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Queuing Models in the Context of Biological Paradigm in Hospitals

B. Batool[1], M. Jamal[2], M. Altaf[3]

[1,2]Mathematics & Statistics Department, PMAS-Arid Agriculture University, Rawalpindi.
[3]Basic Sciences Department, University of Engineering and Technology, Taxila.
[3]dm.altaf@uettaxila.edu.pk

Abstract-In this paper, queuing models in hospitals discipline have been analyzed and simulated. Single queue with multiple servers queuing process are observed by using appropriate probability distributions. The arrival rate calculated by Poisson distribution and service rate is calculated by exponential distribution. The service discipline was based on first come first served. M/M/5 and M/M/8 queuing models are used in order to analyze queuing parameters and performance measures of the system. This paper describes the use of queuing systems to decrease the waiting time of patients in hospitals.

Keywords-Queuing Analysis, Queuing Simulation, M/M/5 Queuing Models, M/M/8 Queuing Models, Performance Measures.

I. INTRODUCTION

Queuing theory is a branch of Operations Research which is fundamentally a relationship among the queues and the service system. Queue can be seemed at any time for getting services. Queue is a chain of those people who wait for their turn to get services. The history of queuing theory goes back nearly 100 years ago, it was introduced in the 19th Century and the first problem of queuing theory was raised by telephone exchange. Later, researchers were inspired and started working on queuing theory problems using probabilities theory and developed the advanced and general models of queuing theory which would be more helpful in many complicated situations. A basic queuing system is generally based on three elements:

i. The first element in this system is arrival process of patients.
ii. The second element is waiting process of patients.
iii. The third element is server and departure process.

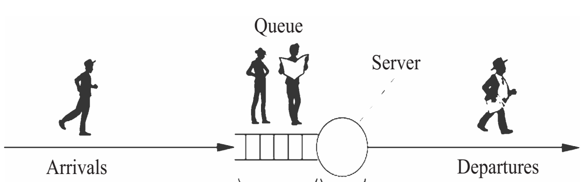

Fig. 1. Basic Queuing System

Queue is a chain of those people who wait for their turn to get services. There are two basic approaches to solve the waiting lines phenomena. Firstly, queuing models are used to examine the queuing system mathematically and are used to measure the performance of the system in which:

I. Average number of patients in the queue.
ii. Average number of servers in the system.
iii. Average waiting time of patients in a queue or system.
iv. Utilization factor of the system.
v. Probability, when queue is full or empty.

Secondly, simulation is the repetition of a real thing completed over time. It gathers the observations to draw interpretation about the real things. When we have multiple queues and infinite population in a queuing system it is difficult to obtain results, then the queuing simulation can be used to get approximations. Due to practical utilization of this queuing theory, simulation modeling of this has been applied on hospital ques.

II. LITERATURE REVIEW

According to literature, in many cases, traditional mathematics are not sufficient to solve the complicated real-life problems. In these circumstances, queuing model is considered most significant implement **to** obtain the better consequences. Therefore, queuing model has attracted the attention of the researchers. Many researchers had done so much work in this field. It has been focused on cost analysis and profit analysis by using the Markovian queuing model with two preference classes and targeted to optimize the total average cost [i]. The simulation of queuing analysis in hospital discipline was studied to focus on the comparison of a single server and multiple servers Queuing models in the field of healthcare department [ii-iii]. Then later main focus of the researchers was to study the queuing theory and waiting lines in Sushi Tei Restaurant [iv]. It has been studied the birth death process with the help of markovian queuing model [v]. The analysis of disappointment arrivals, violation and maintenance customer's violation has been discussed with the help of a single server markovian queuing model [vi]. The use of queuing model to reduce the patients waiting time and got result that the average time of patients was 45 minutes when 9 beds was present there but with the addition of one more bed the waiting time of patients decreased from 45 minutes to 35 minutes [vii]. After this innovations, scientist used queuing model M/M/C and birth death markov process and concluded that waiting time of the customers in a line was less than utilization factor which indicates that customers was no need to wait as they entered in the system was being served quickly. It was also concluded that banking service would be better planned by increasing the servicing facilities. Moreover, these servicing facilities would be more profitable to the bank [viii]. It was studied that students of finance section who had to wait in a queue for long period of time to being served and concluded that by increasing the number of servers, service rate could be improved [ix]. Researchers studied the call center of Nigeria and concluded that by increasing the service facilities, the customers waiting time would be decreased. [x]. Queuing theory has been extensively utilized in industrial settings to analyze how resource-constrained systems respond to various demand levels, and thus is a natural fit for modeling patient flow in a health care setting [xi]. The main emphasis of studies was on the functioning surroundings of a great call center that could be predicted as continuous room with abundant gathering place partitions, in which populace take a seat with headphone in face of computer fatal, provided that tele-services to hidden consumers [xii]. Queues are extremely well-known in our everyday life such as queues are in the education, dramas hall, posting bureau, sickbays, stockpile, accumulate, gasoline drain, bookstores, café, passage James, travel manuscript agency, processor association and telecommunications etc., each and every one face queuing problem [xiii].

