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On Double Framed Soft Rings

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Abstract- In this article the notion of double framed soft ring is familiarized and basic properties of double framed soft rings are discussed. Further behavior of different operations of double framed soft sets is discussed under the environment of rings. Wherever necessary the concepts are elaborated by examples.

Keywords- Soft sets, Soft rings, Double framed soft sets, Double framed rings.

I. INTRODUCTION

In our daily life we face many problems that are so complicated and involve uncertainties that we cannot clearly define a way to overcome these obstacles. Many fields like engineering, computer science, medical science, environmental science and social science are some of the areas of life in which we usually come across such issues. Such type of problems create confusion in our mind and we cannot say anything about their solutions conformably. Day by day these problems have become more complicated. To overcome such type of situations and difficulties Zadeh presented the concept of fuzzy set [1], Pawlak familiarized the notion of rough sets [2] and Molodtsov initiated the notion of soft set (SS) [3]. All these theories create a bit satisfaction to handle such uncertain problems but the theory of soft sets has its own advantages. In his pioneer paper Molodtsov not only introduced the notion of soft sets but also proved its effectiveness by applying this notion to game theory, operations research, measure theory and in Riemann integration. Later on it was Maji et al. [4] who worked on soft sets and gave different operations to soft sets. But Ali et al. [5] pointed out some shortcomings in the operations defined by Maji et al. and improved these operations. Aktas and Cagman [6] presented the notion of soft groups. Shabir and Naz [7] initiated the study of soft topological spaces. Sezgin et al. [8] functioned on soft near rings and Acar et al. [9] functioned on soft rings. Xin and Li [10] defined soft congruence relations over soft rings. The perception of similarities in soft sets is familiarized by Min [11]. Keeping in view the importance of algebraic structures and for more applications of soft sets one can see [12-20].

Keeping in view the advantages and applications of intuitionistic fuzzy sets by Atanassov [21], Jun and Ahn presented the notion of double framed soft set (DFSS). Hadipour [22] used DFSSs in BF-algebras. Khalaf et al. [23] introduced the notion of DFS LA-semigroups. Khan et al. [24] discussed near rings in the view of double framed soft fuzzy sets.

In this paper the notion of double framed soft rings (DFSR) is introduced, basic properties of DFSR are discussed and effect of DFSR is elaborated over the operations of DFSSs.

II. PRILIMANIRIES

In this segment we will discuss the elementary perceptions of rings, SSs and DFSSs. The terms and notions defined in this section are taken from [3, 4, 5, 9, 22, 25, 26].

2.1 Definition:
A set $R \neq \emptyset$ together with binary operations "+ "and " $\cdot$ " defined on R is called a ring if
(i) $(R, +)$ is an abelian group,
(ii) $(R, .)$ is semigroup,
(iii) Distributive laws hold.

2.2 Definition:
A subset $S \neq \emptyset$ of a ring R is called subring of R if S is itself ring under induced binary operations of R.

2.3 Remark:
From now onward in our study R will denote a ring and "SbR(R)" will denote the collection of all subrings of R, if stated otherwise.

2.4 Definition:
Let us denote an initial universe by U, set of parameters by E and let $A \subseteq E$. Then by a SS over U is a pair (F, A), where $F: A \to P(U)$ is a mapping.

2.5 Definition:

A double framed pair $< (\alpha, \beta): E >$ over a universe set U is said to be DFSS where $\alpha: E \to P(U)$ and $\beta: E \to P(U)$ are set valued mappings.

Let θ and φ be two subsets of U. Then the θ –inclusive set and φ-exclusive set of a double framed soft set $< (\alpha, \beta): E >$ is represented and defined as
$i_E(\alpha, \theta) = \{t \in E: \theta \subseteq \alpha(t)\}$ and
$e_E(\beta, \varphi) = \{t \in E: \varphi \supseteq \beta(t)\}$ respectively.

The set $<(\alpha,\beta):E>_{(\theta,\varphi)}=\{t\in E:\theta\subseteq\alpha(t),\varphi\supseteq\beta(t)\}$ is said to be double framed including set of the DFSS $<(\alpha,\beta):E>$. clearly $<(\alpha,\beta):E>_{(\theta,\varphi)}=i_E(\alpha,\theta)\cap e_E(\beta,\varphi)$.

