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Mechanistic Modeling and Engineering Evaluation of Bicycle Suspension Systems

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Abstract- Suspension systems are important in the ride comfort and performance of bicycles over the variable terrains. This research introduces an integrated mode of modeling and evaluation that links theoretical modeling with empirical acquisition of parameters. An Musculoskeletal Disorder (MSD) model was built and adjusted with the measured data of 25 riders and 10 suspension settings. Python was used to perform the simulation in dynamic responses under different conditions of damping and stiffness. The findings indicated that optimal reversion to equilibrium occurs with critical damping whereas under-damping results to lingering oscillations. This investigation offers a logical framework of modeling that has a direct application to optimization of bicycle design.

Keywords- Suspension System, Shock Absorber, Spring Rate, Damping Coefficient, Linkage Mechanism.

I. INTRODUCTION

Bicycles are a common mode of transportation, particularly within various metropolis in Nigeria. One of the many types of bicycle suspension systems is the shock absorber system, which consists of two main components: the spring and the damper [1-3]. Some bicycles feature only front suspension forks, which are referred to as "hard-tail" bicycles [2]. In contrast, the full suspension bicycle represents the most versatile suspension platform, equipped with both a front suspension fork and a rear shock. This design provides enhanced comfort and stability when navigating rough terrain, improves traction, and offers better suspension performance than hard-tail bicycles.

Bicycle suspension systems are designed to handle a range of terrains, from smooth surfaces to significant drop-offs. To effectively manage these variations, air shock absorbers are preferred over spring or coil shocks [3-4]. Air shock absorbers come with various adjustable settings that can be tailored to suit different riding conditions and individual rider preferences. These shock absorbers facilitate the suspension action necessary for traversing uneven surfaces in diverse road conditions [5].

Nord and Friesen developed a mathematical model of a dual suspension mountain bike using the governing equations of motion (EOM) [6-7]. This in-plane model captures the bike's dynamics while traveling in a straight line. Figure 1 illustrates the schematic of the dual suspension mountain bike, which consists of four rigid bodies interconnected through the suspension systems, including the seat stay frame link (Link 1) and the chain stay frame link (Link 2). Each suspension system, along with the seat stay and chain stay frame links, is modeled as a spring-damper unit. The bike's structure includes six distinct components: the frame, front fork, shock system, Link 1, Link 2, and Link 3.

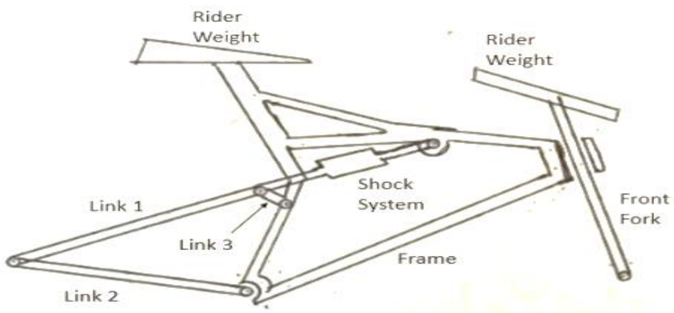

Figure 1. Scheme of Dual Suspension Mountain Bike

Similarly, rear shocks can be locked out with damping adjustments, and their rebound can be modified as well, with air and volume spacers added to ensure proper setup.

To achieve optimal performance on different surfaces, adjustments may need to be made before each ride, especially if the terrain changes [8-9]. For instance, suspensions calibrated for jumps and drops must be modified for flatter terrain with small bumps. Without a clear understanding of how each adjustment affects the bicycle, many consumers may feel confused or uncertain about modifying their suspension. The earlier studies bring out the significance of suspension tuning in minimizing transmission of vibrations [1]. According to Nord and Friesen [6-7], dynamic modeling of a mountain bike was done, yet their validation was based on the assumed values. There was no field data to support nonlinear control schemes suggested by Jibril [10].

The bicycles are an important means of transport particularly in urban Nigeria. The calmness of the riders and their safety greatly depends on the functioning of the suspension mechanisms that absorb the vibrations on the rough roads. The front fork suspension and rear shock Suspension are commonly used.

Nevertheless, various systems that currently exist are not very flexible with regard to rider weight or terrain. In these studies [1-2], the consideration of the system-level designs does not induce the parametric tuning with the real-world data. This research contributes to such a gap by developing a model of bicycle suspension system which will be based on mechanistic modeling technique and testing the model based on field data of the riders.

