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Experimental Evaluation of Mohr Columb Strength Parameter with Grain Size of Granular Soils

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Abstract- Shear strength is an important mechanical property of the soils. Mohr Columb strength model is widely used to mathematically define the shear strength of the soil, which is also of prime importance for the constitutive modelling of the soil. Shear strength of the granular soils can be determined from the angle of internal friction, which is a model parameter. To determine the angle of internal friction, direct shear tests are performed. Such tests are tedious, laborious and require expertise to conduct them. Thus, it is desirable to develop a quick method for the prediction of the angle of internal friction. One such method of quick prediction is to develop the prediction models, correlating quickly assessable properties with tediously determined parameters. In the present study, the authors established the relationship between the grain size of soil particles and the angle of internal friction. Finally, prediction models are established, using grain percentage of sand and percentage of fines as variables. The validity of developed models is also checked on a separate set of soil samples. The prediction models are found to be valid within the ceratin bounds.

Keywords-Shear strength, Angle of internal friction, Size and shape of grains, Prediction models.

I. Introduction

Shear strength of the soil is the maximum resistance that soil can offer without having a failure. Shear strength of the soil is an important mechanical property of soil since it gives an idea about the stability of the soil in various loading conditions. Mohr Columb strength criteria is an important model to establish the shear strength of the soil. Wherein, shear strength of soil depends upon two parameters, first is the cohesion between the soil grains and second is the interlocking or internal friction of soil grains. Any action that tries to disturb the cohesion and internal locking between soil grains leads to shear failure. The governing equation of shear strength of the soil is established based on the idea that shear stress of soil remains proportional to the normal stress and can be formulated as follow:

$$\tau = c + \sigma \tan \varphi \tag{1}$$

where the τ and σ are the shear and normal stress, respectively, the cohesion (c) and angle of internal friction (φ) are the model parameters. In fine-grained soils, shear strength mostly depends on the cohesion between soil particles. Whereas, in the case of granular soils, shear strength depends on the internal locking of grains. This internal locking of grains can be evaluated from the Mohr Columb model parameter called angle of internal friction. Hence angle of internal friction is an important parameter to determine the strength of granular soils. Shear strength of granular soils can be determined from the angle of internal friction. To determine the angle of internal friction, direct shear test or triaxial test is usually applied on soils. It is very important for civil engineering projects to determine the engineering properties of soils. Shear strength is a very important property needed to be evaluated. For quick and economical determination of shear strength of granular soils, there is a need to develop quick and economical methods to determine the angle of internal friction. One very famous method to predict important parameters is to develop predictive models. Physical properties like grain size distribution, moisture content, Atterberg's limit and density of soils are easier and quicker to evaluate. This study is focused on the development of valid relationships between the angle of internal friction and grain size of soil for locally available soils in Pakistan, for the quick prediction of such parameter.

Few researchers have made attempts to develop the predictive models for the angle of internal friction. For instance, Bello[i] analyzed the shear strength compacted lateritic soil samples. Bareither *et al* [ii] examined the physical and geological factors affecting the angle of internal friction of compacted sands and developed a multivariate regression model that can be used to predict friction angle of compacted sands from comparable geological origins based on effective particle size (D_{10}) maximum dry unit weight (γ_{dmax}) and Krumbein roundness (R_s). One of the most commonly cited correlation, wherein φ was predicted based on USCS classification parameters and dry unit weight was reported in a soil mechanics design manual of Naval Facilities Engineering Command USA. (NAVFAC) [iii]. A similar, correlation was presented by Schmertmann [iv] which estimate φ based on relative density. Moreover, the method proposed by Terzaghi *et al.* [v] used a secant definition for the friction angle that is a function of effective normal stress as well as generalized material parameters and porosity. Audnoye presented predictive models for the angle of internal friction for lateritic soil based on fines content in the soil [vi]. Besides, many researchers have also investigated the effect of soil characteristics such as grain size, soil type, and grain matrix characteristics on the shear strength parameters of the soil [vii-xiv]. These studies obviously quantified various impacts of soil characteristics on shear strength parameters of soil. However, further research reinforcement is required in the context of locally available soils in Pakistan to evaluate direct shear strength parameters of soil.