2.6 Example:

Let $U=\{1,2,3,\ldots.30\}$ be the initial universe, $E=\{x_1,x_2,x_3,x_4\}$ be the set of parameters, $\theta=\{8,16,24\}$, $\varphi=\{1,2,3,\ldots,15,18,21,24,27,30\}$ and $<(\alpha,\beta):E>$ be the DFSS over U, where $\alpha:E\to P(U)$ mapping defined by

$$\alpha=\begin{cases}\{1,3,5,\ldots.29\} & if\ x=x_1,\\ \{2,4,6,\ldots\ldots.30\} & if\ x=x_2,\\ \{5,10,15,\ldots,30\} & if\ x=x_3,\\ \{4,8,12,\ldots\ldots28\} & if\ x=x_4,\end{cases}$$

and

$\beta:E\to P(U)$ defined by

$$\beta=\begin{cases}\{1,2,3,\ldots\ldots15\} & if\ x=x_1,\\ \{3,6,9,\ldots\ldots.30\} & if\ x=x_2,\\ \{2,3,5,7,\ldots\ldots29\} & if\ x=x_3,\\ \{6,12,18,24,30\} & if\ x=x_4,\end{cases}$$

Then $i_E(\alpha,\theta)=\{x_2,x_4\}$, $e_E(\beta,\varphi)=\{x_1,x_2,x_4\}$ and $<(\alpha,\beta):E>_{(\theta,\varphi)}=\{x_2,x_4\}$. Now obviously $<(\alpha,\beta):E>_{(\theta,\varphi)}=i_E(\alpha,\theta)\cap e_E(\beta,\varphi)$.

2.7 Definition:

The support of DFSS $<(f,g):A>$ is represented and defined as $Supp<(f,g):A>=\{m\in A, f(m)\neq\emptyset\neq g(m)\}$.

2.8 Definition:

Let $<(f,g):A>$ and $<(h,i):B>$ be two DFSSs over the universal set U. Then we say that $<(f,g):A>$ is a DFS subset of $<(h,i):B>$, if

(i) $A\subseteq B$

(ii) $f(m)\subseteq h(m)$ and $g(m)\supseteq i(m)$, $\forall m\in Supp<(f,g):A>$.

2.9 Definition:

Let $<(f,g):A>$ and $<(h,i):B>$ be two DFSSs over the same universal set U. Then the "AND" product of DFSSs is represented and defined as $<(f,g):A>\wedge<(h,i):B>=<(\lambda,\mu):A\times B>$, where $\lambda(m,n)=f(m)\cap h(n)$ and $\mu(m,n)=g(m)\cup i(n)$, $\forall(m,n)\in A\times B$.

2.10 Example:

Consider U= $\{\bar0,\bar1,\bar2,\bar3\ldots.\overline{29}\}$ with defined addition "$+_{30}$" and multiplication "$\times_{30}$" modulo 30 respectively. Let $A=\{1,2\}$, $B=\{3,4\}$ and $f,g:A\to P(U)$ be defined by

$f(1)=\{\bar0,\bar2,\bar4,\bar6,\bar8,\overline{10},\overline{12},\overline{14},\overline{16},\overline{18},\overline{20},\overline{22},\overline{24},\overline{26},\overline{28}\}$,

$f(2)=\{\bar0,\bar3,\bar6,\bar9,\overline{12},\overline{15},\overline{18},\overline{21},\overline{24},\overline{27}\}$
$g(1)=\{\bar0\}$ and $g(2)=\{\bar0,\bar1,\bar2,\bar3\ldots.\overline{29}\}$.
Now let $h,i:B\to P(R)$ be defined by
$h(3)=\{\bar0\}$, $h(4)=\{\bar0,\bar5,\overline{10},\overline{15},\overline{20},\overline{25}\}$, $i(3)=\{\bar0,\bar1,\bar2,\bar3\ldots.\overline{29}\}$ and $i(4)=\{\bar0,\bar3,\bar6,\bar9,\overline{12},\overline{15},\overline{18},\overline{21},\overline{24},\overline{27}\}$.
Then the "AND" product $<(f,g):A>\wedge<(h,i):B>=<(\lambda,\mu):A\times B>$, where $\lambda(1,3)=\{\bar0\}$, $\lambda(1,4)=\{\bar0,\overline{10},\overline{20}\}$, $\lambda(2,3)=\{\bar0\}$, $\lambda(2,4)=\{\bar0,\overline{15}\}$, $\mu(1,3)=\{\bar0,\bar1,\bar2,\bar3\ldots.\overline{29}\}$, $\mu(1,4)=\{\bar0,\bar3,\bar6,\bar9,\overline{12},\overline{15},\overline{18},\overline{21},\overline{24},\overline{27}\}$, $\mu(2,3)=\{\bar0,\bar1,\bar2,\bar3\ldots.\overline{29}\}$ and $\mu(2,4)=\{\bar0,\bar1,\bar2,\bar3\ldots.\overline{29}\}$.

2.11 Definition:

Let $<(f,g):A>$ and $<(h,i):B>$ be two DFSSs over the same universal set U. Then the "OR" product of DFSSs is represented and defined as $<(f,g):A>\vee<(h,i):B>=<(\lambda,\mu):A\times B>$, where $\lambda(m,n)=f(m)\cup h(n)$ and $\mu(m,n)=g(m)\cap i(n)$ $for\ all\ (m,n)\in A\times B$.

