
(2) Substituting $\alpha = 2$ in theorem 3.1.1 we get
$HR_1(V,x) = 5x^{16} + 2x^{25} + 7x^{36} + x^{16}$
$= 6x^{16} + 2x^{25} + 7x^{36}$.
(3) Using theorem 3.1.1 we have
$R_1{}^m(V,x) = 5x^{\frac{1}{4}} + 2x^{\frac{1}{5}} + 7x^{\frac{1}{6}} + x^{\frac{1}{4}}$
` $= 6x^{\frac{1}{4}} + 2x^{\frac{1}{5}} + 7x^{\frac{1}{6}}$.
(4) Using theorem 3.1.1 and after simplification
$SR_1(V,x) = 6x^{\frac{1}{\sqrt{4}}} + 2x^{\frac{1}{\sqrt{5}}} + 7x^{\frac{1}{\sqrt{6}}}$.

3.1.3 Theorem
The general second Revan polynomial for the molecular structure of $[C_8H_{10}N_4O_2]$ is
$R_2{}^{\alpha}(V,x) = 5x^{3^{\alpha}} + 2x^{6^{\alpha}} + 7x^{9^{\alpha}} + x^{4^{\alpha}}$.
Proof: Using definition of $R_2{}^{\alpha}(V,x)$ and Table 2, we have

$$R_2{}^{\alpha}(V,x) = \sum_{pq\in E(V)} x^{[r_V(p)r_V(q)]^{\alpha}}$$

$$= |RE_{1,3}|x^{[r_V(p)r_V(q)]^{\alpha}} + |RE_{2,3}|x^{[r_V(p)r_V(q)]^{\alpha}} + |RE_{3,3}|x^{[r_V(p)r_V(q)]^{\alpha}} + |RE_{2,2}|x^{[r_V(p)r_V(q)]^{\alpha}}$$

$$= 5x^{(1\times3)^{\alpha}} + 2x^{(2\times3)^{\alpha}} + 7x^{(3\times3)^{\alpha}} + x^{(2\times2)^{\alpha}}$$
$$= 5x^{3^{\alpha}} + 2x^{6^{\alpha}} + 7x^{9^{\alpha}} + x^{4^{\alpha}}.$$

From theorem 3.1.3, we conclude the following results.

3.1.4 Corollary
For the molecular structure of $[C_8H_{10}N_4O_2]$ we compute following Revan polynomials.
(1) $R_2(V,x) = 5x^3 + 2x^6 + 7x^9 + x^4$
(2) $HR_2(V,x) = 5x^9 + 2x^{36} + 7x^{81} + x^{16}$
(3) $R_2{}^m(V,x) = 5x^{\frac{1}{3}} + 2x^{\frac{1}{6}} + 7x^{\frac{1}{9}} + x^{\frac{1}{4}}$
(4) $PR_2(V,x) = 6x^{\frac{1}{\sqrt{3}}} + 2x^{\frac{1}{\sqrt{6}}} + 7x^{\frac{1}{\sqrt{9}}} + x^{\frac{1}{\sqrt{4}}}$.
Proof:
(1) By substituting $\alpha = 1$ in theorem 3.1.3 we obtain
$R_2(V,x) = 5x^3 + 2x^6 + 7x^9 + x^4$.
(2) By Putting $\alpha = 2$ in theorem 3.1.3 we get

$HR_2(V,x) = 5x^9 + 2x^{36} + 7x^{81} + x^{16}.$

(3) Using theorem 3.1.3 we have

$R_2{}^m(V,x) = 5x^{\frac{1}{3}} + 2x^{\frac{1}{6}} + 7x^{\frac{1}{9}} + x^{\frac{1}{4}}.$

(4) Using theorem 3.1.3 and after simplification

$PR_2(V,x) = 6x^{\frac{1}{\sqrt{3}}} + 2x^{\frac{1}{\sqrt{6}}} + 7x^{\frac{1}{\sqrt{9}}} + x^{\frac{1}{\sqrt{4}}}.$

3.1.5 Theorem

The forgotten, harmonic and symmetric division Revan polynomials for the molecular structure of $[C_8H_{10}N_4O_2]$ are