Majority of Pakistani land has alluvial soil. Upper strata of this land contain debris and fine-grained soil while lower strata from 3ft to 4 ft down sandy soil deposits start [xv-xvii]. Thus, in this context, it is imperative to establish a quicker and easier way to determine the shear strength of sandy soil by quantifying the relationship among grain size and shear strength parameter of such soils.

II. Test and Materials

To develop a valid and reliable predictive model, it is imperative to collect soil samples from different locations and of varying characteristics. The variety in the database helps in developing reliable predictive models. Special care was considered to ensure the variety in a database for the development of a predictive model. For this purpose, a number of soil samples were collected from different parts of Pakistan. Natural soil samples were collected from 3-4 ft below the surface.

Fig. 1. District wise soil sample collection map

Total of 68 numbers of soil samples were tested. The physical properties and location were carefully recorded. Figure 1 presents a map of soil sample collection.

A testing plan was designed to conduct different tests on a similar soil sample to determine various engineering characteristics. For instance, all the basic soil material characteristics like moisture content, grain size distribution, liquid limit, plasticity index were determined through laboratory testing as per ASTM standards. To determine the shear strength parameters of soils, direct shear tests were conducted as per ASTM standard.

Direct shear tests were performed by preparing a soil specimen in the shear box. Then the sample was being sheared under different normal stresses at a constant shearing rate of deformation; preferably, three different normal stresses are selected for this purpose. Measurements were taken of horizontal displacement and shearing force. Data of direct sear tests results is presented in Figure 2. Failure envelop is drawn by evaluating maximum shear stress at different normal stress. The slope of failure envelops determine the angle of internal friction (Fig. 3).

For predictive models, one very important aspect is to check their validity on a separate database other than used in the development of these models. For this purpose, a separate set of soil samples were also tested. The testing plan adopted for these samples was the same as for the development of models.