2.12 Example:

Consider U= $\{\bar0,\bar1,\bar2,\bar3\ldots.\overline{29}\}$ with defined addition "$+_{30}$" and multiplication "$\times_{30}$" modulo 30 respectively. Let $A=\{1,2\}$ and $B=\{3,4\}$ and $f,g:A\to P(U)$ by $f(1)=\{\bar0\}$, $f(2)=\{\bar0,\bar1,\bar2,\bar3\ldots.\overline{29}\}$, $g(1)=\{\bar0,\bar3,\bar6,\bar9,\overline{12},\overline{15},\overline{18},\overline{21},\overline{24},\overline{27}\}$ and $g(2)=\{\bar0,\bar1,\bar2,\bar3\ldots.\overline{29}\}$ and $h,i:B\to P(U)$ by $h(3)=\{\bar0,\bar2,\bar4,\bar6,\bar8,\overline{10},\overline{12},\overline{14},\overline{16},\overline{18},\overline{20},\overline{22},\overline{24},\overline{26},\overline{28}\}$, $h(4)=\{\bar0\}$, $i(3)=\{\bar0,\bar5,\overline{10},\overline{15},\overline{20},\overline{25}\}$ and $i(4)=\{\bar0,\bar2,\bar4,\bar6,\bar8,\overline{10},\overline{12},\overline{14},\overline{16},\overline{18},\overline{20},\overline{22},\overline{24},\overline{26},\overline{28}\}$ then the "OR" product of $<(f,g):A>\vee<(h,i):B>=<(\lambda,\mu):C>$ is $\lambda(1,3)=\{\bar0,\bar2,\bar4,\bar6,\bar8,\overline{10},\overline{12},\overline{14},\overline{16},\overline{18},\overline{20},\overline{22},\overline{24},\overline{26},\overline{28}\}$, $\lambda(1,4)=\{\bar0\}$, $\lambda(2,3)=\{\bar0,\bar1,\bar2,\bar3\ldots.\overline{29}\}$, $\lambda(2,4)=\{\bar0,\bar1,\bar2,\bar3\ldots.\overline{29}\}$, $\mu(1,3)=\{\bar0,\overline{15}\}$, $\mu(1,4)=\{\bar0,\bar6,\overline{12},\overline{18},\overline{24}\}$, $\mu(2,3)=\{\bar0,\bar5,\overline{10},\overline{15},\overline{20},\overline{25}\}$ and $\mu(2,4)=\{\bar0,\bar2,\bar4,\bar6,\bar8,\overline{10},\overline{12},\overline{14},\overline{16},\overline{18},\overline{20},\overline{22},\overline{24},\overline{26},\overline{28}\}$.

2.13 Definition:

Let $<(f,g):A>$ and $<(h,i):B>$ be two DFSSs over the same universal set U. Then the union (also called extended union) of DFSSs is denoted and defined as

$< (f,g):A > \cup < (h,i):B > < (f \cup h, g \cap i):A \cup B >$,

where $f \cup h : A \cup B \mapsto P(U)$ is defined by

$$(f \cup h)(m) = \begin{cases} f(m) & if\ m \in A \setminus B \\ h(m) & if\ m \in B \setminus A \\ f(m) \cup h(m) & if\ m \in A \cap B \end{cases}$$

and $g \cap i : A \cup B \mapsto P(R)$ is defined by

$$(g \cap i)(m) = \begin{cases} g(m) & if\ m \in A \setminus B \\ i(m) & if\ m \in B \setminus A \\ g(m) \cap i(m) & if\ m \in A \cap B \end{cases}.$$

2.14 Definition:

The restricted union of two DFSSs $< (f,g):P >$ and $< (h,i):Q >$ over the same universal set U, is represented and defined as

$< (f,g):P > \cup_{\mathcal{r}} < (h,i):Q > = < (\zeta,\eta):Y >$, where

$\zeta(m) = f(m) \cup h(m)$ &

$\eta(m) = g(m) \cap i(m), \forall m \in Y = P \cap Q \neq \emptyset$.

2.15 Definition:

The restricted intersection of two DFSSs $< (f,g):P >$ and $< (h,i):Q >$ over the same universal set U, is represented and defined as

$< (f,g):P > \cap_{\mathcal{r}} < (h,i):Q > = < (\psi,\omega):X >$, Where

$\psi(m) = f(m) \cap h(m)$ &

$\omega(m) = g(m) \cup i(m), \forall m \in X = P \cap Q \neq \emptyset$.