(1) $FR(V,x) = 5x^{10} + 2x^{13} + 7x^{18} + x^8$

(2) $HR(V,x) = 6x^{\frac{1}{2}} + 2x^{\frac{2}{5}} + 7x^{\frac{1}{3}}$

(3) $SDR(V,x) = 5x^{\frac{10}{3}} + 2x^{\frac{13}{6}} + 8x^2$

(4) $IR(V,x) = 5x^{\frac{3}{4}} + 2x^{\frac{6}{5}} + 7x^{\frac{3}{2}} + x.$

Proof: (1) Using definition of $FR(V,x)$ and Table 2, we have

$$FR(V,x) = \sum_{pq\in E(V)} x^{[r_V(p)^2 + r_V(q)^2]}$$

$$= |RE_{1,3}|x^{[r_V(p)^2+r_V(q)^2]} + |RE_{2,3}|x^{[r_V(p)^2+r_V(q)^2]} + |RE_{3,3}|x^{[r_V(p)^2+r_V(q)^2]} + |RE_{2,2}|x^{[r_V(p)^2+r_V(q)^2]}$$

$$= 5x^{(1^2+3^2)} + 2x^{(2^2+3^2)} + 7x^{(3^2+3^2)} + x^{(2^2+2^2)}$$

$$= 5x^{10} + 2x^{13} + 7x^{18} + x^8.$$

(2) Using definition of $HR(V,x)$ and Table 2, we get

$$HR(V,x) = \sum_{pq\in E(V)} \frac{2}{x^{[r_V(p)+r_V(q)]}}$$

$$= |RE_{1,3}|\frac{2}{x^{[r_V(p)+r_V(q)]}} + |RE_{2,3}|\frac{2}{x^{[r_V(p)+r_V(q)]}} + |RE_{3,3}|\frac{2}{x^{[r_V(p)+r_V(q)]}} + |RE_{2,2}|\frac{2}{x^{[r_V(p)+r_V(q)]}}$$

$$= 5\frac{2}{x^{[1+3]}} + 2\frac{2}{x^{[2+3]}} + 7\frac{2}{x^{[3+3]}} + \frac{2}{x^{[2+2]}}$$

$$= 5x^{\frac{1}{2}} + 2x^{\frac{2}{5}} + 7x^{\frac{1}{3}} + 7x^{\frac{1}{2}}$$

$$= 6x^{\frac{1}{2}} + 2x^{\frac{2}{5}} + 7x^{\frac{1}{3}}.$$

(3) Using definition of $SDR(V,x)$ and Table 2, we conclude

$$SDR(V,x) = \sum_{pq\in E(V)} x^{\left[\frac{r_V(p)}{r_V(q)} + \frac{r_V(q)}{r_V(p)}\right]}$$

$$= |RE_{1,3}|x^{\left[\frac{r_V(p)}{r_V(q)} + \frac{r_V(q)}{r_V(p)}\right]} + |RE_{2,3}|x^{\left[\frac{r_V(p)}{r_V(q)} + \frac{r_V(q)}{r_V(p)}\right]} + |RE_{3,3}|x^{\left[\frac{r_V(p)}{r_V(q)} + \frac{r_V(q)}{r_V(p)}\right]} + |RE_{2,2}|x^{\left[\frac{r_V(p)}{r_V(q)} + \frac{r_V(q)}{r_V(p)}\right]}$$

$$SDR(V,x) = 5x^{\left(\frac{1}{3}+\frac{3}{1}\right)} + 2x^{\left(\frac{2}{3}+\frac{3}{2}\right)} + 7x^{\left(\frac{3}{3}+\frac{3}{3}\right)} + x^{\left(\frac{2}{2}+\frac{2}{2}\right)}$$

$$= 5x^{\frac{10}{3}} + 2x^{\frac{13}{6}} + 7x^2 + x^2$$

$$= 5x^{\frac{10}{3}} + 2x^{\frac{13}{6}} + 8x^2.$$

(4) Using definition of $IR(V,x)$ and Table 2, we deduce

$$IR(V,x) = \sum_{pq\in E(V)} x^{\left[\frac{r_V(p)r_V(q)}{r_V(p)+r_V(q)}\right]}$$

$$= |RE_{1,3}|x^{\left[\frac{r_V(p)r_V(q)}{r_V(p)+r_V(q)}\right]} + |RE_{2,3}|x^{\left[\frac{r_V(p)r_V(q)}{r_V(p)+r_V(q)}\right]} + |RE_{3,3}|x^{\left[\frac{r_V(p)r_V(q)}{r_V(p)+r_V(q)}\right]} + |RE_{2,2}|x^{\left[\frac{r_V(p)r_V(q)}{r_V(p)+r_V(q)}\right]}$$