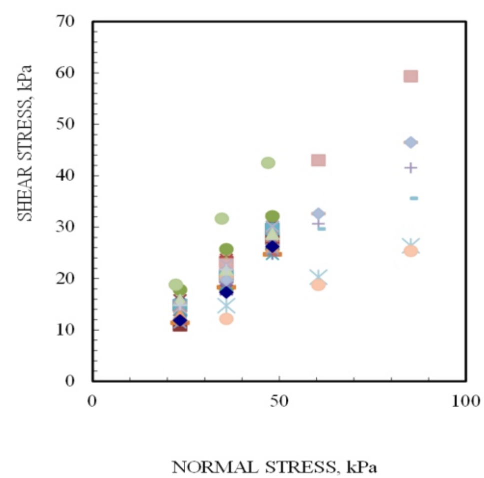

Fig. 2. Direct Shear Test Data

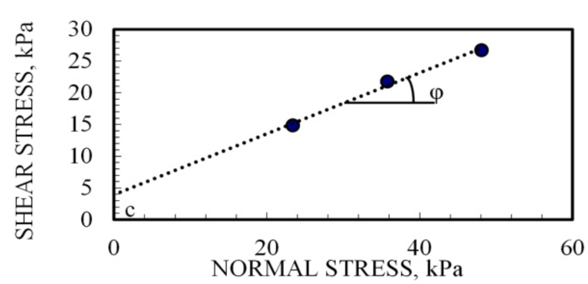

Fig. 3. Direct Shear Parameter Calculation

TABLE I. SUMMARY OF TEST RESULTS

Classification Symbols	**No. of Samples**	**Grain Size Distribution**			**Gradation Coefficients**		**Cohesion, *c***	**The angle of Internal Friction, *φ***
USCS		**Gravel (%)**	**Sand (%)**	**Fines F_{200} (%)**	C_u	C_c	**kPa**	
SW	9	0	95-99	1-5	6.1-7.5	1.1-2.3	0-4.4	31-37
SP	21	0-1	95-98	2-5	0-5.2	0.1-1.7	0-4.1	27-34
SC	4	0	51-76	34-49	2.9-9.01	0.4-1.8	5.2-7.1	24-27.3
SM	10	0-1	53-80	20-47	2.7-11.2	0.4-2.1	0-6.5	22-28.8
SP-SM	19	0	88-94	6-12	2.3-3.4	0.5-1.2	0-3.5	28-32
Other	5	0-62	1-26	15-99	6.1-9.3	1.1-4.1	0.8-9	2-22
Overall Range	68	0-62	1-99	1-99	0-11.2	0.1-4.1	0-9	2-37

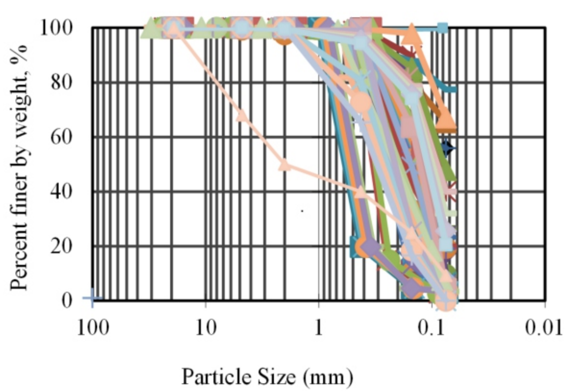

Fig. 4. Particle Size Distribution Curves

III. Test Results and Analysis

Samples were classified as SW, SP, SC, SM and SP-SM as per Unified soil classification system. Soil classification showed that the majority of soil samples were sandy soil. From grain size distribution analysis, it was observed that among 68 soil samples gravel was in the range of 0-62%, Sand 1-99% and Fines 1-99%. To enhance the database of fines, a fine-grained soil sample is also included in this study, that's being a reason for such a high range of fines. Figure 4 presents the particle size distribution curves of all samples. Atterberg's limit test of the soil samples showed that most of the samples were non plastic while the liquid limit was in the range of 0-30%, and plasticity index was from none plastic to 14%. The specific gravity of soil samples was observed to be 2.6-2.76. Since most of the soil samples were classified as non-plastic and sandy soil, the value of cohesion was observed to be very low. For most of the soil samples, c value was nonexistent. However, c value was in the range of 0-9 kPa. A large range of angle of internal friction was observed from the direct shear test on the subjected soil samples. The angle of internal friction was in the range of 2-37. The range of angle of internal friction was in line with the range of angle of internal friction presented in different literature for similar types of soil [xviii-xxvi]. Such a high range of database is very useful for the development of a predictive model. Large range of database can produce reliable predictive models. Summary of test results is presented in Table 1.

IV. Models Development and Discussion

To develop predictive models, it is required to develop a potential relationship between angles of internal friction and quickly characterized the engineering properties of soil. It was observed that for the majority of soil samples value of "c" was zero indicating that soil samples are granular in nature. Grain size and shapes are vital characteristics of granular soil as many engineering properties depend upon them. Keeping this in view, relationships were drawn between the angle of internal friction and sizes of grain particles. Strength of these relationships depends upon scatter along the trend line of relationship. Higher scatter around the trend line indicates poor heath of relationship. The quantitative measurement of such a scatter is given by R^2, coefficient of determination. Higher coefficient of determination means lesser scatter in the database. In statistic, R^2 gives information about the goodness of fit of a model. In regression, the R^2 coefficient of determination is a statistical measure of how well the regression line approximates the real data points [xxvii]. An R^2 of 1 indicates that the regression line perfectly fits the data.

The strength of each relationship is checked by R^2. First attempt was made to develop the relationship between the angle of internal friction and sandy particle sizes. Size of particle ranging from 4.75 mm to 0.075 mm is taken as sand. It was observed that as the percentage of gain size ranging from 4.75-0.075 mm increases, the angle of internal friction also increases. This trend rendering that as the granular particles increases, the angle of internal friction also increases. Granular particles gain a maximum of their shear strength from friction and interlocking between the soil particles within a grain matrix. The R^2 value of 0.713 indicates that relationship drew has good strength; scatter around the predictive curve is at the lower side, as shown in Figure 5.