2.16 Definition:

Let $< (f,g):A >$ and $< (h,i):B >$ be two DFSSs over a same universal set U. Then the extended intersection of DFSSs is defined as $< (f \cap_{\varepsilon} h, g \cup_{\varepsilon} i):A \cup B >$, where $f \cap_{\varepsilon} h : A \cup B \mapsto P(U)$ is defined as

$$(f \cap_{\varepsilon} h)(m) = \begin{cases} f(m) & if\ m \in A \setminus B \\ h(m) & if\ m \in B \setminus A \\ f(m) \cap h(m) & if\ m \in A \cap B \end{cases}$$

And $g \cup_{\varepsilon} i : A \cup B \mapsto P(U)$ is defined by

$$(g \cup_{\varepsilon} i)(m) = \begin{cases} g(m) & if\ m \in A \setminus B \\ i(m) & if\ m \in B \setminus A \\ g(m) \cup i(m) & if\ m \in A \cap B \end{cases}$$

and is denoted as

$< (f,g):A > \cap_{\varepsilon} < (h,i):B > = < (f \cap_{\varepsilon} h, g \cup_{\varepsilon} i):A \cup B >$

III. Double Framed Soft Rings

From now onward R will denote a ring and A, B ... be any subsets of a set of parameters.

3.1 Definition:

A DFSS $< (f,g):A >$ over a ring R is said to be DFSR over R, iff $\forall m \in A$, $f(m)$, $g(m) \in$ SbR(R).

3.2 Example:

Let $(G,+)$ be an abelian group and R be the set of all endomorphisms of $(G,+)$. Then for $\alpha, \beta \in R$, if we define

$(\alpha + \beta)(x) = \alpha(x) + \beta(x)$

$(\alpha.\beta)(x) = \alpha(\beta(x))$.

Then under these defined " + " and " . " R is a ring and also $\{O\}$, center of R (denoted by $C(R)$) and R itself are subrings of R.

Now let $A = \{x,y,z\}$ be a set of parameters and define $f,g: A \to P(R)$ by $f(x) = \{O\}$

$f(y) = C(R), f(z) = R$,

$g(x) = C(R), g(y) = \{O\}\ g(z) = R$,

Then $< (f,g):A >$ is a double framed soft ring over R.

3.3 Remark:

In general "AND" product of two DFSRs is not necessarily a DFSR.

3.4 Example:

Consider the ring $Z_{12} = \{\bar{0},\bar{1},\bar{2},\bar{3},\bar{4},\bar{5},\bar{6},\bar{7},\bar{8},\bar{9},\overline{10},\overline{11}\}$.

Let $A = \{0,2,4\}, B = \{0,5\}$ and $< (f,g):A >$, $< (h,i):B >$ be the DFSRs over Z_{12}. Define the set valued mappings $f,g: A \longrightarrow P(Z_{12})$ by let

$f(0) = \{\bar{0},\bar{1},\bar{2},\bar{3},\bar{4},\bar{5},\bar{6},\bar{7},\bar{8},\bar{9},\overline{10},\overline{11}\}$,

$f(2) = \{\bar{0},\bar{2},\bar{4},\bar{6},\bar{8},\overline{10}\}, f(4) = \{\bar{0},\bar{4},\bar{8}\}, g(0) = \{\bar{0}\}$,

$g(2) = \{\bar{0},\bar{6}\}, g(4) = \{\bar{0},\bar{3},\bar{6},\bar{9}\}$, and $h,i: B \longrightarrow P(Z_{12})$ by $h(0) = \{\bar{0},\bar{6}\}, h(5) = \{\bar{0},\bar{2},\bar{4},\bar{6},\bar{8},\overline{10}\}$, and $i(0) = \{\bar{0},\bar{3},\bar{6},\bar{9}\}, i(5) = \{\bar{0},\bar{4},\bar{8}\}$. Then clearly $< (f,g):A > \wedge < (h,i):B >$ is not a DFSR because $g(4) \cup i(5) = \{\bar{0},\bar{3},\bar{6},\bar{9}\} \cup \{\bar{0},\bar{4},\bar{8}\} = \{\bar{0},\bar{3},\bar{4},\bar{6},\bar{8},\bar{9}\}$, which is not subring of R.

3.5 Theorem:

Let $< (f,g):A >$ and $< (h,i):B >$ be two DFSRs over the same ring R. Then $< (f,g):A > \wedge < (h,i):B >$ is a DFSR, if $g(m) \cup i(n) \in$ SbR(R) $\forall (m,n) \in A \times B$.

Proof.

By using Definition 2.9. "AND" product of DFSSs $< (f,g):A > \wedge < (h,i):B > = < (\lambda,\mu)$:C>, where $\lambda(m,n) = f(m) \cap h(n)$ and $\mu(m,n) = g(m) \cup i(n),\ \forall (m,n) \in A \times B$. Now as $< (\lambda,\mu):A \times B > \neq \emptyset$ is a DFSS over R.

Then $\forall (m,n) \in Supp < (\lambda,\mu)$:

$A \times B > \neq \emptyset, \lambda(m,n) = f(m) \cap h(n) \neq \emptyset$ and $\mu(m,n) = g(m) \cup i(n) \neq \emptyset$.