$$= 5x^{\left(\frac{1\times3}{1+3}\right)} + 2x^{\left(\frac{2\times3}{2+3}\right)} + 7x^{\left(\frac{3\times3}{3+3}\right)} + x^{\left(\frac{2\times2}{2+2}\right)}$$

$$= 5x^{\frac{3}{4}} + 2x^{\frac{6}{5}} + 7x^{\frac{3}{2}} + x.$$

Revan Polynomials for subdivided Aztec diamond **($S.Aztec.D$)**

In this section, we compute the mentioned Revan polynomials for the subdivided aztec diamond ($S.Aztec.D$). The molecular structure form of $S.Aztec.D.$ is shown below.

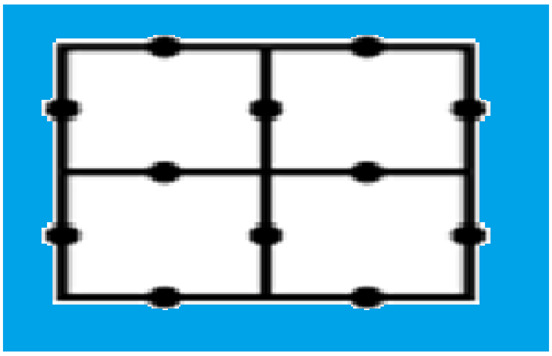

Fig. 2. $S.Aztec.D(\mathbf{1})$

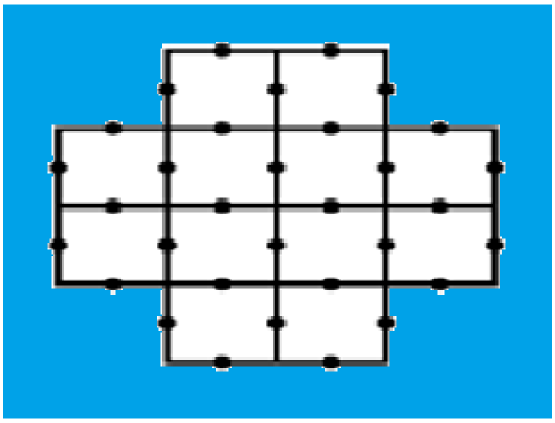

Fig. 3. $S.Aztec.D(\mathbf{2})$

From the Fig. 1 to Fig. 5 we determine the following table.

Table 3: Edge Partitions of $S.Aztec.D.$

Types of Edges $((d_U(p), d_U(q))$	No. of Edges
$(2,2)$	$8m$
$(2,3)$	12
$(2,4)$	$8m^2 + 8 - 12$

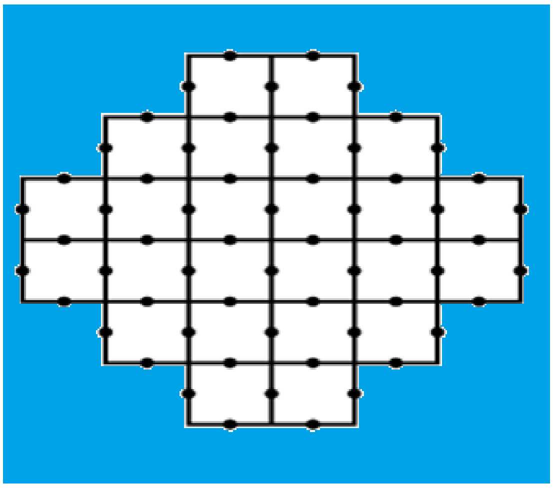

Fig. 4. $S.Aztec.D(3)$

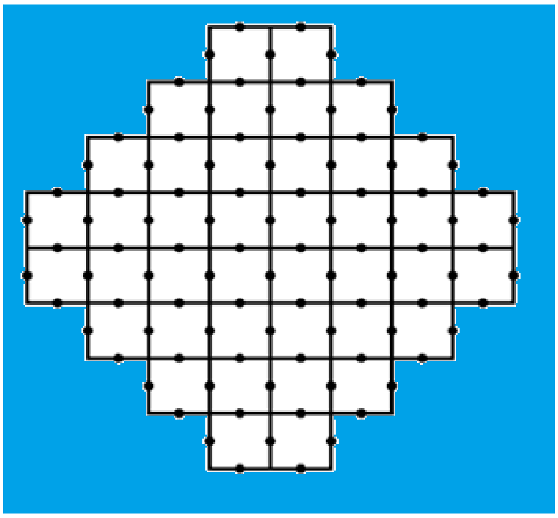

Fig. 5. $S.Aztec.D(4)$

As the vertices of the graph U are of degree 2, 3 or 4. So $\Delta(U) = 4$ and $\delta(U) = 2$. Thus, $r_U(p) = 6 - d_U(p)$. Similarly,$r_U(q) = 6 - d_U(q)$. By using these observations, we establish the following table.