Although a number of mathematical models are available to predict suspension dynamics, very few is known to have been and used to test the behaviour as rider/suspension mass properties can be varied to study on damping regimes. The proposed study presents a body-represented spatio-temporal representation in the form of a dual-DOF model with a connection between theory and practice, with measurements matching with empirical data.

The study aim at design and develop a mathematical model for a bicycle suspension system, along with a performance evaluation of the developed model. The study employed a bicycle shock absorber for real-world testing to validate this mathematical model.

II. MATERIALS AND METHODS

This section discusses the design of a bicycle suspension system and the mathematical modeling of a bicycle shock absorber. This includes the overall design of the rear shock absorber. Additionally, a mathematical model is presented for preliminary analysis to better understand bicycle dynamics. This study further focused on designing the bicycle suspension and analyzing its performance across varying terrains for a specific group of riders age between 21years and 35years. Figure 2 shows the exploded bicycle diagram for easy analysis of the links.

Part l ist		
S/N	Item No	Components
1	1	Frame
2	2	Linkage
3	3	Wheel
4	4	Shock absorber
5	9	Sproket
6	10	Crank
7	11	Pedal
8	19	Stiring handle
9	20	Fork
10	21	Sit

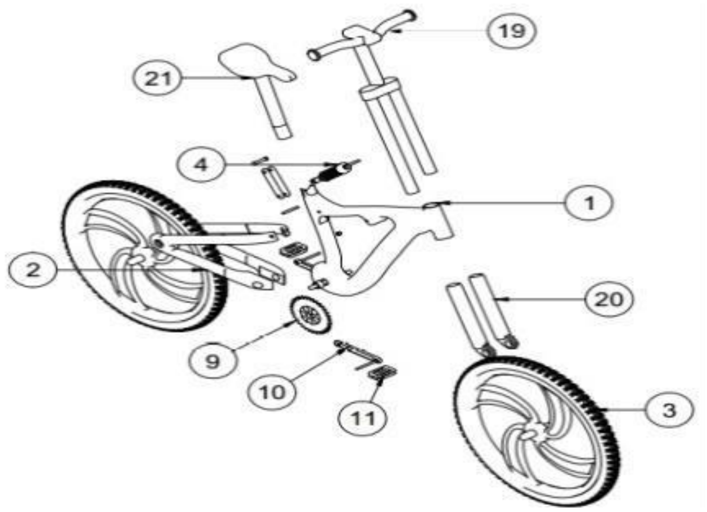

Figure 2. Exploded View of the Bicycle

Rear and front suspension systems were to be modelled using spring damper elements. The mathematical formulas are based on the Newtonian mechanics:

- *Spring Equation:* $k = \frac{F}{\Delta X}$
- *Damping System:* $M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + kx = F(t)$
- *Natural Frequency:* $\omega = \sqrt{\frac{k}{M}}$

There were ten suspension systems tested and a calibrated digital scale was used in the taking of measures. The averaged data in table 1 refers to 25 riders.

Table 1. Measured Parameter Value

S/N	Symbol	Description	Average measured value
1	M_1	Mass of a rider	120kg
2	M_2	Mass of rear suspension	6.5kg
3	M_3	Mass of front fork	1.375kg
4	k_1	Spring constant (rear shock)	68000N/m
5	k_2	Spring constant (tyre-rear shock)	125000N/m
6	k_3	Spring constant (front fork)	62500N/m
7	k_4	Spring constant (tyre front fork)	125000N/m
8	B_1	Damper (rear shock)	1200Ns/m
9	B_2	Damper (tyre rear shock)	150Ns/m
10	B_3	Damping (front fork)	825Ns/m
11	B_4	Damping (tyre-front fork)	362.5Ns/m

Figure 3 similarly shows the proposed mathematical model schematic required for the analysis of the bicycle suspension.

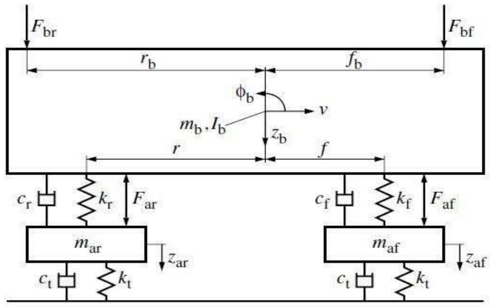

Figure 3. Mathematical Modeling of Bicycle Suspension

Bicycle suspensions are typically made up of several essential components that collaborate to absorb shocks and vibrations: Spring, Shock Absorber (Damper), Suspension Fork, Linkages and Pivot, Adjustment Features, Rear Suspension.