Queuing theory is an important application of birth and death processes. In a queuing system, the patients arrive according to a poison process with the parameter for a birth and death process, inter arrival time of poison process is independently and exponentially distributed. Service time of patients is exponentially distributed with parameter inter arrival time of patients is independent and exponentially distributed. In a birth death process, birth rate is λ and death rate is μ.

In this paper, a new innovative and facile queuing models in hospitals discipline has been analyzed and simulated which have not been quoted in the literature earlier. Single queue with multiple servers queuing process are observed by using appropriate probability distributions.

III. Materials and Methods

Data was taken of two working days in a week from two different public hospitals in Rawalpindi. Data was taken from gynae OPD. For queuing model, the following assumptions were satisfied through the data:

i. Arrival time of patients follows a poison process with parameter λ.
ii. Time of Inter arrival of poison process was independently and exponentially distributed.
iii. Service time of the patients was exponentially distributed with parameter μ.
iv. Inter arrival time of patients was independent and it was exponentially distributed.
v. The queue was infinite.
vi. Every patient received desire service.
vii. Service discipline adopted first come, first served.

A. OPD gynae patient's registration department and queuing model

In this department, M/M/s queuing model was observed in which there was single queue and multiple servers. In M/M/s queuing model, the first M/M signify the arrival and departure distribution of the patients which follows to a poison distribution and exponential distribution. The s represents the number of servers.

B. Labor room and queuing model

In this department, M/M/s queuing model was observed in which there is single queue and multiple servers. In M/M/s queuing model, the first M/M represent the arrival and departure distribution of the patients which follows to a poison distribution and s represents the number of servers. M/M/s queuing model was used for queuing analysis.

a) Single queue

It was observed there was a single queue which was endless, there was no restriction unlimited patients could be get services. Service discipline was first come first served for the patients as shown in Fig. 2.

b) Multiple servers

Multiple servers and single queue queuing model is considered as M/M/5 and M/M/8. In which 5 and 8 represent the number of servers as shown in Fig. 2.

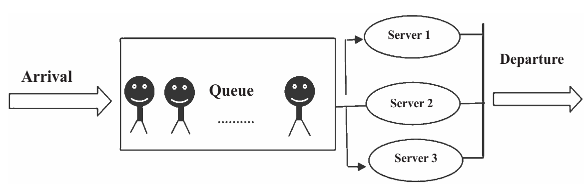

Fig. 2. Single queue, multiple servers

C. Parameters of the queuing models

N = All number of patients.
s = Number of servers.
λ = Entrance rate of the patients per hour.
μ = Service rate of the patients per hour.
$s\mu$ = Service rate when $s > 1$.
ρ = Overall system utilization factor. $\rho = \frac{\lambda}{\mu}$

P_0 = Steady state probability of all idle doctors.
L_q = Average number of patients in the queue.
L_s = Average number of patients in the system.
W_s = Patient's mean awaiting time in the system per hour.
W_q = Patient's mean awaiting time in a queue per hour.

D. Performance Measures

Average number of patients in the line is measured as:

$$L_q = \left[\frac{\left(\frac{\lambda}{\mu}\right)^s \mu\lambda}{(s-1)!(\mu s-\lambda)^2} \right] P_0$$

Patient's mean waiting time in the queue is measured as:

$$W_q = \frac{L_q}{\lambda}$$

Average numbers of patients waiting in the system is measured as:

$$L_s = L_q + \frac{\lambda}{\mu}$$

Patient's mean waiting time in the system is measured as:

$$W_s = \frac{L_s}{\lambda}$$

IV. Results and Discussion

We have used WinQsb software to compute the performance measures and to estimate the parameters of the models using queuing analysis and queuing simulation of single queue and multiple servers queuing models at public hospitals. Obtaining results are compared. By using MATLAB programming for hospital 1 & hospital 2, concluded that by increasing the number of servers the patient's mean waiting time could be reduced.