Table 4: Revan Edge Partition of $S.Aztec.D$.

Types of Edges $((r_U(p), r_U(q))$	No. of Edges $(\lvert RE \rvert)$
$(4,4)$	$8m$
$(4,3)$	1 12
$(4,2)$	$8m^2 + 8 - 12$

3.2.1 Theorem: The first general Revan polynomial for the $S.Aztec.D$ is

$$R_1{}^{\alpha}(U,x) = 8mx^{8^{\alpha}} + 12x^{7^{\alpha}} + (8\,m^2 + 8m - 12)x^{6^{\alpha}}.$$

Proof: Using definition of Revan polynomial and Table 2, we have

$$R_1{}^{\alpha}(U,x) = \sum_{pq\in E(U)} x^{[r_U(p)+r_U(q)]^{\alpha}}$$

$$= |RE_{4,4}|x^{[r_U(p)+r_U(q)]^{\alpha}} + |RE_{4,3}|x^{[r_U(p)+r_U(q)]^{\alpha}} + |RE_{4,2}|x^{[r_U(p)+r_U(q)]^{\alpha}}$$

$$= 8mx^{(4+4)^{\alpha}} + 12x^{(4+3)^{\alpha}} + (8\,m^2 + 8m - 12)x^{(4+2)^{\alpha}}$$

$$= 8mx^{8^{\alpha}} + 12x^{7^{\alpha}} + (8\,m^2 + 8m - 12)x^{6^{\alpha}}.$$

From 3.2.1 theorem, we get the following consequences.

3.2.2 Corollary

The first Revan, first hyper Revan, modified first Revan and sum connectivity polynomials for the molecular structure of $S.Aztec.D.$ are

(1) $R_1(U,x) = 8mx^8 + 12x^7 + (8\,m^2 + 8m - 12)x^6$

(2) $HR_1(U,x) = 8mx^{64} + 12x^{49} + (8\,m^2 + 8m - 12)x^{36}$

(3) $R_1{}^{m}(U,x) = 8mx^{\frac{1}{8}} + 12x^{\frac{1}{7}} + (8\,m^2 + 8m - 12)x^{\frac{1}{6}}$

(4) $SR_1(U,x) = 8mx^{\frac{1}{\sqrt{8}}} + 12x^{\frac{1}{\sqrt{7}}} + (8\,m^2 + 8m - 12)x^{\frac{1}{\sqrt{6}}}.$

Proof:

(1) By taking $\alpha = 1$ in theorem 3.2.1 we get

$R_1(U,x) = 8mx^8 + 12x^7 + (8\,m^2 + 8 - 12)x^6.$

(2) Substituting $\alpha = 1$ in theorem 3.2.1 we obtain

$HR_1(U,x) = 8mx^{64} + 12x^{49} + (8\,m^2 + 8m - 12)x^{36}.$

(3) Using theorem 3.2.1 we conclude

$R_1{}^{m}(U,x) = 8mx^{\frac{1}{8}} + 12x^{\frac{1}{7}} + (8\,m^2 + 8m - 12)x^{\frac{1}{6}}.$

(4) Using theorem 3.2.1and after simplification

$SR_1(U,x) = 8mx^{\frac{1}{\sqrt{8}}} + 12x^{\frac{1}{\sqrt{7}}} + (8\,m^2 + 8m - 12)x^{\frac{1}{\sqrt{6}}}.$

3.2.3 Theorem:

The general second Revan polynomial for $S.Aztec.D.$ is

$$R_2{}^{\alpha}(U,x) = 8mx^{16^{\alpha}} + 12x^{12^{\alpha}} + (8\,m^2 + 8m - 12)x^{8^{\alpha}}.$$