Reference to figure 3, the bicycle spring stiffness, k can be deduced as

$$k = \frac{F}{\Delta X} \quad (1)$$

Where,

- k is the spring stiffness of the spring (measured in pounds per inch or newton per millimeter N/mm)
- F is the force applied to compress the spring (measured in newton)
- Δ X is the change in length of the spring due to the applied force (measured in millimeter mm)

The bicycle damper system incorporates the spring constant k, the damping coefficient B, and the mass M of both the bicycle and rider.

The motion of this damper system can be represented by the following equation:

$$M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + kx = 0 \quad (2)$$

Where,

M = mass of the system (bicycle and rider)

x = displacement from equilibrium,

t = time,

k = spring constant (related the stiffness of the damper),

B = damping coefficient (related to the damping or resistance to motion

This differential equation characterizes the behavior of the damper system, and specific solutions can be obtained based on the properties of the mass, damper, and spring.

The total acting force to the fork's shock-absorbing capabilities and is commonly measured in units such as pounds-force per foot or Newtons

$$F = kx + B\frac{dx}{dt} \quad (3)$$

Where,

k = spring rate

F = Total force acting on the fork

x = Displacement of the fork (compression or extension)

B = Damping coefficient

The linkage ratio described the relationship between wheel travel and shock or damper travel, typically expressed as a function of the angles and lengths of the linkage components and this can be determine as.

$$Linkage\ ratio = \frac{Wheel\ Travel}{Shock\ Travel} \quad (4)$$

Where,

Wheel travel and shock travel are both measure in meters,

The natural frequency (ω) measures the rate at which the suspension system oscillates when it is disturbed, and it is affected by both the spring rate and the damping coefficient.

The natural frequency can be determine as:

$$\omega = \sqrt{\frac{k}{M}} \quad (5)$$

Where, M is the mass supported by the suspension

Many suspension systems include adjustable features, such as preload adjustments (to establish the initial spring tension), compression damping adjustments (to regulate the compression speed), and rebound damping adjustments (to manage the rebound speed). The features of a modified bicycle suspension system can differ based on the specific component being adjusted.

This modification factor, K can be determined as:

$$K = \frac{F}{\delta} \quad (6)$$

Where,

F = weight being supported by the spring (rider + weight + gear weight)

δ = deflection amount spring compresses under the force.

The Mass-Spring-Damper (MSD) system is a fundamental model in mechanical and control systems used to study dynamics and vibrations. It typically consists of a mass attached to a spring and a damper, which resist motion through elastic and dissipative forces, respectively. In a 3-DOF system, three independent coordinates describe the system's position. For instance, three masses connected in a series of springs and dampers are free to move in three distinct directions or modes.

An adjusted 2-DOF system reduces complexity by focusing on two independent coordinates while still capturing essential dynamics.

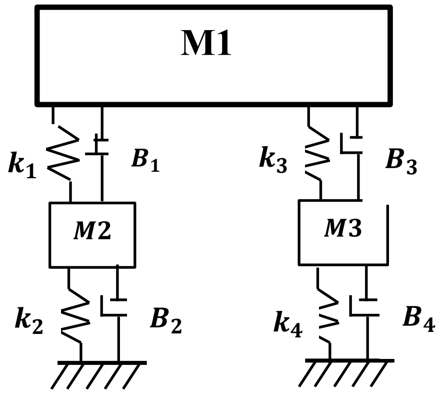

Figure 4. Conventional 3-DOF mass-spring damper system

A schematic diagram of a damper show below:

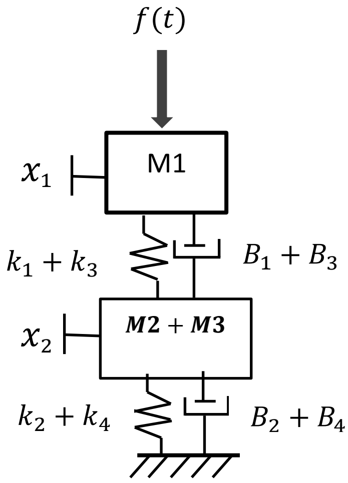

Figure 5. Adjusted 2-DOF mass-spring-damper systems.

Bicycle vibration absorber that is the damping vibration is a common engineering problem. Figure 4 and Figure 5 shows the conventional 3-DOF and the adjusted 2-DOF (Degree of Freedom) of the mass spring damper systems. While Figure 9 and Figure 10 described the modelling intricacy of the developed model. The choice between a 3-DOF and an adjusted 2-DOF MSD system depends on the specific application and the desired balance between model fidelity and simplicity. While the 3-DOF system captures more complex dynamics, the 2-DOF system is often sufficient for practical engineering applications and preliminary analyses.