TABLE I
PARAMETERS AND PERFORMANCE MEASURES OF THE PATIENTS USING M/M/5 QUEUING ANALYSIS IN THE HOSPITAL 1 FOR (DAY 1 AND DAY 2)

Parameters and performance measures	Symbols	Day 1	Day 2
		M/M/5	M/M/5
Arrival rate of patients	λ	34 / hr.	49 / hr.
Service rate of patients	μ	08 / hr.	10 / hr.
Service rate for 5 servers	$s\mu$	40 / hr.	50 / hr.
Utilization of overall organization	ρ	85%	98.00%
Probability that all doctors are idle	P_0	0.84%	0.08%
Average number of patients in the system	L_s	7.95	51.4656
Average number of patients in the queue	L_q	3.70	46.56
Average time patients spend in the system	W_s	0.2341	1.0503
Average time patients spend in the queue	W_q	0.1091	0.9503

TABLE II
PARAMETERS AND PERFORMANCE MEASURES OF THE PATIENTS USING M/M/5 QUEUING SIMULATION IN THE HOSPITAL 1 FOR (DAY 1 AND DAY 2)

Days \ Parameters		ρ	L_s	L_q	W_s	W_q
Day 1	$\lambda = 34$, $\mu = 8$	87.43 %	7.04	2.67	0.201 /hr.	0.076 /hr.
Day 2	$\lambda = 49$, $\mu = 10$	98.95 %	38.86	33.9	0.775 /hr.	0.677 /hr.

Results shown in Table I are obtained by assuming the system in steady state to estimate the parameters performance of M/M/5 queuing model of two working days in a week for hospital 1.

In day 1, the number of patients arrived in a hospital are 34 per hour from which 8 patients availed the services. The average numbers of patients in queue served by 5 doctors are L_q = 3.70 whereas average numbers of patients in the system are L_s = 7.95 per hour. Hence the average numbers of patients spent W_q = 0.1091 per hour in the queue waiting to be attended by the doctors and average numbers of patients in the system before joining the queue are W_s = 0.2341 per hour waiting in the queue to be served and spent time after being served before departure. In day 2, the number of patients arrived in a hospital are

TABLE III
PARAMETERS AND PERFORMANCE MEASURES OF THE PATIENTS USING M/M/8 QUEUING ANALYSIS IN THE HOSPITAL 2 FOR (DAY 1 AND DAY 2)

Parameters and performance measures	Symbols	Day 1	Day 2
		M/M/8	M/M/8
Arrival rate of patients	λ	104 / hr.	71 / hr.
Service rate of patients	μ	14 / hr.	14 / hr.
Service rate for 8 servers	$s\mu$	112 / hr.	112 / hr.
Utilization of overall organization	ρ	92.85%	63.39%
Probability that all doctors are idle	P_0	0.02%	0.60%

Average number of patients in the system	L_s	17.58	5.37
Average number of patients in the queue	L_q	10.15	0.30
Average time patients spend in the system	W_s	0.1691	0.0758
Average time patients spend in the queue	W_q	0.0977	0.0043

49 per hour from which 10 patients availed the services. The average numbers of patients in queue served by 5 doctors are L_q = 46.56 whereas average numbers of patients in the system are L_s= 51.47 per hour. Hence the average numbers of patients spent W_q= 0.9503 per hour in the queue waiting to be attended by the doctors and average numbers of patients in the system before joining the queue are W_s = 1.0503 per hour waiting in the queue to be served and spent time after being served before departure.

Results shown in Table II are obtained by queuing simulation performed on EXCEL by estimating the parameters of queuing model. Simulation runs for 1000 times per hour for hospital 1. In day 1, the average numbers of patients in queue served by 5 doctors are L_q = 2.67 whereas average numbers of patients in the system are L_s = 7.04 per hour. Hence the average numbers of patients spent W_q = 0.076 per hour in the queue waiting to be attended by the doctors and average numbers of patients in the system before joining the queue are W_s= 0.201 per hour waiting in the queue to be served and spent time after being served before departure. In day 2, the average numbers of patients in queue served by 5 doctors are L_q = 33.91 whereas average numbers of patients in the system are L_s= 38.86 per hour. Hence the average numbers of patients spent W_q= 0.677 per hour in the queue waiting to be attended by the doctors and average numbers of patients in the system before joining the queue are W_s= 0.775 per hour waiting in the queue to be served and spent time after being served before departure.

So it can be observed that the results obtained from the queuing simulations are better than the queuing model.

TABLE IV
PARAMETERS AND PERFORMANCE MEASURES OF THE PATIENTS USING M/M/8 QUEUING SIMULATION IN THE HOSPITAL 2 FOR (DAY 1 AND DAY 2)

Days \ Parameters		ρ	L_s	L_q	W_s	W_q
Day 1	λ = 104 μ = 14	96%	14.26	2.67	0.135 /hr.	0.062 /hr.
Day 2	λ = 71 μ = 14	67.14%	5.81	0.44	0.781 /hr.	0.005 /hr.