Proof: By $R_2{}^{\alpha}(U,x)$ and from Table 4, we have

$$R_2{}^{\alpha}(U,x) = \sum_{pq\in E(U)} x^{[r_U(p)r_U(q)]^{\alpha}}$$

$$= |RE_{4,4}|x^{[r_U(p)r_U(q)]^{\alpha}} + |RE_{4,3}|x^{[r_U(p)r_U(q)]^{\alpha}} + |RE_{4,2}|x^{[r_U(p)r_U(q)]^{\alpha}}$$

$$= 8mx^{(4\times4)^{\alpha}} + 12x^{(4\times3)^{\alpha}} + (8\,m^2 + 8m - 12)x^{(4\times2)^{\alpha}}$$

$$= 8mx^{16^{\alpha}} + 12x^{12^{\alpha}} + (8\,m^2 + 8m - 12)x^{8^{\alpha}}.$$

Theorem 3.2.3, give us the following results in the form of corollary

3.2.4 Corollary

The following polynomials for the molecular structure of $S.Aztec.D.$ are determined as

(1) $R_2(U,x) = 8mx^{16} + 12x^{12} + (8\,m^2 + 8m - 12)x^8$

(2) $HR_2(U,x) = 8mx^{256} + 12x^{144} + (8\,m^2 + 8m - 12)x^{64}$

(3) $R_2{}^m(U,x) = 8mx^{\frac{1}{16}} + 12x^{\frac{1}{12}} + (8\,m^2 + 8m - 12)x^{\frac{1}{8}}$

(4) $PR_2(U,x) = 8mx^{\frac{1}{\sqrt{16}}} + 12x^{\frac{1}{\sqrt{12}}} + (8\,m^2 + 8m - 12)x^{\frac{1}{\sqrt{8}}}.$

Proof:

(1) By putting $\alpha = 1$ in theorem 3.2.3 we get

$$R_2(U,x) = 8mx^{16} + 12x^{12} + (8\,m^2 + 8m - 12)x^{8}.$$

(2) Substituting $\alpha = 2$ in theorem 3.2.3 we obtain

$$HR_2(U,x) = 8mx^{256} + 12x^{144} + (8\,m^2 + 8m - 12)x^{64}.$$

(3) From theorem 3.2.3 we get

$$R_2{}^m(U,x) = 8mx^{\frac{1}{16}} + 12x^{\frac{1}{12}} + (8\,m^2 + 8m - 12)x^{\frac{1}{8}}.$$

(4) Using theorem 3.2.3 and after simplification

$$PR_2(U,x) = 8mx^{\frac{1}{\sqrt{16}}} + 12x^{\frac{1}{\sqrt{12}}} + (8\,m^2 + 8m - 12)x^{\frac{1}{\sqrt{8}}}.$$

3.2.5 Theorem

The following polynomials for the molecular structure of $S.Aztec.D.$ are determined as

(1) $FR(U,x) = 8mx^{32} + 12x^{25} + (8\,m^2 + 8m - 12)x^{20}$

(2) $HR(U,x) = 8mx^{\frac{1}{4}} + 12x^{\frac{2}{7}} + (8\,m^2 + 8m - 12)x^{\frac{1}{3}}$

(3) $SDR(U,x) = 8mx^{2} + 12x^{\frac{25}{7}} + (8\,m^2 + 8m - 12)x^{\frac{5}{2}}$

(4) $IR(U,x) = 8mx^{2} + 12x^{\frac{12}{7}} + (8\,m^2 + 8m - 12)x^{\frac{4}{3}}.$

Proof: (1) Using definition of $FR(U,x)$ and Table 4, we have

$$FR(U,x) = \sum_{pq\in E(T_1)} x^{[r_U(p)^2 r_U(q)^2]}$$

$$= |RE_{4,4}|x^{[r_U(p)^2 r_U(q)^2]} + |RE_{4,3}|x^{[r_U(p)^2 r_U(q)^2]} + |RE_{4,2}|x^{[r_U(p)^2 r_U(q)^2]}$$

$$= 8mx^{(4^2+4^2)} + 12x^{(4^2+3^2)} + (8\,m^2 + 8m - 12)x^{(4^2+2^2)}$$

$$= 8mx^{32} + 12x^{25} + (8\,m^2 + 8m - 12)x^{20}.$$

(2) Using definition $HR(U,x)$ and Table 4, we conclude

$$HR(U,x) = \sum_{pq\in E(U)} \frac{2}{x^{[r_U(p)+r_U(q)]}}$$

$$= |RE_{4,4}|\frac{2}{x^{[r_U(p)+r_U(q)]}} + |RE_{4,3}|\frac{2}{x^{[r_U(p)+r_U(q)]}} + |RE_{4,2}|\frac{2}{x^{[r_U(p)+r_U(q)]}}$$