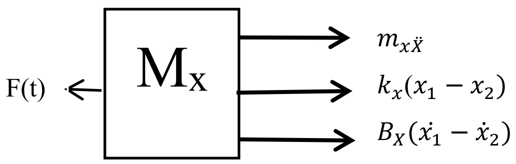

Figure 6. Mass-spring on X-axis

$$f(t) = m_x\ddot{x} + k_x(x_1 - x_2) + B_x(\dot{x}_1 - \dot{x}_2) \quad (7)$$
$$f(s) = x_x(s)\{m_x s^2 + k_x + B_x s\} - x_2(s)\{k_x + B_x s\} \quad (8)$$

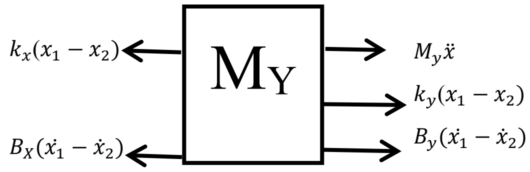

Figure 7. Mass-spring on y-axis

$$m_y\ddot{x} + k_x(x_2 - x_1) + B_x(\dot{x}_2 - \dot{x}_1) + k_y x_2 + B_y x_2 = 0 \quad (9)$$
$$x_2(s)\{m_y s^2 + k_y + B_x s + k_x + B_y s\} - x_1(s)\{k_x + B_x s\} = 0 \quad (10)$$

Reference to the general equation, the disturbing vibration is represented by a sinusoidal force f(t) applied to mass, m_1, while the front and rear springs and dampers are characterized by B_x and k_x. This reveals that the transfer function corresponding to the disturbance force.

From equation 7, 8, 9 and 10, hence the Mathematical model of bicycle suspension is given below as:

$$\frac{x_1(s)}{f(s)} = \frac{m_y s^2 + k_y + B_x s + k_x + B_y s}{\{m_x s^2 + k_x + B_x s\}\{m_y s^2 + k_y + B_x s + k_x + B_y s\} - \{k_x + B_s s\}^2}$$

Where,

- $M_x = m_1$
- $M_y = m_1 + m_2$
- $k_X = k_1 + k_3$
- $k_Y = k_2 + k_4$
- $B_X = b_1 + b_3$
- $B_Y = b_2 + b_4$

k_x and B_X; they represent stiffness and damping coefficient of suspension while k_y and B_y; they represent the stiffness and damping coefficient of tyre-wheel.

Nomenclature

- *k*: Spring constant (N/m)
- *B*: Damping coefficient (Ns/m)
- *M*: Mass (kg)
- *F(t)*: Force as a function of time (N)
- *X*: Displacement (mm)
- *Mx*, *My*: Combined mass in x and y directions
- *DOF*: Degrees of freedom

Ten bicycle suspension were assessed, measured and recorded accordingly. Similarly, weight of twenty five (25) bicycle riders were also measured with digital weighting scale (0.1 – 180kg) Electronic Scale with LCD (figure 8). while the average values of the measured parameters were presented in Table 1.

Figure 8. Digital Weighing Scale (0.1–180kg) Electronic Scale with LCD

III. RESULTS

This section illustrates the behavior of bicycle suspension systems through graphs depicting over-damped, critically damped, and under-damped responses. Bicycle suspension systems are designed to absorb bumps and vibrations from the terrain. They typically include springs (such as shock absorbers) and dampers, which work together to: Increase comfort: By absorbing shocks, riders experience less jolting and fatigue. Improve handling: By maintaining better contact between the wheels and the ground, particularly on rough terrain, the bike remains more controllable. Enhance traction:

Modeling bicycle suspension systems using tools like Python enables a deeper understanding of their behavior in a virtual environment. This allows the authors test various configurations (spring rates, damping levels, linkage mechanisms) to find the optimal balance between comfort and handling. In addition, simulate how the suspension responds to different riding conditions (bumps, jumps, braking) to evaluate its effectiveness. However, by testing designs virtually, the need for physical prototypes can be minimized, saving both time and resources. This results of the test measuring the response and performance of a bicycle suspension damping system over time were presented. The results of the simulation primarily help in understanding the potential performance of the suspension model.