As $f(m), h(n), g(m)$ and $i(n) \in$ SbR(R). As Intersection of subrings is a subrings so $\lambda(m,n) \in$ SbR(R) and $\mu(m,n) \in$ SbR(R) as given that $g(m) \cup i(n) \in$ SbR(R), if so, $< (\lambda,\mu):A \times B >$ is a double framed soft ring over R, $\forall (m,n) \in Supp < (\lambda,\mu):A \times B >$.

$\Rightarrow <(f,g):A> \wedge <(h,i):B> = <(\lambda,\mu):A\times B>$ is a DFSR over R.

3.6 Remark:

In general "OR" product of two DFSR is not necessarily a DFSR.

3.7 Example:

Consider the ring $Z_{15} = \{\bar{0},\bar{1},\bar{2} \ldots \overline{14}\}$. Let $A = \{1,2,3,4\}$, $B = \{5,6,7\}$, $<(f,g):A>$ and $<(h,i):B>$ be the DFSRs over Z_{15} with the set valued mappings $f,g:A \longrightarrow P(Z_6)$ defined by,
$f(1) = \{\bar{0}\} = f(3)$,
$f(2) = \{\bar{0},\bar{1},\bar{2} \ldots \overline{14}\}$, $f(4) = \{\bar{0},\bar{3},\bar{6},\bar{9},\overline{12}\}$,
$g(1) = \{\bar{0}\}$, $g(2) = \{\bar{0},\bar{3},\bar{6},\bar{9},\overline{12}\}$, $g(3) = \{\bar{0},\bar{5},\overline{10}\}$,
$g(4) = \{\bar{0},\bar{1},\bar{2} \ldots \overline{14}\}$ and $h,i:B \longrightarrow P(Z_{15})$ defined by
$h(5) = \{\bar{0},\bar{1},\bar{2} \ldots \overline{14}\}$, $h(6) = \{\bar{0},\bar{3},\bar{6},\bar{9},\overline{12}\}$,
$h(7) = \{\bar{0},\bar{5},\overline{10}\}$, and $i(5) = \{\bar{0},\bar{1},\bar{2} \ldots \overline{14}\}$,
$i(6) = \{\bar{0},\bar{5},\overline{10}\}$, $i(7) = \{\bar{0},\bar{3},\bar{6},\bar{9},\overline{12}\}$. Then clearly $<(f,g):A> \wedge <(h,i):B>$ is not a DFSR because
$f(4) \cup h(7) = \{\bar{0},\bar{3},\bar{6},\bar{9},\overline{12}\} \cup \{\bar{0},\bar{5},\overline{10}\} = \{\bar{0},\bar{3},\bar{5},\bar{6},\bar{9},\overline{10},\overline{12}\}$, which is not subring.

3.8 Theorem:

Let $<(f,g):A>$ and $<(h,i):B>$ be two DFSRs over the same ring R. Then $<(f,g):A> \vee <(h,i):B>$ is a DFSR, if $f(m) \cup h(n) \in$ SbR(R), $\forall (m,n) \in A \times B$.

Proof.

By using Definition 2.11 "OR" product of DFSSs $<(f,g):A> \vee <(h,i):B> = <(\lambda,\mu):A\times B>$, where $\lambda(m,n) = f(m) \cup h(n)$ and
$\mu(m,n) = g(m) \cap i(n)$, $\forall (m,n) \in A \times B$. Now as $<(\lambda,\mu):A\times B> \neq \emptyset$ is a DFSR over R.
Then $\forall (m,n) \in Supp <(\lambda,\mu):A\times B> \neq \emptyset, \lambda(m,n) = f(m) \cup h(n) \neq \emptyset$ and $\mu(m,n) = g(m) \cap i(n) \neq \emptyset$.
As $f(m)$, $h(n)$, $g(m)$ and $i(n) \in$ SbR(R). As intersection of subrings is a subrings so $\mu(m,n) \in$ SbR(R) and $\lambda(m,n) \in$ SbR(R) because given that $g(m) \cup i(n) \in$ SbR(R) so, $<(\lambda,\mu):A\times B>$ is a DFSR over R, $\forall$
$(m,n) \in Supp <(\lambda,\mu):A\times B>. \Rightarrow <(f,g):A> \vee <(h,i):B> = <(\lambda,\mu):A\times B>$ is a DFSR over R.

3.9 Remark:

Usually the union of two DFSRs over a ring R is not necessarily a DFSR over R. We explain it by the given example.

