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Advanced Cryptography Through the Exponential Functions and Modular Techniques

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Abstract- A new approach to cryptography, leveraging the natural transformations over an exponential function has been presented in this study. This context improves the information security by applying advanced encryption and digital signature techniques to confirm confidentiality, integrity, and verification. The proposed system allows secure transactions, reliable communication channels, and best protection against unauthorized approach. Exponential functions introduce an added layer of complexity and hence improves encryption strength so as to make them valued for securing sensitive data in different fields such as finance, healthcare, and national security. In this study, the Natural Transformation has been introduced that aids a different term in cryptography to secure data and enhance security for sensitive information. Encryption and decryption have been performed using an advanced mathematical method that includes modular operations, including $mod26$, in addition to an extended technique using $mod68$ for enhanced security. Capital alphabets has been assigned to numeric value for $mod26$ and for hard security.

Keywords- Plain Text, Cipher Text, Encryption Function, Decryption Function, Natural Transformation and Key Algorithm.

I. INTRODUCTION

Cryptography has progressed in last few years, addressing the increasing need for secure communication in the digital stage of development. Traditional and modern cryptographic methods have incorporated mathematical and physical principles to ensure data confidentiality, integrity, and authentication. Recent advancements in cryptography have explored advanced methods such as natural transformations and exponential functions to improve security mechanisms.

Natural transformations are considered more due to its applications in resolving mathematical models [1]. They efficiently used these transformations for solving the fractional models. Scientists have explored their potential use in cryptography based on similar mathematical concepts. Furthermore, [2] introduced N-transform properties while highlights their application in engineering and computational problems. In addition, [3] proposed a changed cryptographic algorithm based on natural transformation and exponential functions that demonstrates enhanced encryption and decryption performance. The Laplace transform is a commonly used mathematical tool, has also been very useful addition in cryptographic techniques. Similarly, [4] developed cryptography method while using the Laplace transformation and established its efficacy in securing data. Likewise, [5] proposed an asymmetric key cryptographic method that leverage out the Laplace transform to improve encryption and decryption procedures.

Classical cryptographic procedures like Caesar cipher provides foundational insights, but modern adaptations such as three-pass protocols implementation introduced by [6] improves the security and confidentiality. In addition the crypto analysis research such as that proposed by [7] has meaningfully contributed in understanding the strengths and weaknesses of several cryptographic approaches. The integration of chaotic systems and quantum mechanics has also advanced the cryptographic field. In this field, [8] introduced a chaotic cryptography system for making condensed ciphertexts, demonstrating the potential of chaos philosophy in secure communications. Moreover, quantum cryptography has been explored by [9] and it offers unparalleled security by leveraging the philosophies of quantum mechanics. Regardless of these improvements, some cryptographic implementations remain defenseless, as highlighted by [10], who identified the mismanagement of pairings in cryptographic structures. Efficient algorithms had been developed to exploit the periodic properties of tropical irreducible matrices that allows private and shared keys to be computed from community information.

Furthermore, [11] designed three attack algorithms which can limit the user-equivalent secluded key by repeating through the controls of the public matrix. Using this key will allows to originate the shared undisclosed key amongst the collaborating parties.[12] investigated the possibility to reduce the security concerns in IoT fog systems. In their survey article, they discussed innovative approaches to handle the challenges faced in this paradigm. They presented and discussed the problems of IoT-Fog architecture with examples. The works are characterized into resource management, architectural bases, and directing protocols that combine privacy, security, and resource efficacy.

The community key cryptosystem is extremely endangered by the quantum computer. Hence, in post-quantum cryptographic time, it is important to study protected public key cryptography with quantum computing. [13] introduced an Oracle assumption regarding numerous exponentiations, which can be derived from the tropical matrix. A mixture encryption system together with message authenticating codes, symmetric encryption, and hash function has been designed. The study of [14] presented a speedy text-encryption strategy that is based on a combination of elliptic curve and wavelet decomposition that provides an improved security. In three phases of cryptography, first one transforms the characters to mathematical values through ASCI code and hence eliminate trends in input by using a diffusion set of rules. A set of points at the elliptic curves at the second phase deforms with a randomized plotting breaking in a substantial degree of nonlinearity. In the third phase, the representation is changed with the wavelet built wherever the max-plus algebra was considered, that results in a ciphertext with high diffusion.