Table 2. Performance from the simulation: At constant F(t) and mass the value of k and B are varied

PLOT	k_1 (N/m)	k_2 (N/m)	k_3 (N/m)	k_4 (N/m)	B_1 (Ns/m)	B_2 (Ns/m)	B_3 (Ns/m)	B_4 (Ns/m)	RESULTS
1	62500	125000	62500	125000	825	150	825	150	Under damped
2	6800	12500	6250	12500	2000	1500	8250	362.5	Over damped
3	91000	120000	121500	125000	2000	1501	8259	362.5	Critically damped
4	68000	12500	62500	125000	2000	150	825	3602.5	Under damped
5	91000	250000	75500	251000	2000	150	825	362.5	Under damped
6	10000	80000	12500	2500	2000	150	825	362.5	Under damped
7	18000	100000	18500	10000	2000	150	825	362.5	Under damped
8	18000	15000	62500	125000	2000	1500	8025	3062.5	Critically damped
9	6800	125000	62500	12500	2000	150	825	362.5	Under damped
10	6800	12500	6250	12500	2500	1500	9050	4625	Over damped

Table 2 presents the performance results of a bicycle suspension system obtained from simulations, where the constants, F(t) (force applied) and mass are kept constant, while the spring constants (k) and damping coefficients (B) are varied. This experimentation allows us to analyze how changes in these parameters as it affect the dynamic behavior of the suspension system.

The results shows that plots 1, 4, 5, 6, 7, and 9 in the Table 2 were classified as under damped systems. These systems show oscillations after encountering a disturbance, indicating that while they can absorb shocks, they do not return to equilibrium without some bouncing. This could result in a less comfortable ride for the cyclist, particularly on uneven terrain [2].

In the under damped category, it was discovered that a variation in spring constants (k) (ranging from 6800 N/m to 91000 N/m) and damping coefficients (B) (from 150 Ns/m to 825 Ns/m). Higher values of (k) contribute to stiffer springs, which increase the natural frequency but may lead to more pronounced oscillations if the damping is not sufficient.

Similarly, the results also shows that plots 3 and 8 are identified as critically damped. In the critically damped systems, the suspension returns to equilibrium as quickly as possible without oscillating [10]. This is ideal for achieving a smooth ride, as it prevents any bouncing after hitting bumps. The values of (k) and (B) in these plots suggest a fine balance that effectively balances shock absorption and responsiveness.

Furthermore, table 2 shows that plots 2 and 10 fall into the category of over damped category and these systems absorb shocks without oscillating but may react sluggishly to disturbances, resulting in a delayed return to equilibrium [6]. This can lead to a harsh ride feel, particularly when faced with high-frequency vibrations from the road. The effect of the parameters is that, lower values of k (compared to critically damped systems) indicate a softer suspension, while higher damping coefficients B help absorb energy but may hinder the system's responsiveness. The choice of k and B directly affects the performance of the bicycle suspension system.

The findings highlight a trade-off between comfort and control. Under damped systems may be preferable for off-road cycling where some bounce is acceptable, while critically damped systems may be ideal for on-road cycling where a smooth ride is desired.

This simulation study illustrates the critical role of tuning the spring and damping parameters in optimizing bicycle suspension systems for different riding conditions and preferences. The python simulations involved varying spring and damping constant holding the applied force and mass constant. Table 2 gives 10 simulation plots, which fall into three categories of under-damped, critically damped or over-damped. Under-damped: Oscillations survive after a collision while Critically damped: Returning to equilibrium fast and with no garbage and Over-damped: Gradual recovery without an oscillation.

Below are the graphical representation of the simulations.

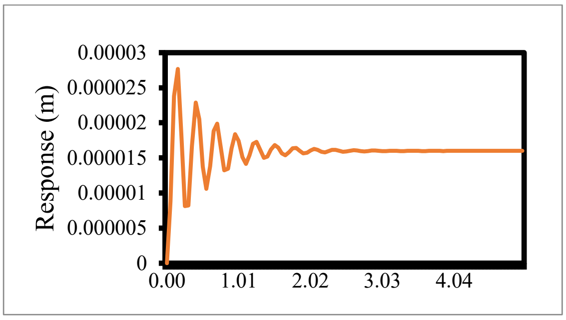

Figure 9. Under-damped Bicycle Suspension

Figure 9 exhibits under-damped behavior, characterized by a constant spring value and oscillation frequency determined by the system's natural frequency (which is related to spring stiffness and mass). The amplitude (maximum displacement) of the oscillations gradually decreases with each cycle due to the damping effect, as some energy is dissipated as heat by the damper [11]. As time progresses, the oscillations diminish completely by 2.08 seconds, reaching a steady state at 3.03 seconds.