Similarly, the effect of fines on the angle of internal friction was determined by drawing a relationship between them. Particles having size lesser than 0.075mm are called fines. These particles differ both in size and shape from granular particles. The shape of fine particles is flaky while the shape of granular particles is angular to sub-angular [xxviii-xxix]. The strength of the relationship between fines and angle of internal friction was also determined by the coefficient of determination. Relationship drawn between the angle of internal friction and fine contents shows that with an increase in the percentage of fines angle of internal friction tends to decrease. So it is evident that with an increase in finer particles angle of internal friction decreases.

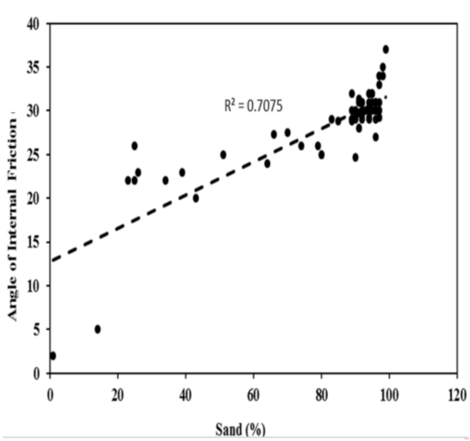

Fig. 5. Relationship between Angle of Internal Friction and Sand (%)

Because interlocking and friction between particles are lesser for fine particles than the granular particles.

The effect of fines and coarser soil grains on the angle of internal friction can be explained by the shape of these particles [xxx]. The shape of sandy grains is angular to sub angular [xvi]. Edges of bigger sand particles may be rounded due to the transportation process. The shear strength in these particles appears due to friction or interlocking between these grains due to irregular shape and interlocking of these particles [xvi, xxx].

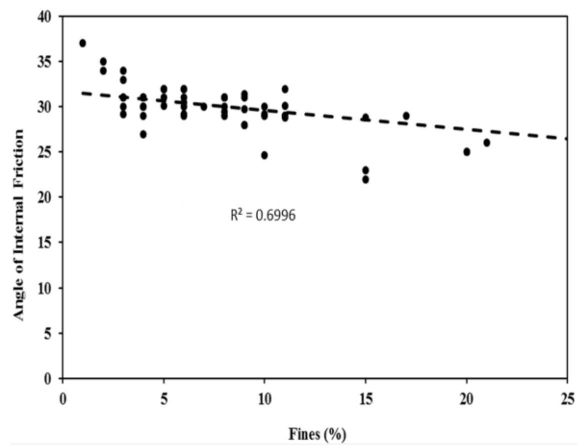

Fig. 6. Relationship between Fines (%) and Angle of internal friction

The angle of internal friction is the quantification of this friction or interlocking between grains [xxx]. Fines consist of silt and clay. Silts are very fine quartz, and some flaky shaped very fine micaceous minerals, while the shape of clay is the flakiest or most needle type. Friction or interlocking between these fine-grained soil fragments is very low to none existent. Shear strength of fine-grained soil is governed by gel formed around these particles called cohesion [xxxi].

Two models were developed one based on the sand percentage in a soil sample and other based on fines content in a soil sample to predict the angle of internal friction.

$$\varphi = 0.19 \text{ Sand } (\%) + 12.75 \quad (2)$$

$$\varphi = -0.20 \text{ Fines } (\%) + 31.75 \quad (3)$$

To develop a universal prediction model, multi linear regression analysis was done. This analysis was carried out using a statistical tool available in SPSS. The basis of multi linear regression analysis is to use more than one predictor or variables.