Mathematical cryptology is a subfield of cryptography and focuses on emerging and applying encryption approaches to encode and decode information. Mathematicians designed robust protocols, implementing encryption techniques, and employed decryption approaches to recover the original data in an understandable form. The whole cryptographic process is grounded in mathematical transformations that provides the basis for its functionality and effectiveness.

The term natural transformation in the current study refers to mathematical transform used in the planned cryptographic scheme to change the numerical depiction of plaintext. The equivalent inverse transformation is then used in decryption to recover the actual coefficients. In this framework, the use of modular arithmetic and further advanced mathematical techniques has expanded importance and allowing the secure encoding of data. The mapping of numerical domains such as the first twenty-five positive integers with the English alphabet demonstrates the application of mathematical transformations in cryptography. In this study an innovative method in cryptography has been proposed while highlighting the role of mathematical transformations in enhancing information security. The first 25 positive digits are: $Z^+ = \{1,2,3,\ldots,25\}$.

For higher security and protection, first sixty-eight positive integer numbers have been used $Z^+ = \{0,1,2,3,\ldots,67\}$

II. MATERIALS AND METHODS

Plain Text: Plain text refers to a message expressed in simple, human language that can be easily understood by the sender and anyone else who encounters it. For example, the word 'IMAGINATION' can be considered plain text.

Cipher Text: When a plain text message is encrypted using an appropriate method, making it unreadable by anyone and far from human-readable form, it is called cipher text. For example, the cipher text of 'IMAGINATION' is 'QOAYSAAUGEA

Encryption Function: Data can be converted into an unreadable format using encryption techniques, ensuring that only those with permission can access the original data. Its importance has become crucial for securing communications. There are several methods to encrypt data, but the exponential encryption method through natural transformation had been used in this study.

Decryption Function: In cryptography, decryption is the process of applying a decryption function or method to convert cipher text (the encrypted version of data) back to its original plaintext format. This function typically involves applying the inverse operations of the encryption algorithm, which essentially reverses the encryption process.

Natural Transformation Based on Mathematical Transformation: The natural transformation can be defined as: "It is a Mathematical function that is used in cryptography to determine the results of the function".

Let f (t) be the function; then the natural transformation of this function is written as:

$$N\{f(t)\} = \int_0^\infty f(ut)e^{-st}dt = R(u,s) \qquad (1)$$

The inverse transformation can be written as

$$N^{-1}\{R(u,s)\} = f(t)$$

Here $f(t)$ and $R(u,s)$ are a couple of Natural transformations.

A transformation that occurs naturally is linear, which means that the transformation of a parameter $'t'$ can be expressed as a linear function. Mathematically the transformation is written as

$$N\{\alpha f(t) + \beta g(t)\} = \int_0^\infty \{\alpha f(t) + \beta g(t)\}e^{-st}dt$$

and

$$N\{\alpha f(t) + \beta g(t)\} = \alpha N\{f(t)\} + \beta N\{g(t)\} \qquad (2)$$

Some Useful Results of Natural Transformation:

Natural transformations provide a way to relate two functions by defining a family of morphisms that

satisfy certain coherence conditions.

Let $f(t)=e^{at}$, where α is a constant and $t \geq 0$ then the Natural Transformation can be written as

$$N(e^{\alpha t})=\int_0^{\infty} e^{\alpha u t} e^{-st} dt.$$

And also, it can write as by applying Natural Transformation,

$$N(e^{at})=\frac{1}{s-au}.$$

Exponential Function:

$N(e^{at})=\frac{1}{s-au}$ (3)

Algebraic Function:

$N(t^{\alpha})=\frac{\alpha! u^{\alpha}}{s^{\alpha+1}}, \quad \alpha \in N$

(4)

Algebraic Function of nth Order: $N\{t^n\}=\frac{u^n n!}{s^{n+1}}, \quad n \in N$ (5)

Encryption Approach*:*

$L_{(n,1)}=M_{(n,1)}-26K_{(n,1)} \quad for$ n=0,1,2,3..., *(6)*

Here,

$$M(n,1) = 2^{2n+1}(2n+2)(2n+3)L(n,0).$$

Extended Encryption Approach:

$L_{(n,1)}=M_{(n,1)}-68K_{(n,1)} \quad for$ n=0,1,2,3..., *(7)*

$L_{(n,1)}=M_{(n,1)}-67K_{(n,1)} \quad for$ n=0,1,2,3...,

For generating key algorithm

For encryption

$K(n,1)=\frac{M(n,1)-L(n,1)}{26} \quad for\ n=0,1,2,3\ldots$ *(8)*

For extended encryption

$K(n,1)=\frac{M(n,1)-L(n,1)}{68} \quad for\ n=0,1,2,3\ldots$ *(9)*

Steps in Encryption Algorithm and Decryption Algorithm are as follows:

Input: Plaintext to be converted.