3.10 Example:

Consider the ring $R = \{\bar{0},\bar{1},\bar{2},\bar{3},\bar{4},\bar{5}\}$

+	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$
$\bar{0}$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$
$\bar{1}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	$\bar{0}$
$\bar{2}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	$\bar{0}$	$\bar{1}$
$\bar{3}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	$\bar{0}$	$\bar{1}$	$\bar{2}$
$\bar{4}$	$\bar{4}$	$\bar{5}$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$
$\bar{5}$	$\bar{5}$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$

×	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$
$\bar{0}$	$\bar{0}$	$\bar{0}$	$\bar{0}$	$\bar{0}$	$\bar{0}$	$\bar{0}$
$\bar{1}$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$
$\bar{2}$	$\bar{0}$	$\bar{2}$	$\bar{4}$	$\bar{0}$	$\bar{2}$	$\bar{4}$
$\bar{3}$	$\bar{0}$	$\bar{3}$	$\bar{0}$	$\bar{3}$	$\bar{0}$	$\bar{3}$
$\bar{4}$	$\bar{0}$	$\bar{4}$	$\bar{2}$	$\bar{0}$	$\bar{4}$	$\bar{2}$
$\bar{5}$	$\bar{0}$	$\bar{5}$	$\bar{4}$	$\bar{3}$	$\bar{2}$	$\bar{1}$

and let $A = B = \{x,y,z\} \subseteq E$. Now define the mappings $f,g:A \rightarrow P(R)$ by $f(x) = \{\bar{0}\}$, $f(y) = \{\bar{0},\bar{3}\}$,
$f(z) = \{\bar{0},\bar{2},,\bar{4}\}$, $g(x) = \{\bar{0},\bar{3}\}$, $g(y) = \{\bar{0},\bar{2},4\}$,
$g(z) = \{\bar{0},\bar{1},\bar{2},\bar{3},\bar{4},\bar{5}\}$ and $h,i:B \rightarrow P(R)$ by
$h(x) = \{\bar{0},\bar{3}\}$, $h(y) = \{\bar{0},\bar{2},4\}$, $h(z) = \{\bar{0}\}$,
$i(x) = \{\bar{0},\bar{2},4\}$, $i(y) = \{\bar{0},\bar{2},4\}$, $i(z) = \{\bar{0},\bar{1},\bar{2},\bar{3},\bar{4},\bar{5}\}$.
Then Obviously $<(f,g):A>$ and $<(h,i):B>$ are DFSRs over R. Now by the Definition 2.13 DFSRs $<(f,g):A>$ and $<(h,i):B>$, we have
$<(f,g):A> \cup <(h,i):A> = <(f \cup h, g \cap i):A \cup B> = <(j,k):A \cup B>$
where
$j(x) = f(x) \cup h(x) = \{\bar{0}\} \cup \{\bar{0},\bar{2}\} = \{\bar{0},\bar{2}\}$,
$j(y) = f(y) \cup h(y) = \{\bar{0},\bar{3}\} \cup \{\bar{0},\bar{2},4\} = \{\bar{0},\bar{2},\bar{3},\bar{4}\}$
$j(z) = f(z) \cup h(z) = \{\bar{0},\bar{2},4\} \cup \{\bar{0}\} = \{\bar{0},\bar{2},4\}$
$k(x) = g(x) \cap i(x) = \{\bar{0},\bar{3}\} \cap \{\bar{0},\bar{2},4\} = \{\bar{0}\}$
$k(y) = g(y) \cap i(y) = \{\bar{0},\bar{2},4\} \cap \{\bar{0},\bar{2},4\} = \{\bar{0},\bar{2},4\}$
$k(z) = g(z) \cap i(z) = R \cap R = R$
Here $j(y) = \{\bar{0},\bar{2},\bar{3},\bar{4}\}$ is not a subring. So we conclude that union of two DFSRs is not a double framed soft ring.

3.11 Theorem:

The union of two DFSRs $<(f,g):A>$ and $<(h,i):B>$ over the same ring R is DFSR over R. If $f(m)$ is a subring of $h(m)$ or $h(m)$ is a subring of $f(m)$ $, \forall m \in A \cap B$.

Proof.

Suppose $f(m)$ is a subring of $h(m)$ or $h(m)$ is a subring of $f(m)$, then by either case $f(m) \cup h(m)$ is a subring of R. Next consider $(g \cap i)(m)$. If $m \in A \setminus B$, then $(g \cap i)(m) = g(m) \in$ SbR(R) and if $m \in B \setminus A$, then $(g \cap i)(m) = i(m) \in$ SbR(R). Furthermore if $m \in A \cap B$, then $(g \cap i)(m) = g(m) \cap i(m) \in$ SbR(R) because intersection of any number of subrings of ring R is again subring of the ring R. Hence in either case $(g \cap i)(m) \in$ SbR(R). Hence $<(f,g):A> \cup <(h,i):B>$ is DFSR over R.

3.12 Remark:

In general restricted union of two DFSRs is not necessarily a DFSR over a ring R.