Results shown in Table III are obtained by assuming the system in steady state to estimate the parameters performance of M/M/8 queuing model of five working days in a week for hospital 2. In day 1, the number of patients arrived in a hospital are 104 per hour from which 14 patients availed the services. The average numbers of patients in queue served by 8 doctors are L_q = 10.15 whereas average numbers of patients in the system are L_s= 17.58 per hour. Hence the average numbers of patients spent W_q= 0.0977 per hour in the queue waiting to be attended by the doctors and average numbers of patients in the system before joining the queue are W_s= 1691 per hour waiting in the queue to be served and spent time after being served before departure. In day 2, the number of patients arrived in a hospital are 71 per hour from which 14 patients availed the services. The average numbers of patients in queue served by 8 doctors are L_q = 5.37 whereas average numbers of patients in the system are L_s = 0.30 per hour. Hence the average numbers of patients spent W_q = 0.0043 per hour in the queue waiting to be attended by the doctors and average numbers of patients in the system before joining the queue are W_s= 0.0758 per hour waiting in the queue to be served and spent time after being served before departure.

Results shown in Table 4 are obtained by queuing simulation performed on EXCEL by estimating the parameters of queuing model. Simulation runs for 1000 times per hour for hospital 2. In day 1, the number of patients arrived in a hospital are 104 per hour from which 14 patients availed the services. The average numbers of patients in queue served by 8 doctors are L_q = 2.67 whereas average numbers of patients in the system are L_s = 14.26 per hour. Hence the average numbers of patients spent W_q = 0.062 per hour in the queue waiting to be attended by the doctors and average numbers of patients in the system before joining the queue are W_s= 0.135 per hour waiting in the queue to be served and spent time after being served before departure. In day 2, the average numbers of patients in queue served by 8 doctors are L_q = 0.44 whereas average numbers of patients in the system are L_s= 5.81 per hour. Hence the average number of patients spent W_q= 0.005 per hour in the queue waiting to be attended by the doctors and average numbers of patients in the system before joining the queue are W_s= 0.781 per hour waiting in the queue to be served and spent time after being served before departure.

So it can be observed that the results obtained from the queuing simulations are better than the queuing model.

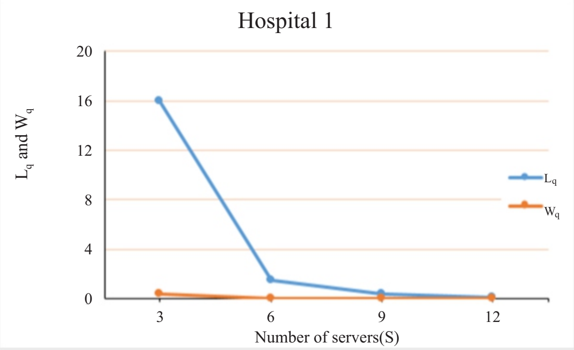

Fig. 3. Average waiting time of patients in a queue and mean queue length against number of servers

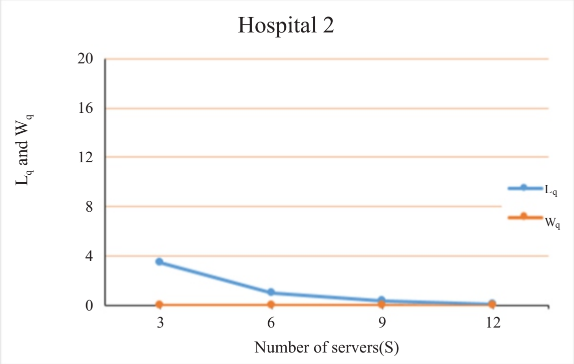

Fig. 4. Average waiting time of patients in a queue and mean queue length against number of servers

V. CONCLUSIONS

Queues are big problem in hospitals. Patients have to wait for long time under some undesirable conditions. In this study, queuing model was applied in two different public hospitals and the parameters of the queuing M/M/5 and M/M/8 are estimated and the performance of the system also checked by two different queuing analysis and queuing simulation approaches and obtaining results are compared. It is concluded that the patients mean waiting time in hospital 2 as compared to hospital 1 is less because the number of servers in hospital 2 are more as compared to hospital 1 and we also conclude that by using queuing simulation MATLAB programing, we can decrease the patients mean waiting time in hospital 1 by increasing the number of servers.

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