1. Convert the plaintext to numerical values
2. Represent the message as numerical values
3. Apply the specific transformation based on mathematical operation
4. Create transformed values by using the transformation.
5. Reduce transformed values
6. Generate the ciphertext
7. Generate the key values

Output: Ciphertext and its equivalent key evidence.

Steps in Decryption Algorithm

Input: Ciphertext and required key information

1. Convert ciphertext letterings into their labelling numerical observations
2. Put the inverse of recommended transformation.
3. The actual numerical characters recovered
4. Change the recovered characters back into alphabetical characters.
5. Obtain the actual plaintext.

III. RESULTS AND DISCUSSION

*Encryption Approach***:** If the plain text in the form of "IMAGINATION", then by the use of natural transformation, the cipher text is "QOAYSAAUGEA"

If

$f(t)=t^2e^{2t}$

Now assign the value of alphabets from the table

8 12 0 6 8 13

0 19 8 14 13

Suppose that

$$\begin{aligned} L_{(0,0)}=8, L_{(1,0)}=12, L_{(2,0)}=0, L_{(3,0)}=6, L_{(4,0)} \\ =8, L_{(5,0)}=13, L_{(6,0)} \\ =0,\ L_{(7,0)}=19, L_{(8,0)} \\ =8,\ L_{(9,0)}=14, L_{(10,0)} \\ =13, L_{(n,0)}=0\ ,\forall\ n \geq 11 \end{aligned}$$

We have function

$f(t)=t^2e^{2t}$

Alphabets in the form of coefficients of series

$$f(t)=\sum_{n=0}^{\infty} L_{(n,0)} \frac{2^n t^{n+2}}{n!}$$

$$\begin{aligned} =L_{(0,0)}\frac{2^0t^2}{0!}+L_{(1,0)}\frac{2^1t^3}{1!}+L_{(2,0)}\frac{2^2t^4}{2!} \\ +L_{(3,0)}\frac{2^3t^5}{3!}+L_{(4,0)}\frac{2^4t^6}{4!} \\ +L_{(5,0)}\frac{2^5t^7}{5!}+L_{(6,0)}\frac{2^6t^8}{6!} \\ +L_{(7,0)}\frac{2^7t^9}{7!}+L_{(8,0)}\frac{2^8t^{10}}{8!} \\ +L_{(9,0)}\frac{2^9t^{11}}{9!}+L_{(10,0)}\frac{2^{10}t^{12}}{10!} \end{aligned}$$

Now

$$\begin{aligned} =(8)\frac{2^0t^2}{0!}+(12)\frac{2^1t^3}{1!}+(0)\frac{2^2t^4}{2!}+(6)\frac{2^3t^5}{3!} \\ +(8)\frac{2^4t^6}{4!}+(13)\frac{2^5t^7}{5!} \\ +(0)\frac{2^6t^8}{6!}+(19)\frac{2^7t^9}{7!} \\ +(8)\frac{2^8t^{10}}{8!}+(14)\frac{2^9t^{11}}{9!} \\ +(13)\frac{2^{10}t^{12}}{10!} \end{aligned}$$

Then

$$\begin{aligned} =(t^2)\frac{8}{0!}+(t^3)\frac{24}{1!}+(t^4)\frac{0}{2!}+(t^5)\frac{48}{3!} \\ +(t^6)\frac{128}{4!}+(t^7)\frac{416}{5!}+(t^8)\frac{0}{6!} \\ +(t^9)\frac{2432}{7!}+(t^{10})\frac{2048}{8!} \\ +(t^{11})\frac{7168}{9!}+(t^{12})\frac{13312}{10!} \end{aligned}$$