3.13 Example.

Consider the ring $Z_{12} = \{\bar{0},\bar{1},\bar{2} \ldots \overline{11}\}$ and let $A = \{0,2,4,6,8\}$, B= $\{1,2,3,4,5\}$, $<(f,g):A>$ and $<(h,i):B>$ be the DFSRs over Z_{12} with set valued mappings $f,g:A \longrightarrow P(Z_{12})$ defined by, let $f(0) = \{\bar{0}\}$,
$f(2) = \{\bar{0},\bar{2},\bar{4},\bar{6},\bar{8},\overline{10}\}$, $f(4) = \{\bar{0},\bar{3},\bar{6},\bar{9}\}$, ,
$f(6) = \{\bar{0},\bar{3},\bar{6},\bar{9}\}$, $f(8) = \{\bar{0},\bar{1},\bar{2} \ldots \overline{11}\}$,
$g(0) = g(4) = g(8) = \{\bar{0}\}$,
$g(2) = g(6) = \{\bar{0},\bar{1},\bar{2} \ldots \overline{11}\}$ and $h,i:B \longrightarrow P(Z_6)$ defined by $h(1) = \{\bar{0}\}$, $h(2) = \{\bar{0},\bar{3},\bar{6},\bar{9}\}$,
$h(3) = \{\bar{0},\bar{1},\bar{2} \ldots \overline{11}\}$, $h(4) = \{\bar{0},\bar{2},\bar{4},\bar{6},\bar{8},\overline{10}\}$,
$h(5) = \{\bar{0},\bar{1},\bar{2} \ldots \overline{11}\}$ and
$i(1) = i(3) = i(5) = \{\bar{0},\bar{1},\bar{2} \ldots \overline{11}\}$, $i(2) = i(4) = \{\bar{0}\}$.
Then $<(f,g):A> \cup_r <(h,i):B>$ is not a DFSR because

$f(4) \cup h(4) = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}\} \cup \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}, \overline{10}\} = \{\bar{0}, \bar{2}, \bar{3}, \bar{4}, \bar{6}, \bar{8}, \bar{9}\}$, which is not subring of Z_{12}.

3.14 Theorem:

Let $< (f,g): A >$ and $< (h,i): B >$ be two DFSRs over the same ring R, Then the restricted union of $< (f,g): A >$ and $< (h,i): B >$ is a DFSR over R, if $f(m) \cup h(m) \in \mathrm{SbR(R)}, \forall\, m \in A \cap B$.

Proof.

By using Definition 2.14,
$< (f,g): A > \cup_r < (h,i): A > = < (\lambda,\mu): A \cap B >$; where $\lambda(m) = f(m) \cup h(m)$ and $\mu(m) = g(m) \cap i(m)\ \forall\, m \in A \cap B$.

Clearly $< (\lambda,\mu): A \cap B >$ is a DFSS over R. If $m \in Supp < (\lambda,\mu): A \cap B >$ then $\lambda(m) = f(m) \cup h(m)$ is non empty and $\mu(m) = g(m) \cap i(m)$ is also non empty. As $f(m), h(m), g(m)$ and $i(m) \in \mathrm{SbR(R)}$ with $f(m) \cup h(m) \in \mathrm{SbR(R)}$ and intersection of any number of subrings of ring R is a subring of the ring R. So $\forall\, m \in Supp < (\lambda,\mu): A \cap B >$, $\lambda(m) = f(m) \cup h(m) \in \mathrm{SbR(R)}$, and $\mu(m) = g(m) \cap i(m) \in \mathrm{SbR(R)}$. Hence $< (f,g): A > \cup_r < (h,i): B > = < (\lambda,\mu): A \cap B >$ is a DFSRs over R.

3.15 Remark:

In general restricted intersection of two DFSRs is not necessarily a DFSR over a ring R.

3.16 Example.

Consider the ring $Z_{18} = \{\bar{0}, \bar{1}, \bar{2} \dots \overline{17}\}$, with $A = \{1,2,3,4\}$, B= $\{4,5,6,7\}$ and $< (f,g): A >$ and $< (h,i): B >$ be the DFSRs over Z_{18} with defined set valued mappings $f, g: A \longrightarrow P(Z_{18})$ by, let
$f(1) = \{\bar{0}, \bar{1}, \bar{2} \dots \overline{17}\}$, $f(2) = \{\bar{0}\}$,
$f(3) = \{\bar{0}, \bar{1}, \bar{2} \dots \overline{17}\}$, $f(4) = \{\bar{0}\}$,
$g(1) = \{\bar{0}\}$, $g(2) = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \overline{10}, \overline{12}, \overline{14}, \overline{16}\}$, $g(3) = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\}$,
$g(4) = \{\bar{0}, \bar{6}, \overline{12}\}$ and $h, i: B \longrightarrow P(Z_6)$ defined by
$h(4) = h(5) = \{\bar{0}, \bar{1}, \bar{2} \dots \overline{17}\}$,
$h(6) = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \overline{10}, \overline{12}, \overline{14}, \overline{16}\}$,
$h(7) = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\}$ and $i(4) = \{\bar{0}, \bar{9}\}$,
$i(5) = \{\bar{0}, \bar{6}, \overline{12}\}$,
$i(6) = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}, \overline{12}, \overline{15}\}$, $i(7) = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \overline{10}, \overline{12}, \overline{14}, \overline{16}\}$. Then
$< (f,g): A > \cap_r < (h,i): B >$ is not a DFSR because $g(4) \cup i(4) = \{\bar{0}, \bar{6}, \overline{12}\} \cup \{\bar{0}, \bar{9}\} = \{\bar{0}, \bar{6}, \bar{9}, \overline{12}\}$, which is not subring of R.