$$HR(U,x) = 8m\frac{2}{x^{[4+4]}} + 12\frac{2}{x^{[4+3]}} + (8\,m^2 + 8m - 12)\frac{2}{x^{[4+2]}}$$

$$= 8mx^{\frac{1}{4}} + 12x^{\frac{2}{7}} + (8\,m^2 + 8m - 12)x^{\frac{1}{3}}.$$

(3) Using definition of $SDR(U,x)$ and Table 4, we conclude

$$SDR(U,x) = \sum_{pq\in E(U)} x^{\left[\frac{r_U(p)}{r_U(q)}+\frac{r_U(q)}{r_U(p)}\right]}$$

$$SDR(U,x) = |RE_{4,4}|x^{\left[\frac{r_U(p)}{r_U(q)}+\frac{r_U(q)}{r_U(p)}\right]} + |RE_{4,3}|x^{\left[\frac{r_U(p)}{r_U(q)}+\frac{r_U(q)}{r_U(p)}\right]} + |RE_{4,2}|x^{\left[\frac{r_U(p)}{r_G(q)}+\frac{r_U(q)}{r_U(p)}\right]}$$

$$= 8mx^{\left(\frac{4}{4}+\frac{4}{4}\right)} + 12x^{\left(\frac{4}{3}+\frac{3}{4}\right)} + (8\,m^2 + 8m - 12)x^{\left(\frac{4}{2}+\frac{2}{4}\right)}$$

$$= 8mx^{2} + 12x^{\frac{25}{7}} + (8\,m^2 + 8m - 12)x^{\frac{5}{2}}.$$

(4) Using definition $IR(U,x)$ and Table 4, we get

$$IR(U,x) = \sum_{pq\in E(U)} x^{\left[\frac{r_U(p)r_U(q)}{r_U(p)+r_U(q)}\right]}$$

$$= |RE_{4,4}|x^{\left[\frac{r_U(p)r_G(q)}{r_U(p)+r_U(q)}\right]} + |RE_{4,3}|x^{\left[\frac{r_U(p)r_U(q)}{r_U(p)+r_U(q)}\right]} + |RE_{4,2}|x^{\left[\frac{r_U(p)r_G(q)}{r_U(p)+r_U(q)}\right]}$$

$$= 8mx^{\left(\frac{4\times4}{4+4}\right)} + 12x^{\left(\frac{4\times3}{4+3}\right)} + (8\,m^2 + 8m - 12)x^{\left(\frac{4\times2}{4+2}\right)}$$

$$= 8mx^{2} + 12x^{\frac{12}{7}} + (8\,m^2 + 8m - 12)x^{\frac{4}{3}}.$$

IV. GRAPHICAL ANALYSIS

We have sketched the behaviors of all Revan polynomials for the [$C_8H_{10}N_4O_2$] and $S.Aztec.D.$ networks. It explores how the Revan polynomials change its behavior with the increase in network size and structure. Graphical representations of some Revan Polynomials for $S.Aztec.D.$ (U) and [$C_8H_{10}N_4O_2$] (V) are given below in Fig 6 to Fig 19.