Using natural transformation on f(t),

$$N\{t^n\} = \frac{n!\,u^n}{s^{n+1}}$$

$$N\{f(t)\} = \left(\frac{2!\,u^2}{s^3}\right)\frac{8}{0!} + \left(\frac{3!\,u^3}{s^4}\right)\frac{24}{1!} + \left(\frac{4!\,u^4}{s^5}\right)\frac{0}{2!} + \left(\frac{5!\,u^5}{s^6}\right)\frac{48}{3!} + \left(\frac{6!\,u^6}{s^7}\right)\frac{128}{4!} + \left(\frac{7!\,u^7}{s^8}\right)\frac{416}{5!} + \left(\frac{8!\,u^8}{s^9}\right)\frac{0}{6!} + \left(\frac{9!\,u^9}{s^{10}}\right)\frac{2432}{7!} + \left(\frac{10!\,u^{10}}{s^{11}}\right)\frac{2048}{8!} + \left(\frac{11!\,u^{11}}{s^{12}}\right)\frac{7168}{9!} + \left(\frac{12!\,u^{12}}{s^{13}}\right)\frac{13312}{10!}$$

Here first security process starts if we put u=1, it becomes as:

$$L_{(s,1)} = \left(\frac{16}{s^3}\right) + \left(\frac{144}{s^4}\right) + \left(\frac{0}{s^5}\right) + \left(\frac{960}{s^6}\right) + \left(\frac{3840}{s^7}\right) + \left(\frac{17472}{s^8}\right) + \left(\frac{0}{s^9}\right) + \left(\frac{175104}{s^{10}}\right) + \left(\frac{184320}{s^{11}}\right) + \left(\frac{788480}{s^{12}}\right) + \left(\frac{1757184}{s^{13}}\right)$$

Now adjusting these values, we get

16 144 0 960 3840 17472 0 175104 184320 788480 1757184

To mod 26, we get the values

16 14 0 24 18 0 0 20 6 4 0

Finally, get the cipher text by comparing these values with the table of alphabet

"(QOAYSAAUGEA)"

This is the cipher text that obtained from simple text.

Key Algorithm: We generate the key from (8) as,

0 5 0 36 147 672 0 6734 7089 30326 67584

Decryption Approach: We have originated

16 144 0 960 3840 17472 0 175104 184320 788480 1757184

We can write it as,

$$R_{(s,u)} = \left(\frac{2!\,u^2}{s^3}\right)\frac{8}{0!} + \left(\frac{3!\,u^3}{s^4}\right)\frac{24}{1!} + \left(\frac{4!\,u^4}{s^5}\right)\frac{0}{2!} + \left(\frac{5!\,u^5}{s^6}\right)\frac{48}{3!} + \left(\frac{6!\,u^6}{s^7}\right)\frac{128}{4!} + \left(\frac{7!\,u^7}{s^8}\right)\frac{416}{5!} + \left(\frac{8!\,u^8}{s^9}\right)\frac{0}{6!} + \left(\frac{9!\,u^9}{s^{10}}\right)\frac{2432}{7!} + \left(\frac{10!\,u^{10}}{s^{11}}\right)\frac{2048}{8!} + \left(\frac{11!\,u^{11}}{s^{12}}\right)\frac{7168}{9!} + \left(\frac{12!\,u^{12}}{s^{13}}\right)\frac{13312}{10!}$$

Applying inverse Natural Transformation

$$f(t) = (8)\frac{2^0t^2}{0!} + (12)\frac{2^1t^3}{1!} + (0)\frac{2^2t^4}{2!} + (6)\frac{2^3t^5}{3!} + (8)\frac{2^4t^6}{4!} + (13)\frac{2^5t^7}{5!} + (0)\frac{2^6t^8}{6!} + (19)\frac{2^7t^9}{7!} + (8)\frac{2^8t^{10}}{8!} + (14)\frac{2^9t^{11}}{9!} + (13)\frac{2^{10}t^{12}}{10!}$$

From the above equation the coefficients are,

8 12 0 6 8 13 0 19 8 14 13

By comparing from standard table we have "IMAGINATION"

Modified Extended Encryption Approach: If the plain text in the form of **"Learning Never Ends!"** Then by the use of Natural Transformation the cipher text is,

"**WUYMUAk6?E2occ2AAwks"**

By using the function: $f(t) = t^2e^{2t}$

Assign the value of alphabets from the table

11 30 26 43 39 34 39 32 52 13 26 47 26 43 52 4 39 29 44 55

Now suppose that

$L_{(0,0)} = 11$ $L_{(1,0)} = 30$ $L_{(2,0)} = 26$ $L_{(3,0)} = 43$ $L_{(4,0)} = 39$ $L_{(5,0)} = 34$ $L_{(6,0)} = 39$ $L_{(7,0)} = 32$ $L_{(8,0)} = 52$ $L_{(9,0)} = 13$ $L_{(10,0)} = 26$ $L_{(11,0)} = 47$ $L_{(12,0)} = 26$ $L_{(13,0)} = 43$ $L_{(14,0)} = 52$ $L_{(15,0)} = 4$ $L_{(16,0)} = 39$ $L_{(17,0)} = 29$ $L_{(18,0)} = 44$ $L_{(19,0)} = 55$;