3.17 Theorem:

Let $< (f,g): A >$ and $< (h,i): B >$ be two DFSRs over the same ring R. Then the restricted intersection $< (f,g): A > \cap_r < (h,i): A >$ is a DFSR over R, if $g(m) \cup i(m) \in \mathrm{SbR(R)}, \forall\, m \in A \cap B$.

Proof.

By using Definition 2.15,
$< (f,g): A > \cap_r < (h,i): B > = < (\lambda,\mu): A \cap B >$; $\lambda(m) = f(m) \cap h(m)$ and $\mu(m) = g(m) \cup i(m)\ \forall\, m \in A \cap B$.

Clearly $< (\lambda,\mu): A \cap B >$ is a DFSS over R. If $m \in Supp < (\lambda,\mu): C >$ then $\lambda(m) = f(m) \cap h(m)$ is non empty and $\mu(m) = g(m) \cup i(m)$ is also non empty. As $f(m), h(m), g(m)$ and $i(m) \in \mathrm{SbR(R)}$, with intersection of subrings of subring of ring R is a subring of the ring R and $g(m) \cup i(m) \in \mathrm{SbR(R)}$. So $\lambda(m) = f(m) \cap h(m) \in \mathrm{SbR(R)}$ and $\mu(m) = g(m) \cup i(m) \in \mathrm{SbR(R)}, \forall\, m \in Supp < (\lambda,\mu): A \cap B$.

Hence $< (f,g): A > \cap_r < (h,i): B >$ is a DFSR over R.

3.18 Remark:

In general extended intersection of two DFSRs is not necessarily a DFSR over a ring R.

3.19 Example:

Consider the ring $Z_6 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$ and let $A = \{1,2,3\}$, $B = \{2,3,4\}$ and $< (f,g): A >$ and $< (h,i): B >$ be two DFSRs over Z_6, with set valued mappings $f, g: A \longrightarrow P(Z_6)$ defined by $f(1) = \{\bar{0}\}$,
$f(2) = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$, $f(3) = \{\bar{0}\}$,
$g(1) = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$, $g(2) = \{\bar{0}, \bar{2}, \bar{4}\}$,
$g(3) = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$, and $h, i: B \longrightarrow P(Z_6)$ defined by
$h(2) = \{\bar{0}\}$, $h(3) = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$,
$h(4) = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$, and $i(2) = \overline{\{0, 3\}}$,
$i(3) = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$, $i(4) = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$. Then
$< (f,g): A > \cap_\varepsilon < (h,i): A \cup B >$ is not DFSR because $g(2) \cup i(2) = \{\bar{0}, \bar{2}, \bar{4}\} \cup \{\bar{0}, \bar{3}\} = \{\bar{0}, \bar{2}, \bar{3}, \bar{4}\}$, which is not subring of R.

3.20 Theorem:

The extended intersection of DFSRs $< (f,g): A >$ and $< (h,i): B >$ over the same ring R is DFSR over R, if $g(m)$ is a subring of $i(m)$ or $i(m)$ is a subring of $g(m), \forall m \in A \cap B$.

Proof.

Suppose $g(m)$ is a subring of $i(m)$ or $i(m)$ is a subring of $g(m)\ \ \forall m \in A \cap B$, then by either case $g(m) \cup i(m) \in \mathrm{SbR(R)}$. Now consider $(f \cap h)(m)$. If $m \in A \setminus B$, then $(f \cap h)(m) = f(m)$ and if $m \in B \setminus A$, then $(f \cap h)(m) = h(m)$. In either case $(f \cap h)(m) \in \mathrm{SbR(R)}$. Now if $m \in A \cap B$ then $(f \cap h)(m) = f(m) \cap h(m)$, then $(f \cap h)(m) \in \mathrm{SbR(R)}$, intersection of subrings of ring R is subring of the ring R. Next consider $(g \cup i)(m)$. If $m \in A \setminus B$, then $(g \cup i)(m) = g(m)$ and if $m \in B \setminus A$, then $(g \cup i)(m) = i(m) \in \mathrm{SbR(R)}$. Furthermore if $m \in A \cap B$, then $(g \cup i)(m) \in \mathrm{SbR(R)}$ because $(g \cup i)(m) = g(m) \cup i(m)$ and $g(m) \cup i(m) \in \mathrm{SbR(R)}$. Hence $< (f,g): A > \cap_\varepsilon < (h,i): B >$ is DFSR over R.

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