Following the predictive model was adopted after multi linear regression analysis using both Sand (%) and Fines (%) as a predictor for the angle of internal friction.

$$\varphi = 0.105 \text{ Sand}(\%) - 0.104 \text{ F}_{200} (\%) + 21.42 \quad (4)$$

R	R^2	Standard Error of Estimate
0.86	0.74	2.80

Since the database used in the development of models, contains more concentrated high values of angle of internal friction and a concentrated range of variables, thus high values of constants are obtained in the models. The high value of constants in the equations suggests that the developed models are valid for a certain range of variables. Generally, such kind of empirical correlations could only be valid within the certain bounds [v].

V. The Validity of Developed Models & Predictive Curve

The validity of any model can be checked by comparing the real scale test result with the predicted angle of internal friction. Difference between the test result and the predicted result determine the validity of predictive models. The validity of the predictive model can also be presented on a graph by plotting experimental value on x-axis and predicted model on the y-axis. Equality line shows the same value of prediction and the actual test result. Deviation around equality line shows the error in the prediction of the model. To check the reliability of developed models, separate soil samples were tested. Percentage of sand in these samples was varying from 66-98 %, and fines were varying from 2-34 %. The angle of internal friction was observed to be in the range of 24.7-35. The validity of all three models was checked against this real scale test result data, as shown in figure 7. It was observed that all models predicted the angle of internal friction with very less deviation from the actual test result. Prediction of all models showed high reliability while Eq. 4 has the highest reliability among all. In addition, Model no. 3 is a multi-linear regression model. It has two predictor or variables prediction of the angle of internal friction. A simple predictive curve using model no 3 was also drawn, as shown in figure 8. Practitioners can easily and quickly predict the angle of internal friction from particle size results.

VI. Conclusion and Limitation

The angle of internal friction depends upon size as well as the shape of grains of soil. With the increase in finer fraction of soil grains, the angle of internal friction tends to decrease due to less resistance between particles. The angle of internal friction can be well predicted from the fraction of different soil grains. Three very simple predictive models were presented based on Sand percentage and fines percentage. All models were found to be valid with ±7% error against real scale test result data.

Since the locally available soils from Pakistan are used in this study, thus the predictive models are more valid for Pakistani soils. To obtain global predictive models, further studies are required based on the more enhanced database. Furthermore, these models are only valid for a certain range of the variable. Such predictive models can only be used for preliminary prediction of the angle of internal friction, albeit they cannot be used as a replacement of real scale tests.