Graphical behavior for $[C8H10N4O2]$ (V) *in Fig 6 to 12*

$R_1{}^\alpha(G,x)$

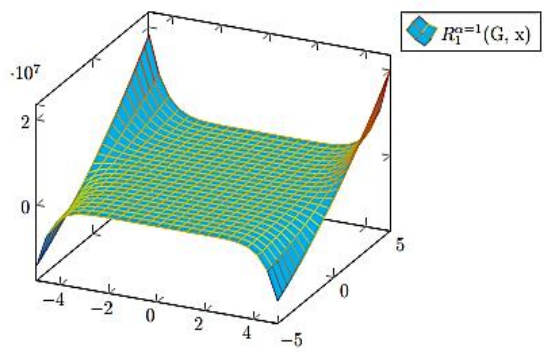

Fig 6

${R_2}^{\alpha}(G,x)$

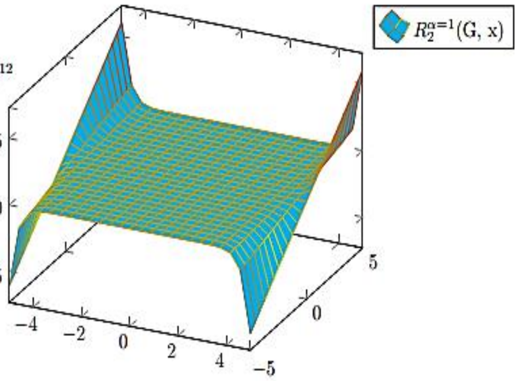

Fig 7

$HR_1(G,x)$

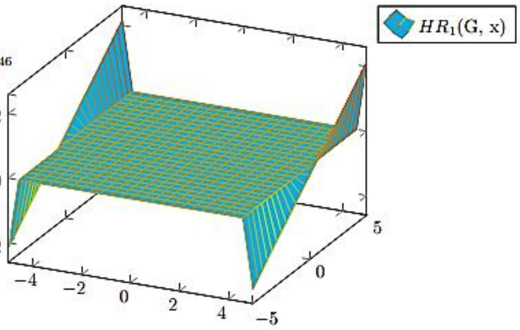

Fig 8

$HR_2(G,x)$

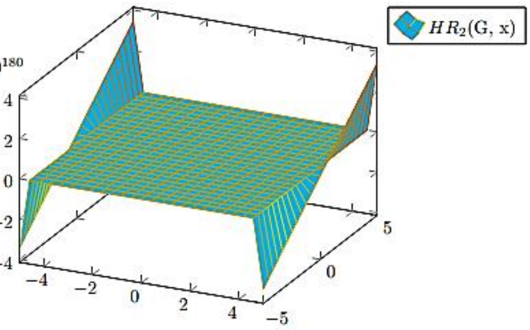

Fig 9

$FR(G,x)$

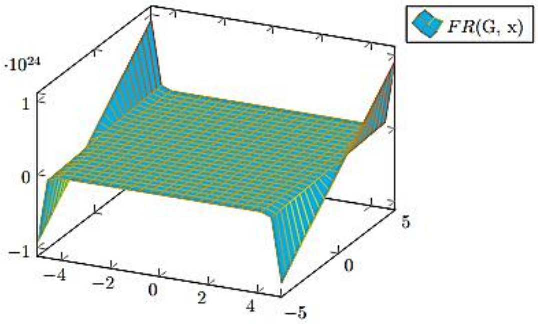

Fig 10

$SDR(G,x)$

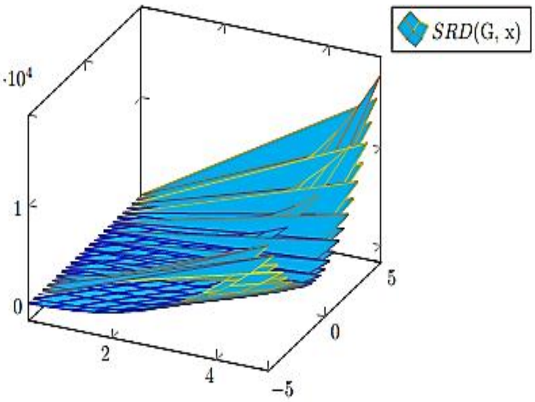

Fig 11

$IR(G,x)$

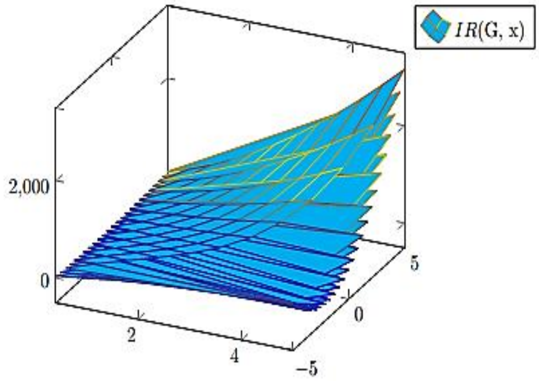

Fig 12

Graphical behavior for $[C8H10N4O2](V)$ *in Fig 13 to 19*