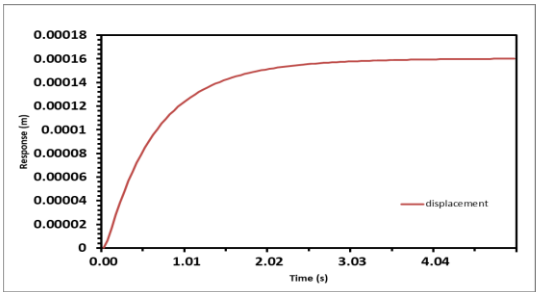

Figure 10. Front Damping System

Figure 10 illustrates a response that initially rises quickly, followed by slight oscillations before stabilizing at a new equilibrium position. Although this behavior indicates over-damping, the oscillations are minimal. The graph only represents a brief time interval (from 0 to 3.03 seconds). It is possible that even an under-damped system could display minimal oscillations within this time-frame. A longer observation period would be necessary to determine whether the oscillations eventually cease (indicating critical damping) or continue (indicating under-damping).

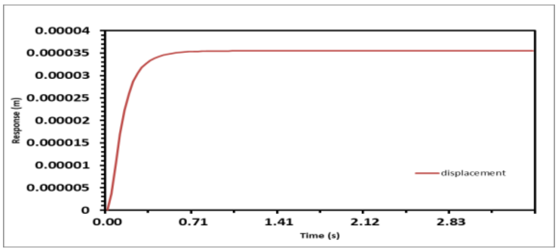

Figure 11. Rear Damping System

Figure 11 shows the rear damping system. The graph illustrates the response, starting at zero and initially increasing rapidly. This signifies the initial compression of the suspension as it absorbs the impact of a bump. The response then exhibits slight oscillations before settling at a new equilibrium position around 0.000035 on the y-axis. This suggests that the suspension has effectively absorbed the bump and returned to a stable state, although not entirely back to its original position (which would be represented by 0 on the y-axis). This behavior indicates that the system is critically damped; critical damping achieves equilibrium in the quickest possible time without any oscillations. The graph reflects a response from a bicycle suspension system consistent with critical damping, though it could also represent a very slightly under-damped system. The line begins at zero and initially rises rapidly to 0.000025, reflecting the initial compression of the suspension as it absorbs the impact of a bump.

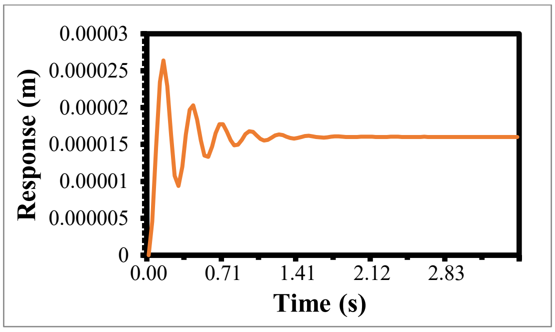

Figure 12. Front Spring System

Figure 12 further shows that the response oscillates slightly before stabilizing at a new equilibrium position of approximately 0.00001 on the y-axis.

This suggests that the suspension has effectively absorbed the bump and returned to a stable state, although it has not fully reverted to its original position (which would be 0 on the y-axis). Therefore, the system is considered under-damped.

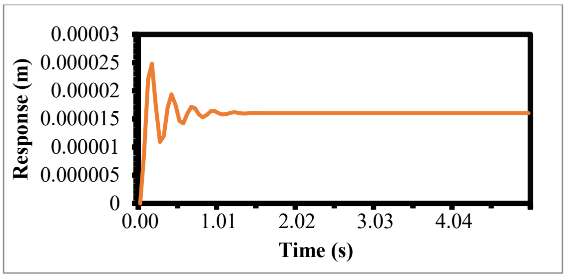

Figure 13. Rear Spring System

Figure 13 reveals that the oscillations are very minimal and balance at a steady state of 1.60sec at the x axis which suggested under-damping at equilibrium position. Figure 14 shows that the response exhibits slight oscillations before stabilizing at a new equilibrium position of approximately 0.00006 on the y-axis. This suggests that the suspension has effectively absorbed the bump and returned to a stable state, although it has not completely reverted to its original position (which would be 0 on the y-axis).