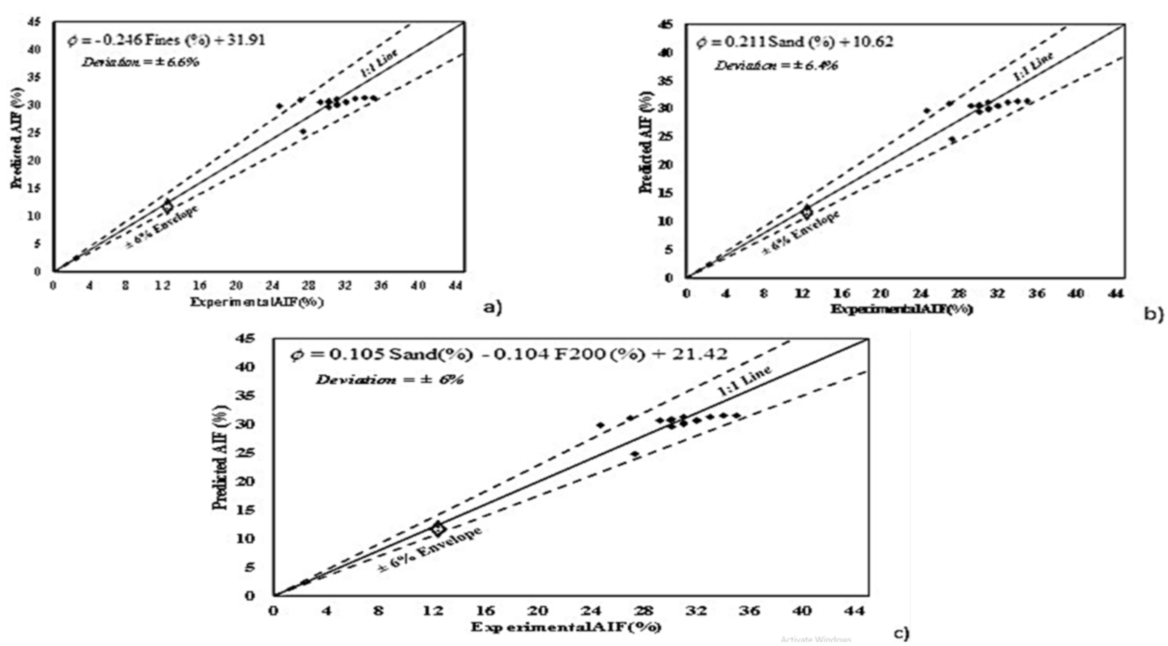

Fig. 7. Validity of developed models (a) Eq. 2 (b) Eq.3 (c) Eq.4

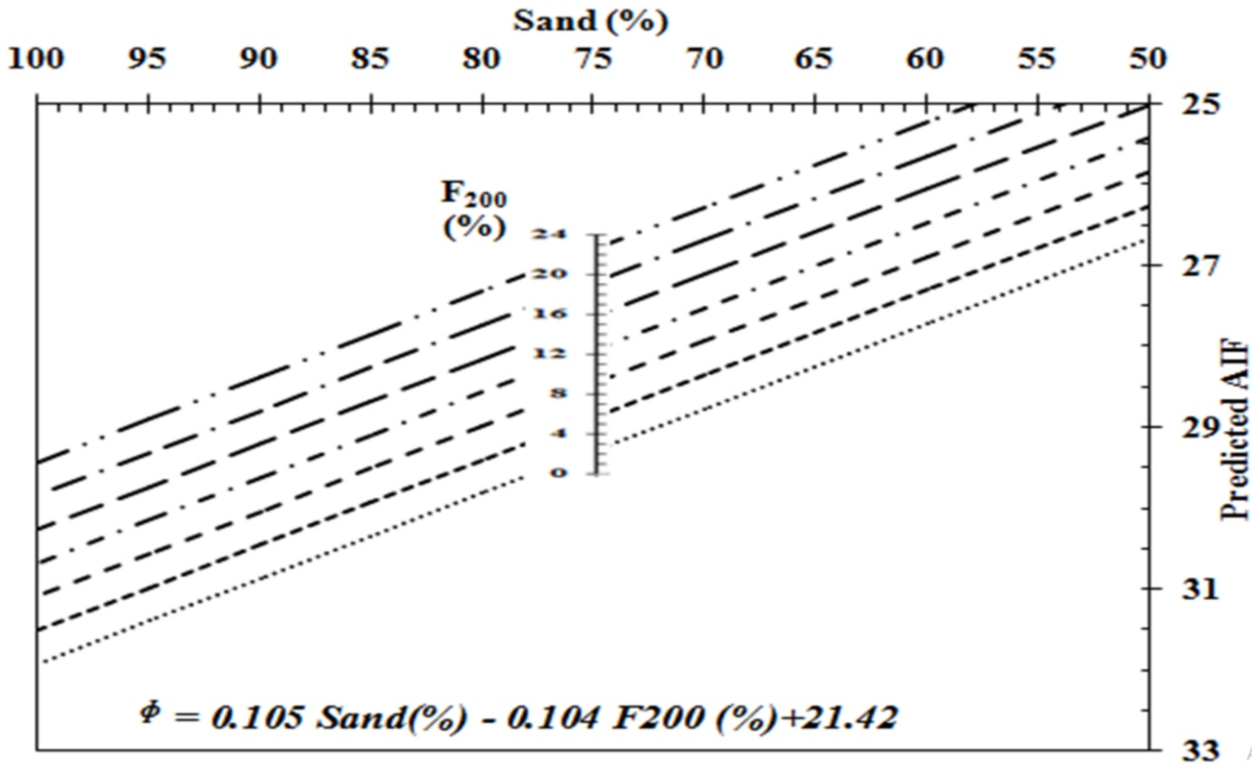

Fig. 8. Prediction curves

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