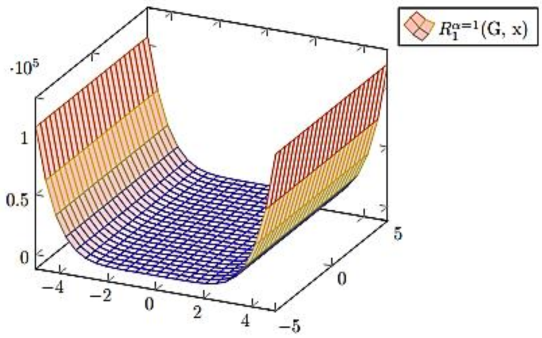

Fig 13

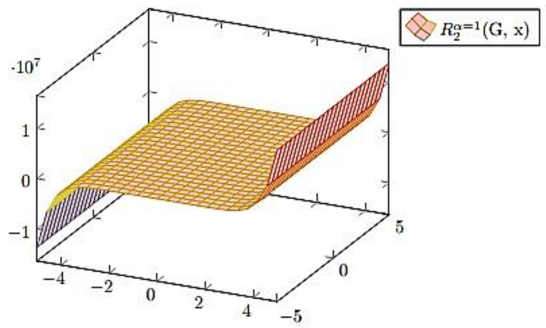

Fig 14

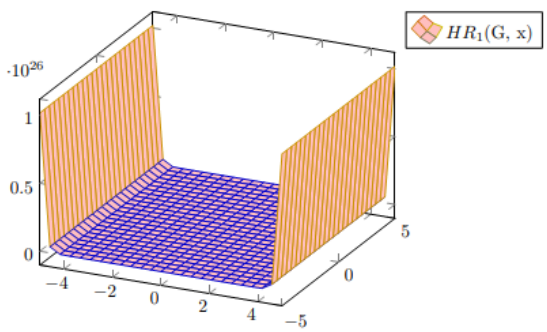

Fig 15

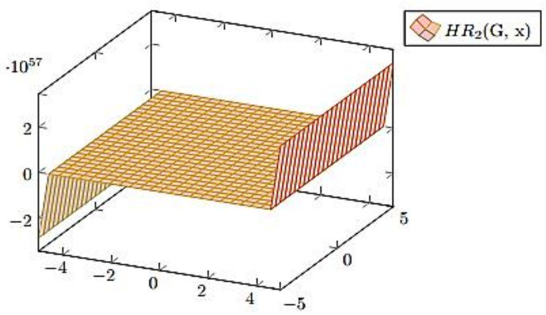

Fig 16

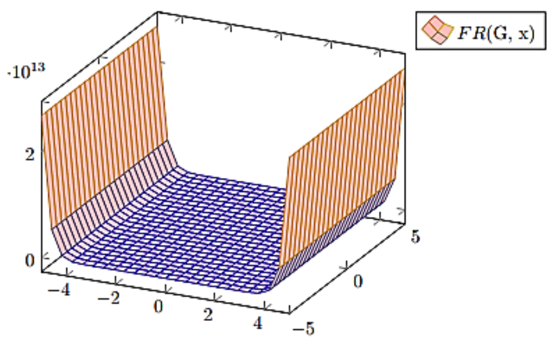

Fig 17

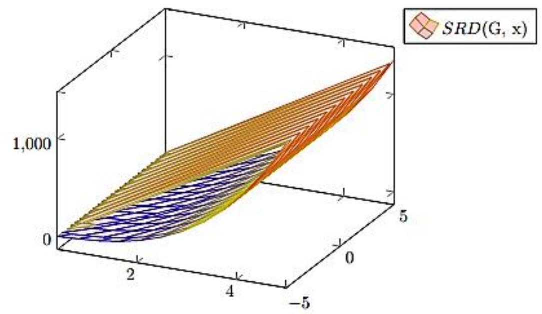

Fig 18

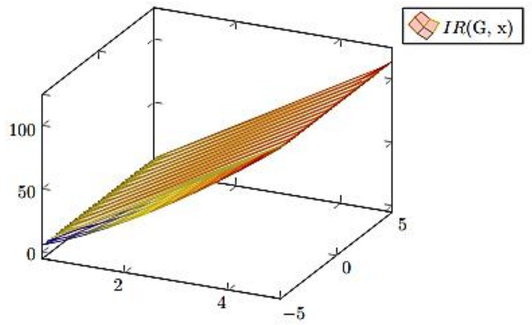

Fig 19

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Conflict of Interest: The authors declare no conflict of interest.

Data Availability: All data used are included inside the article.

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