$L_{(n,0)} = 0$,$\forall\, n \geq 20$

We have a function

$f(t) = t^2e^{2t}$

Alphabets in the form of coefficients of series

$$f(t) = \sum_{n=0}^{\infty} L_{(n,0)}\frac{2^nt^{n+2}}{n!}$$

$$L_{(0,0)}\frac{2^0t^2}{0!} + L_{(1,0)}\frac{2^1t^3}{1!} + L_{(2,0)}\frac{2^2t^4}{2!} + L_{(3,0)}\frac{2^3t^5}{3!} + L_{(4,0)}\frac{2^4t^6}{4!} + L_{(5,0)}\frac{2^5t^7}{5!} + L_{(6,0)}\frac{2^6t^8}{6!} + L_{(7,0)}\frac{2^7t^9}{7!} + L_{(8,0)}\frac{2^8t^{10}}{8!} + L_{(9,0)}\frac{2^9t^{11}}{9!} + L_{(10,0)}\frac{2^{10}t^{12}}{10!} + L_{(11,0)}\frac{2^{11}t^{13}}{11!} + L_{(12,0)}\frac{2^{12}t^{14}}{12!} + L_{(13,0)}\frac{2^{13}t^{15}}{13!} + L_{(14,0)}\frac{2^{14}t^{16}}{14!} + L_{(15,0)}\frac{2^{15}t^{17}}{15!} + L_{(16,0)}\frac{2^{16}t^{18}}{16!} + L_{(17,0)}\frac{2^{17}t^{19}}{17!} + L_{(18,0)}\frac{2^{18}t^{20}}{18!} + L_{(19,0)}\frac{2^{19}t^{21}}{19!}$$

Using natural transformation on f(t)

$$N\{f(t)\} = (\frac{2!\,u^2}{s^3})\frac{11}{0!} + (\frac{3!\,u^3}{s^4})\frac{60}{1!} + (\frac{4!\,u^4}{s^5})\frac{104}{2!} + (\frac{5!\,u^5}{s^6})\frac{344}{3!} + (\frac{6!\,u^6}{s^7})\frac{624}{4!} + (\frac{7!\,u^7}{s^8})\frac{1088}{5!} + (\frac{8!\,u^8}{s^9})\frac{2496}{6!} + (\frac{9!\,u^9}{s^{10}})\frac{4096}{7!} + (\frac{10!\,u^{10}}{s^{11}})\frac{13312}{8!} + (\frac{11!\,u^{11}}{s^{12}})\frac{6656}{9!} + (\frac{12!\,u^{12}}{s^{13}})\frac{26624}{10!} + (\frac{13!\,u^{13}}{s^{14}})\frac{96256}{11!} + (\frac{14!\,u^{14}}{s^{15}})\frac{106496}{12!} + (\frac{15!\,u^{15}}{s^{16}})\frac{352256}{13!} + (\frac{16!\,u^{16}}{s^{17}})\frac{851966}{14!} + (\frac{17!\,u^{17}}{s^{18}})\frac{131072}{15!} + (\frac{18!\,u^{18}}{s^{19}})\frac{2555904}{16!} + (\frac{19!\,u^{19}}{s^{20}})\frac{3801088}{17!} + (\frac{20!\,u^{20}}{s^{21}})\frac{11534336}{18!} + (\frac{21!\,u^{21`}}{s^{22}})\frac{28835840}{19!}$$

Here first security process starts if we put u=1, it becomes as:

$$L_{(s,1)} = \left(\frac{22}{s^3}\right) + \left(\frac{360}{s^4}\right) + \left(\frac{1248}{s^5}\right) + \left(\frac{6880}{s^6}\right) + \left(\frac{18720}{s^7}\right) + \left(\frac{45696}{s^8}\right) + \left(\frac{139776}{s^9}\right) + \left(\frac{294912}{s^{10}}\right) + \left(\frac{1198080}{s^{11}}\right) + \left(\frac{732160}{s^{12}}\right) + \left(\frac{3514368}{s^{13}}\right) + (\frac{15015936}{s^{14}}) + (\frac{19382272}{s^{15}}) + (\frac{73973760}{s^{16}}) + (\frac{204472320}{s^{17}}) + (\frac{35651584}{s^{18}}) + (\frac{782106624}{s^{19}}) + (\frac{1299972096}{s^{20}}) + (\frac{4383047680}{s^{21}}) + (\frac{12111052800}{s^{22}})$$