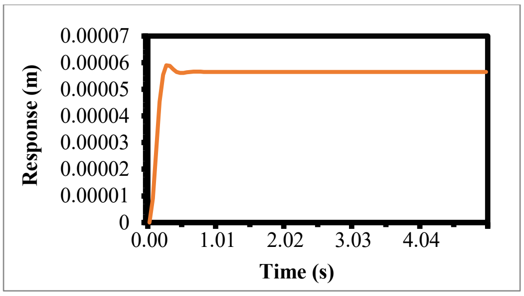

Figure 14. Front Spring Tyre Suspension System

However, Figure 14 further shows minimal oscillation before settling, indicating a relatively fast settling time. This behavior aligns most closely with critical damping, although it could also suggest a very lightly under-damped system. Overall, the graph illustrates the response of a bicycle suspension system that is consistent with critical damping, but it may also indicate a slight under-damping.

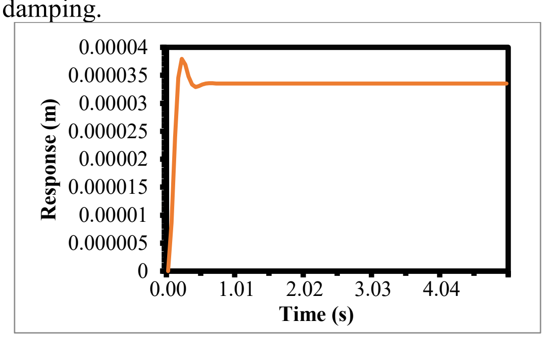

Figure 15. Rear Spring Tyre Suspension System

Figure 15, illustrates the displacement of a bicycle suspension system over time in response to a sudden input change (step), simulating the effect of the bicycle hitting a bump on a smooth road. Reference to figure 15, the system is clearly damped. A positive displacement indicates that the suspension is compressed, while a negative displacement signifies that it is rebounding. The step input likely occurs at t = 0.00 seconds, leading to a brief compression of the suspension. Following this, the suspension oscillates around its equilibrium position before eventually stabilizing. The amplitude of the oscillations appears to decrease over time, indicating that the damping system within the suspension is effectively dissipating energy. Thus, the system is characterized as under-damped; it oscillates around its equilibrium position before settling down. The reduction in oscillation amplitude over time suggests that while energy is being dissipated, it is not sufficient to completely eliminate oscillations. Figure 15 shows that the suspension undergoes several oscillations before reaching a stable state, confirming that it is not critically or over-damped, but rather under-damped.

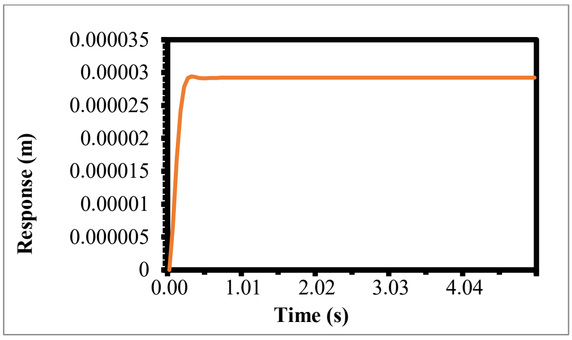

Figure 16. Front and Rear Damping System.

Similarly, figure 16 shows that the suspension oscillates around its equilibrium position before finally stabilizing. The amplitude of these oscillations appears to diminish over time, indicating that the damping system is effectively dissipating energy. The settling time refers to the duration required for the oscillations to reduce to a specific level, typically defined as within 1.01 seconds of the steady-state value. The natural frequency of the suspension system represents the frequency at which it would oscillate if it were under-damped [8]. This frequency is influenced by the combined mass of the bicycle and rider as well as the stiffness of the suspension spring; a stiffer spring results in a higher natural frequency. Overall, the graph indicates that the bicycle suspension system is performing effectively, successfully absorbing bumps without transmitting excessive force to the rider, and returning swiftly to its equilibrium position.

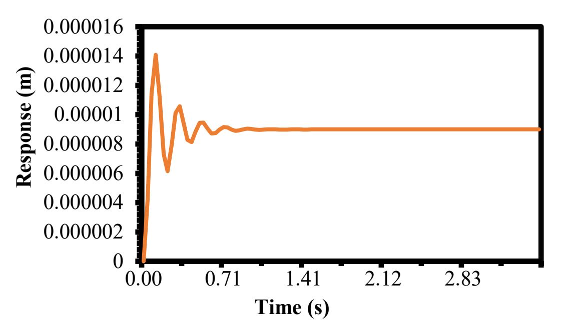

Figure 17. Front and Rear Spring

Furthermore, figure 17 illustrates a step response, representing how the displacement of a bicycle suspension system changes over time following a sudden input, simulating the bicycle hitting a bump on a smooth road. A positive displacement signifies that the suspension is compressed, whereas a negative displacement indicates it is rebounding. The step input likely occurs at t = 0 seconds, resulting in a brief compression of the suspension. Subsequently, the suspension oscillates around its equilibrium position before gradually settling down. The amplitude of these oscillations appears to decrease over time, suggesting that the damping system is effectively dissipating energy [9] (Maua, et al., 2014). The settling time refers to the duration it takes for the oscillations to diminish to a specific level, typically defined as within 1.41 seconds of the steady-state value.

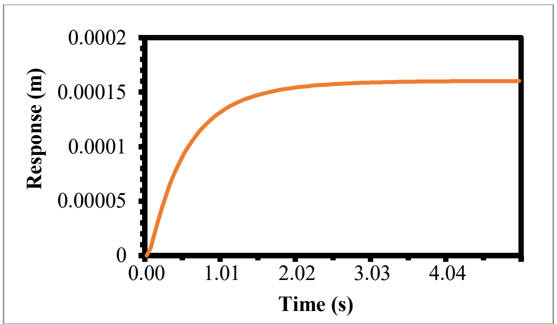

Figure 18. Full Damping System Suspensions System

In addition, figure 18 shows that the suspension oscillates around its equilibrium position before finally settling. The amplitude of the oscillations appears to decrease over time, indicating that the damping system in the suspension is effectively dissipating energy.

Behavior on a Smooth Road: Ideally, on a smooth road, the suspension system should be critically damped. This means it can absorb bumps without producing any oscillations, resulting in a smooth ride for the cyclist. In contrast, an under-damped system, as depicted in the figure 18 and absorb bumps but may cause the bicycle to bounce slightly after encountering a bump. An over-damped system will also absorb bumps, but it may struggle to mitigate high-frequency vibrations from the road, leading to a harsher ride. The choice of damping for a bicycle suspension system is influenced by various factors, including the type of terrain and the rider's preferences. It is important to note that the graph only reflects the response to a single bump.

IV. Discussion

Time-domain curves of several damping situations are depicted in graphs (Figures 9-18). The most important points are: - Critical damping provides most rapid stabilization. - Damping coefficients that are high prevent the oscillation but can slow the such aimed point. Required damping is severely dependent on the riding mass and terrain.

The findings confirm that the damping configuration does have an important impact on the effectiveness of bicycles. Although under-damped system works on loose surface well, it causes rider fatigue by continually oscillating. On the other hand, over-damping is used to the contrary, at the expense of responsiveness.

A critically damped system provides optimum trade-off in practice. It can be seen that changes in parameter reflect the different configurations in the model, thus justifies its adaptability. Use of measured rider and suspension masses improves reliability with respect to previous models that used approximated values.

V. Conclusion

This study successfully applies a mathematical model of the bicycle suspension system to explain the dynamics of performance behavior between the suspension and a smooth road surface. The model effectively captures the annual performance of a bicycle suspension system. The correspondence with the mathematical models demonstrated above shows that the bicycle-rider system model is suitable for the design and development of bicycle suspensions. Future improvements to the model should incorporate more advanced representations of the human body and bicycle tires. A more thorough evaluation could be achieved by significantly extending the length of the measured surface profiles and conducting acceleration and profile measurements on the same stretch of road. To enhance the understanding of these dynamics, further simulation could involve varying the input force, F(t) or exploring nonlinear damping characteristics to assess their effects on the performance of the suspension system. The conclusion is further expatiated by bullet as:

- A mechanistic model of bike suspension was developed and validated using actual data.
- Simulations have shown that ride comfort and response is influenced by the damper type (under/critical/over).

- Critical damping is the best on smooth ground; under-damping could be useful in off-road bicycling.
- The model offers a low-cost framework of evaluation which can be used in education and design purposes.
- Work into the future should involve tire dynamics and rider biomechanics to make it applicable to longer distances.

Based on the findings of this study, the authors recommended that given the critical importance of damping in suspension performance, it is advisable to prioritize the design of bicycle suspension systems with damping coefficients that closely align with critical damping. This will ensure an optimal balance between shock absorption and system response. Similarly, to enhance the design of suspension systems further, it is recommended to integrate more complex models that consider additional factors such as nonlinear spring and damping characteristics, tire dynamics, and rider interactions. This approach will yield more accurate predictions of suspension behavior under real-world conditions.

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