Now adjusting these values, we get

22 360 1248 6880 18720 45696 139776 294912 1198080 732160 3514368 15015936 19382272 73973760 204472320 35651584 782106624 1299972096 4383047680 12111052800

To mod 68, we get the values

22 20 24 12 20 0
36 64 56 4 60 40
28 28 60 0 0 48
36 44

Finally, we get the cipher text by comparing these values with extended table of alphabets

"WUYMUAk6?E2occ2AAwks"

Extended Key Approach: By using (9) we have key

0 5 18 101 275 672 2055 4336 17618 10767 51681 220822 285033 1087849 3006945 524288 11501568 19117236 64456583 178103717

Extended Decryption Approach: If the cipher text in the form of **"WUYMUAk6?E2occ2AAwks"** Then convert it into the simple text like **"Learning Never Ends!"** By using the inverse mathematical natural transformation

We have originated

22 360 1248 6880 18720 45696 139776 294912 1198080 732160 3514368 15015936 19382272 73973760 204472320 35651584 782106624 1299972096 4383047680 12111052800

It can also be written as,

$$R_{(s,u)} = \left(\frac{2!\,u^2}{s^3}\right)\frac{11}{0!} + \left(\frac{3!\,u^3}{s^4}\right)\frac{60}{1!} + \left(\frac{4!\,u^4}{s^5}\right)\frac{104}{2!} + \left(\frac{5!\,u^5}{s^6}\right)\frac{344}{3!} + \left(\frac{6!\,u^6}{s^7}\right)\frac{624}{4!} + \left(\frac{7!\,u^7}{s^8}\right)\frac{1088}{5!} + \left(\frac{8!\,u^8}{s^9}\right)\frac{2496}{6!} + \left(\frac{9!\,u^9}{s^{10}}\right)\frac{4096}{7!} + \left(\frac{10!\,u^{10}}{s^{11}}\right)\frac{13312}{8!} + \left(\frac{11!\,u^{11}}{s^{12}}\right)\frac{6656}{9!} + \left(\frac{12!\,u^{12}}{s^{13}}\right)\frac{26624}{10!} + \left(\frac{13!\,u^{13}}{s^{14}}\right)\frac{96256}{11!} + \left(\frac{14!\,u^{14}}{s^{15}}\right)\frac{106496}{12!} + \left(\frac{15!\,u^{15}}{s^{16}}\right)\frac{352256}{13!} + (\frac{16!\,u^{16}}{s^{17}})\frac{851966}{14!} + (\frac{17!\,u^{17}}{s^{18}})\frac{131072}{15!} + (\frac{18!\,u^{18}}{s^{19}})\frac{2555904}{16!} + (\frac{19!\,u^{19}}{s^{20}})\frac{3801088}{17!} + (\frac{20!\,u^{20}}{s^{21}})\frac{11534336}{18!} + (\frac{21!\,u^{21}}{s^{22}})\frac{28835840}{19!}$$

By applying inverse Natural Transformation, we have the following coefficients

11 30 26 43 39
34 39 32 52 13 26
47 26 43 52 04
39 29 44 55

From the extended table of alphabets via comparing these standards, the resulting plain text is
"Learning Never Ends!"

IV. CONCLUSION

This study provides an interesting conclusion showing the important feature of exponential cryptography algorithm that it is unaffected to sophisticated attacks. This approach remains secure even when exposed to advanced cryptanalysis systems and proved by several analysis. Its stability enhanced from the power of natural transformation principles and the complex nature of exponential functions. It creates strong blocks contrary to any effort by adversaries to breakdown the encryption**.** As Compared to classical ciphers substitutions, the method used in this study does not replace each plaintext with a fixed ciphertext symbol. Plaintext characters are assigned numerical digits first and then represented as a coefficient. The proposed encryption process applies the arithmetical transformation and sectional reduction to individually corresponding coefficient for plaintexts containing n-symbols. The key-generation process transforms the values related to the plaintext. If the mathematical operations consider unit-cost processes, hence the number of actions increases linearly with the length of plaintext resulting in operational complexity. The exponential transformation, however, produce large middle numerical values; hence, the computational cost depends on bit-length.

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